Liquidity Management in Banking: the Role of Leverage*

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October 9, 2017

Abstract

This paper examines potential impacts of banks' leverage on their incentives to manage their liquidity. We analyse a model where banks control their liquidity risk by managing their liquid asset positions. In the basic framework, a model with a single bank, where the possibility of selling long-term assets when in need of liquidity is not taken into account, we find that the bank chooses to prudently manage its liquidity risk only when its leverage is low. In a model with multiple banks and a secondary market for long-term assets, we find that a banking system where banks are highly leveraged can be prone to liquidity crises. Our model predicts a typical pattern of liquidity crises that is consistent with what was observed during the 2007-2009 crisis.

JEL Classification: D82, G21.

Keywords: Leverage, Liquidity Risk, Moral Harzard, Cash-In-The-Market Pricing.

*The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England or its committees. We are grateful to Urs Birchler, Gianni De Nicolo, Hans Gersbach, Michel Habib, Bruno Parigi, Sebastian Pfeil for helpful comments and suggestions. We owe special thanks to Jean-Charles Rochet for thoroughly reading our manuscript at several stages, as well as for his valuable comments. A significant part of this research was done when the second author was affiliated with the University of Zurich and received the funding from the ERC (grant agreement 249415-RMAC), from NCCR FinRisk (project Banking and Regulation) and from Swiss Finance Institute (project "Systemic Risk and Dynamic Contract Theory".

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1 Introduction

Liquidity risk management failures at banks played a key role in the global financial crisis of 2007-2009. In response the Basel Committee has proposed two new regulatory liquidity standards to complement its revised capital requirement framework for promoting banking sector stability. This raises a new question on the interaction between the two main regulatory tools: are liquidity requirements and capital requirements substitutes or complements?

In this paper, we highlight a channel through which the two requirements can be seen as substitute. The key channel is the impact of banks' leverage on their incentives to manage their liquidity risk appropriately.

The Basel Committee's main objective with the Liquidity Coverage Ratio (LCR) is to ensure that banks have sufficient high quality liquid assets (HQLA) to survive a liquidity shock. LCR can be seen as a direct approach to regulate banks' liquidity risk profile. A less direct alternative to this approach could be to incentivise banks to hold a suitable amount of liquid asset on their own accord. The question is: how?

This paper suggests that capital requirements can be an effective tool to achieve this. The role of capital requirements as an incentive device is well known and is one of the main rationales to justify its existence. Policy makers and academics have historically focused on the effects of capital requirements on banks' choice of credit risk. In this paper, we show that this logic can be extended to liquidity risk and that a suitably designed capital requirement can induce banks to prudently manage their liquidity risk.

We develop a framework where banks control their liquidity risk by managing their liquid asset positions. The context we have in mind is one of banks that are financed by debt and equity. Due to investors' demand of liquid investment, banks can issue only short-term debt and thus expose themselves to a liquidity risk that stems from the maturity mismatch between asset payoffs and desired redemptions. To be insured against this risk, banks need to carry some liquid assets on their balance sheet. We study the bank's optimal liquidity holdings and how they are affected by the bank's leverage.

We assume specifically that banks have choice between two types of assets. The first one is a kind of liquid reserves that have a net return of zero. The second asset is a constant return to scale investment project (long-term asset) that produces a random cash flow only after two periods. Although the latter is more profitable than the former, its capacity to generate liquidity in some future states of nature may be restricted. Inspired of the recent crisis, we model the liquidity shock by the arrival, at an intermediate date, of some new information about the quality of the project. When good news are revealed, the liquidity raised by pledging the project's cash flows is sufficient to cover the banks' refinance demand. However, if bad news are disclosed, the project has limited pledgeability, which may lead to the banks' closure if they do not hold liquid reserves ex-ante.

In practice when in need of liquidity, banks typically have three options: either they use their ex-ante liquidity holdings or they borrow against the future cash flows generated by their long-term assets or they sell these assets in the secondary market. We first consider, in the basic model, the case of an individual bank that could raise liquidity using just the two first options. Our focus is then on the bank's precautionary motive for liquidity holdings, i.e. holding liquidity to be insured against liquidity risk. Our main finding is that the bank hold adequate liquidity to protect itself against this risk if and only if its leverage ratio is low. The intuition lies in the fact that when leverage is high, the bank's exposure to liquidity risk is large. Buying insurance is then too costly, which induces the bank to forgo the insurance option and gamble. In our simple setup, there exists a threshold of leverage below which the bank will choose to manage its liquidity risk prudently, which implies that a properly designed capital requirement is sufficient to induce a better liquidity risk management. We are not claiming that this is a general result. All we are claiming is that a restriction on banks' leverage can have positive impact on their incentives to manage their liquidity. Hence, liquidity requirement and capital requirement need to be jointly designed in an optimal way to avoid overregulation.

Next, we extend our basic setting into a multi-banks contexts that allow to take into consideration the possibility of asset sales. Precisely, we analyze a three-bank setting in which banks that are in need of liquidity can sell their long-term asset in the secondary market. We assume that because of asset specificity, the only potential purchasers of one bank's asset are other banks. Hence, the market price depends on the overall amount of liquidity available in the system for asset purchases.

Allowing for asset sales has two interesting implications. First, given that the market price of long-term asset depends on the aggregate liquidity of the banking system, the distribution of leverage in the system should be matter for banks' liquidity profile. Moreover, it is also interesting to see how the impact of banks' leverage on the banks' choice of liquidity holdings has consequences on deleveraging and fire sales in the banking sector during liquidity crises. The second implication lies in the fact that there exists an additional reason for banks to hold liquidity beside the precautionary motive. The idea is that banks that survive the liquidity shock have opportunity to buy assets put for sale by banks that have liquidity demand. If such assets are sold at price below their fundamental value, banks that do have enough liquidity stand to make windfall profits from purchasing assets.

By characterizing the rational expectation equilibria, we derive a set of results that shed light on these issues. We find that funding liquidity and market liquidity of long-term asset are positively related. We also find that a banking system that consists of highly leveraged banks can be prone to liquidity crises. Our model predicts a typical pattern of the crises: High leverage results in low ex-ante liquidity holdings of banks. Then, when a liquidity shock is realized, many banks have trouble in honoring their debt obligations and thus have to sell their investment at fire-sale prices, which causes the failure of banks that are in need of liquidity. This pattern is consistent with what was observed during the 2007-2009 crisis.

The organization of the paper is as follows. After discussing the related literature in the next section, we describe, in Section 3, the basic model. In Section 4 we analyse the banks' optimal liquidity holdings and the effect of bank's leverage on this choice. Section 5 consider multiple bank setting and the consequences of permitting asset sales. Finally we conclude in Section 6.

2 Related Literature

To the best of our knowledge, the present paper is the first one that addresses the impact of banks' leverage on the banks' incentives to manage their liquidity risk. Still the insights on which our model builds are related to various literatures.

The idea that the liability structure of a bank may have effect on its asset composition is linked to the large literature that evaluates the foundation for the imposition of capital regulation. See, among others, Rochet (1992), Besanko and Kanatas (1996), Blum (1999), Repullo (2004)[1]. This literature studies how the incentives of banks to take excessive risk can be curbed by requiring banks to maintain an adequate capital ratio. While the focus of this literature is the impact on the banks' incentives to take credit risk, our paper aims to examine the effect on their incentives to manage their liquidity risk.

In our paper, the reason for banks to hold liquidity is based on two assumptions: (i) ex-ante uncertainty about the liquidity needs; (ii) limited pledgeability due to asymmetric information. Those two assumptions are similar to those used by Hölmstrom and Tirole (1998) to analyse the liquidity demand of corporate sector and the role of government in supplying liquidity. The main difference lies in the fact that in Hölmstrom and Tirole (1998), liquidity shocks arise as production shocks to the firms' technologies. The size of the shocks is exogenous and especially independent of the firms' balance sheet characteristic. We rather derive liquidity needs as being determined in equilibrium by asset-liability mismatch. Such difference explains why in Hölmstrom and Tirole (1998), the firms' liquidity demand does not depend on their liability structure whereas in our framework it does. We believe that liquidity shocks arising from technology shocks as in Hölmstrom and Tirole (1998) are suitable for non-financial enterprise while our formulation is more reasonable in the context of financial institutions.

The present paper is also related to several contributions that use the "cash-in-the-market-pricing" mechanism proposed by Allen and Gale (1994, 2004, 2005) to understand the financial fragility. Bolton et al. (2011) construct a framework to analyse the optimal composition of inside liquidity (i.e. the cash reserves held by financial intermediaries themselves to meet their liquidity demand) and outside liquidity (i.e. the liquidity holdings of other investors with a longer horizon). They examine the asset allocation between cash and long-term investment of two types of agents, short and long-run investors. Short-run investors (SRs) can be hit by a liquidity shock that takes the form of a late maturity of their investment. The key novelty of their analysis is the focus on the timing of liquidity trades. They assume that SRs had the choice of either immediately responding to the liquidity shock by selling their assets at fire-sale prices, or taking a chance that the shock might be short-lived at the risk of having to raise liquidity at a later date under much worse condition. They then analyse how the expectation about the timing of liquidity trades affects the investment decision of SRs. Our paper instead points to the effect of the banks' leverage on their choice of investment.

A more closely related to our work is the paper of Acharya and Viswanathan (2011b) that builds a model to understand the de-leveraging of the financial sector during crises. They examine how adverse shocks that materialize in good economic times, represented by high expectations about economic fundamentals, lead to greater de-leveraging and asset price deterioration. In their framework, banks issue short-term debts to finance

[1] For an excellent review of this literature, see Freixas and Rochet (2008), VanHoose (2007).

their investment in long-term risky assets. In response to a liquidity shock, banks raise new debt and, if necessary, sell their assets. Good expectations about the quality of those assets enable low-capital banks to be funded ex-ante and the resulting distribution of leverage in the economy can potentially lead to more serious fire-sale problems when adverse shocks arise in good times. Our multiple banks setup with asset trading is in fact inspired of Acharya and Viswanathan (2011b)'s setting. The main difference is that we allow banks to hold liquidity to self-insure against liquidity shock, which enables us to shed light on how banks' incentives to manage liquidity risk are affected by their liability structure.

Several papers study the banks' choice of investment between liquid and illiquid assets. They differ in the determinants they focus on. Acharya et al. (2011a) examine the effect of policy interventions to resolve bank failure on ex-ante bank liquidity. Malherbe (2014) provides a model in which the fear of future market illiquidty due to adverse selection may trigger hoarding behavior today. Heider et al. (2015) analyse banks' liquidity holdings to shed light on how banks' private information about the risk of their assets affects the trading and pricing of liquidity in the interbank market. Acharya et al. (2015) studies how the liquidity choices of firms are shaped by the risk-sharing opportunities in the economy. Our paper considers the effect of banks' liability structure on their liquidity choices.

Finally, some papers address the optimal design of bank liquidity requirements. Calomiris et al. (2014) develop a theory of liquidity requirements which focuses on the role of cash in incentivizing banks to properly manage their default risk. They argue that because cash is riskless asset and cash holdings are observable, banks can commit to exert effort on risk management by holding a sufficient amount of cash. Hence, in Calomiris et al. (2014), a liquidity requirement that takes the form of a narrow cash reserve requirement can be used to provide banks incentives to reduce credit risk. The present paper considers the traditional role of cash holdings in limiting the liquidity risk and examine whether the banks' incentives to manage this risk are affected by their capitalisation.

Walther (2015) constructs a model to analyse how financial regulation in the form of restriction on maturity mismatch can be used to avoid socially wasteful fire sales. It is found that there exists situations where fire sales arise in the decentralized competitive equilibrium. In that case, imposing a linear constraint on banks is sufficient to restore efficiency. Such constraint can be implemented by regulatory tools such as Net Stable Funding Ratio or Liquidity Coverage Ratio. In Walther (2005), banks are ex-ante identical and banks' short-term debt takes the form of collateralized debt with exogenous hair-cut. In contrast, in our model banks are ex-ante heterogenous and the interest rate on the short-term debt is derived in equilibrium depending on the banks' choices of asset composition. Walther (2005) does not examine how the banks' decisions on maturity mismatch are influenced by their capital ratio as we do in the present paper.

Finally, Santos and Suarez (2016) present a model where the rationale for bank liquidity standards is an improvement of the efficiency of the decision of the lender of last resort. They consider a dynamic model in which receiving liquidity support from the lender of last resort may help banks to cope with investor runs. In their setting, holding liquidity is costly because it forces banks to forgo valuable investment opportunities, but it can be efficient. The reason is that, when a run happens, liquidity holding increases the time available before the lender of last resort must decide on supporting the bank, which facilitates the arrival of information on the bank's financial condition and improves the efficiency of the decision taken by the lender of last resort.

3 The basic model

In this section, we describe the problem of an individual bank that seeks to manage its liquidity risk. We consider an economy that lasts for three dates $t = 0, 1, 2$. There is a bank with balance sheet of size normalized to 1. We assume that the bank is funded at date 0 by equity (of amount E) and short-term debt (of amount $1 - E$). The face value of short-term debt repaid at date 1 is denoted by D.

The bank has access to two investment opportunities. The first one is a storage technology, referred to as cash, that has a net return of zero. The second investment opportunity is a constant return to scale project, referred to as long-term asset, that requires a start-up investment at date $t = 0$ and generates an uncertain cash flow at date $t = 2$. Figure 1 summarizes the payoff structure of the project. Precisely, if the bank invests 1 at date 0 in this project, it receives $\tilde{y} > 0$ at date 2 with probability θ, and zero with the complement probability. $\tilde{y}$ is not known at date 0 but will be revealed at date 1. At date 0, we only know that there are two possible states at date 1: *high or low state.* In the high state, which happens with probability α, $\tilde{y}$ is equal to y_H whereas in the low state, $\tilde{y}$ takes a lower value y_L. We assume that the realization of $\tilde{y}$ is observable but not verifiable. Therefore, the short-term debt repayment cannot be contingent on such information.

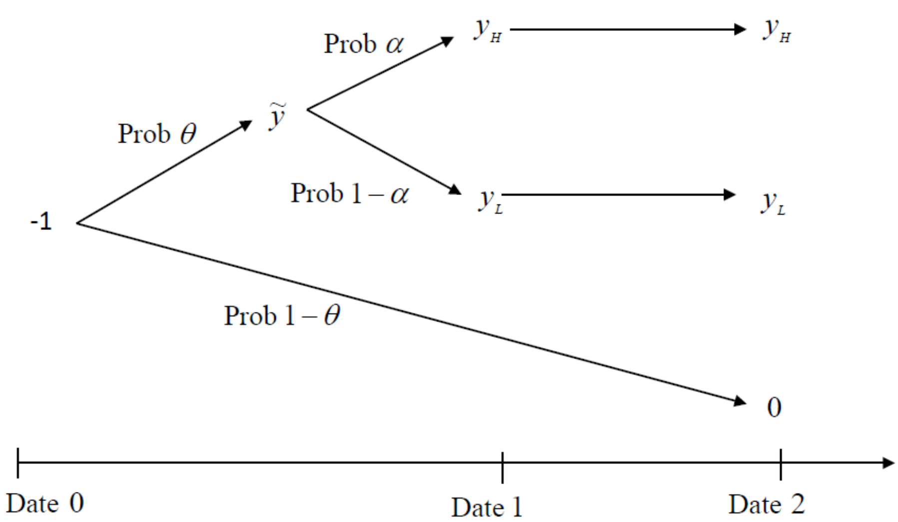


Figure 1: The risky investment opportunity

Assumption 1. *The investment project has positive NPV:*

$$\mathbb{E}(\theta\tilde{y}) = \alpha\theta y_H + (1-\alpha)\theta y_L > 1$$

Observe that on average, investing in the project is more profitable than holding cash. However, given the mistmach of the timing between the bank's liquidity needs and the project's cash flow, the bank may optimally choose to invest a positive amount in the storage technology.

At date 1, the bank has two sources of liquidity to repay its short-term debt. The first one is the amount of cash it holds from date 0. The second one is the new borrowing it can make by pledging the date 2 - cash flow generated by the project. The extent to which the bank can pledge its future cash flow may be constrained by a moral hazard problem. Specifically, we assume that between date 1 and date 2, after raising new funds and before the project's cash flow is realized, the bank can switch investment to another asset with probability of success θ_1 and success cash flow y_1.

Assumption 2. *The moral hazard problem matters only in the low state:*

$$\theta_1 < \theta, \ y_H > y_1 > y_L \text{ and } \frac{1}{2}\theta y_L > \theta_1 y_1$$

Hence, the new asset has a lower success probability but its success return is higher than that of the project in the low state. Assuming that $\theta_1 y_1 < \frac{1}{2}\theta y_L$ ensures that investing in the θ_1-asset is a negative net present value investment for the bank. The main implication of Assumption 2 is that while in the high state the bank can pledge the full value of its long-term asset to outside investors, the bank's borrowing capacity in the low state is strictly lower than the expected present value of its future cashflows. We could alternatively interpret the asset switch as the fact that the bank refrains from monitoring. Precisely, if a high value of $\tilde{y}$ is realized at date 1, the quality of the investment project happens to be very good and no intermediate monitoring needs to be done. However, when a low value is realized, the project's return depends on the bank's monitoring activities. If the bank refrains from them, it can save on the monitoring cost and thus, receive more in case of success. Nevertheless, the success probability will be reduced. The realization of the low state can be seen as the materialization of a liquidity shock that put constraints on the amount of liquidity the bank can raise and makes the rollover of its short-term debt problematic.

If the bank cannot raise enough liquidity, it will be closed and the bank's investment project is liquidated. We assume that the liquidation value is equal to ℓ, which is independent of the state. ℓ can be interpreted as the minimum possible value of the asset (for financial assets) or as the resale price (for physical assets)

Assumption 3. *The value of the invesment project to the bank's financiers is less than the value to the bank:*

$$\ell < \theta y_L$$

Assumption 3 is justified in situations where the management of the bank's assets requires sophisticated expertise that the bank's financiers do not have.

Assumption 4.

$$\alpha\theta y_H + (1-\alpha)\,\ell > 1$$

Assumption 4 means that the expected payoff of the project, even if it is liquidated early, is positive[2]. This assumption implies that at date 0, it is still worth for the bank to invest in the project even if the bank may be closed if the liquidity shock is realized.

The timing of the model, which is summarized in Figure 2, is as follows:

[2]Note that Assumption 1 is automatically satisfied if both Assumptions 3 and 4 are true.

- At date 0, the bank chooses the composition of its assets. Denote by c the amount of cash it holds. Thus, $1 - c$ will be invested in the project.
- At date 1, first, the value of $\tilde{y}$ is observed. Then, the bank tries to raise funds to pay back its short-term debt. If the bank cannot raise sufficient liquidity, it is liquidated.
- At date $\frac{3}{2}$, between date 1 and date 2, the bank may switch its investment to the θ_1-asset.
- At date 2, the project's cash flow is realized and payments are settled.

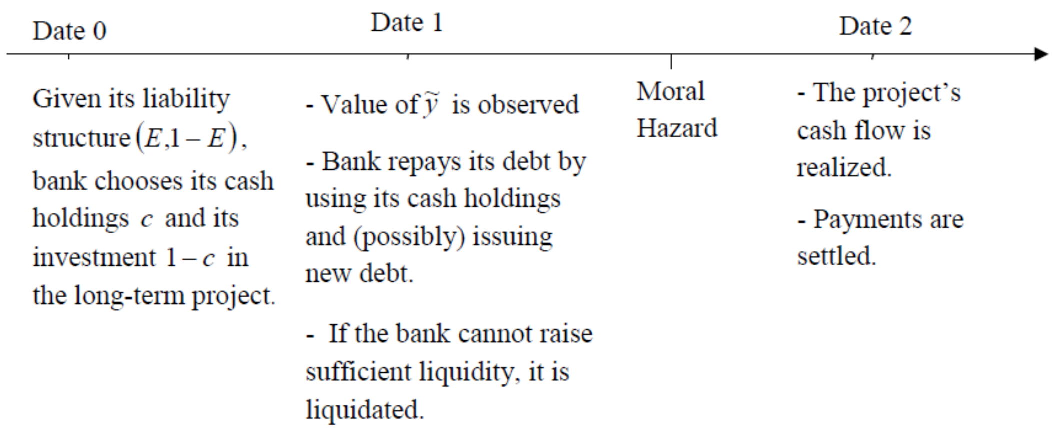


Figure 2: The timeline

Before proceeding with the analysis of the bank's optimal cash holdings, some additional remarks are in order. First, we take as given the bank's maturity mismatch: bank's creditors want short-term debts, while borrowers need long-term credit. We assume that the bank cannot issue long-term debt.

Second, does our bank's short-term debt correspond to wholesale or retail debt? One of the main differences between the two types is the fact that retail deposits are insured but wholesale deposits are not. This difference implies that the repayment promised to retail depositors does not depend on the bank's choice of assets while the repayment to wholesale creditors does. In this model, motivated by the financial crisis 2007 - 2009, we refer to the wholesale debts such as the ones held by Money Market Funds. The debt repayment is thus endogenously determined in our framework by the break-even condition of the bank's debtholders.

Concerning our formulation of the liquidity shock, note that in the present framework, this shock does not come from the uncertainty about the amount of short-term debt that need to be repaid, as commonly assumed in the model with retail deposits. Indeed, in our setup, the bank knows exactly as of date 0 how much debt it has to repay. What it does not know is its funding capacity at the time the repayment is made. If everything goes well, i.e. there are good news about the quality of the bank's asset, the bank's borrowing capacity is not constrained and thus, the need to refinance the short-term debt does not create any problem to the bank. However, when bad news about the bank's investment are revealed, its capacity to raise funds is restricted. As a consequence, the bank may fail to roll-over its debt. The above-described scenario is analogous to what happened in the 2007-2009 crisis. Prior to the crisis, banks financed a growing portion of their subprime mortgage loans with short-term debts such as repos or asset-backed commercial papers (ABCP). Everything worked very well until an increase in subprime mortgage defaults was first noted in February 2007, which was followed by a deterioration of the banks' short-term funding market. Many banks then experienced difficulties in rolling-over their short-term debts.

4 Analysis

We now analyse the bank's optimal investment decision. Our main objective is to study how much cash the bank will hold on its balance sheet and how this decision is affected by the bank's leverage. We will proceed in two steps. First, we determine the bank's borrowing capacity at date 1. Then, we examine its optimal cash holdings at date 0.

4.1 Borrowing Capacity

At date 1, the bank has to repay its short-term debt D. It has c units of cash, which implies that its liquidity needs are $D-c$. The bank can raise this amount by issuing new debt repaid at date 2. We now determine how much the bank can borrow at date 1 by pledging the future cash flow generated by its long-term asset.

If the high state is realized at date 1, the moral hazard problem does not matter, the bank can pledge the full value of its asset to investors, i.e. they can borrow up to θy_H, and there is no problem in rolling over its short-term debt.

If the low state is realized, the incentive compatibility condition which ensures that the bank does not switch to the riskier asset is as follows:

$$\theta\left(y_L - f\right) \geq \theta_1\left(y_1 - f\right)$$

where f is the face value of the new debt issued against one unit of long-term asset. After simplification, this yields:

$$f \leq \frac{\theta y_L - \theta_1 y_1}{\theta - \theta_1} = f^*$$

f^* represents the maximum cash flow that can be pledged to outside investors (i.e. f^* is the maximum pledgeable income). The bank's maximum borrowing capacity (per unit of long-term asset) in the low state is thus θf^*. Notice that $\theta f^* < \theta y_L$. We make an additional assumption as follows:

Assumption 5.

$$\ell < \theta f^*$$

Assumption 5 ensures that new borrowing is a better way to raise liquidity for the bank than partial liquidation of its long-term asset. The following lemma summarizes the bank's situation at date 1:

Lemma 1. *At date* 1*:*

(i) *If* $\frac{D-c}{1-c} \leq \theta f^*$*, the bank can always roll over its debt.*

(ii) *If* $\frac{D-c}{1-c} > \theta f^*$*, the bank is liquidated when being hit by a liquidity shock (i.e. when the low state is realized).*

We refer to the first situation as the one where the bank is liquid. The second situation is referred to as the case where the bank is illiquid.

4.2 Optimal Cash Holding Policy

In the next step, we study the bank's decision regarding the amount of cash held. Given two possible situations of the bank at date 1, we will first determine how much cash the bank holds in each situation. Then, we characterize the optimal cash policy of the bank.

If the bank chooses c so that it will be liquid at date 1, the amount of cash held by the bank is determined by the following program[3]:

$$\Pi^{li} = \underset{0 \leq c \leq 1}{Max} \left\{ \alpha\theta \left[(1-c)\, y_H - f_H \right] + (1-\alpha)\, \theta \left[(1-c)\, y_L - f_L \right] \right\}$$

where f_s, $s = H, L$ is the face value of the new debt issued at date 1 in the state s:

$$f_s = \frac{D-c}{\theta} \text{ for all } s$$

subject to the break-even condition of short-term investors:

$$\alpha D + (1-\alpha)\, D = 1 - E \tag{1}$$

and the liquidity condition:

$$\frac{D-c}{1-c} \leq \theta f^* \tag{2}$$

Plugging (1) into (2) and into the objective function, we can rewrite the above program as follows:

$$\Pi^{li} = \underset{0 \leq c \leq 1}{Max} \left\{ \alpha\theta y_H + (1-\alpha)\, \theta y_L - 1 + E - c\left(\alpha\theta y_H + (1-\alpha)\, \theta y_L - 1 \right) \right\} \tag{3}$$

subject to

$$(1 - E - \theta f^*) \leq c\,(1 - \theta f^*) \tag{4}$$

This program makes clear the trade-off driving the bank's cash holding decision. The cost of holding cash is the foregone return of the long-term asset, which explains why the term "$c\,(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1)$" is deducted from the bank's expected profit. The

[3] The superscript "li" refers to liquidity.

benefit of holding cash is to provide insurance against the liquidity shock at date 1, which is reflected in Constraint (4). Note that this constraint matters only if $\theta f^* < 1$. One unit of cash at date 0 generates one unit of liquidity at date 1 whereas the amount of liquidity raised against one unit of long-term asset is θf^*. Clearly, holding cash makes sense only when $\theta f^* < 1$. We make the following assumption to ensure the role of cash in our model:

Assumption 6.

$$\theta f^* < 1$$

At the optimum, the bank holds an amount of cash that is just sufficient to overcome the liquidity shock, i.e. $c^{li} = \max\left(\frac{1-E-\theta f^*}{1-\theta f^*}, 0\right)$. Note that when E is high enough (i.e. $E \geq 1-\theta f^*$), the liquidity shock is low, the bank is liquid even though it holds zero cash. Hence, the bank's expected profit when choosing to be liquid at date 1 is:

$$\Pi^{li} = \alpha\theta y_H + (1-\alpha)\,\theta y_L - 1 + E - \max\left(\frac{1-E-\theta f^*}{1-\theta f^*}, 0\right)(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1)$$

We now turn to the amount of cash the bank holds if it chooses to be illiquid at date 1. The bank problem in this case is written as follows[4]:

$$\Pi^{illi} = \underset{0\leq c\leq 1}{Max}\left\{\alpha\theta\left[(1-c)\,y_H - \frac{D-c}{\theta}\right]\right\}$$

subject to the break-even condition of short term investors:

$$\alpha D + (1-\alpha)\,(c + (1-c)\,\ell) = 1 - E \tag{5}$$

and the illiquidity condition:

$$\frac{D-c}{1-c} > \theta f^*$$

Plugging (5) into the objective function, we get:

$$\Pi^{illi} = \underset{0\leq c\leq 1}{Max}\left\{\alpha\theta y_H + (1-\alpha)\,\ell - 1 + E - c\,(\alpha\theta y_H + (1-\alpha)\,\ell - 1)\right\}$$

subject to

$$(1 - E - \theta f^*) > c\,(1-\theta f^*)$$

Hence, $c^{illi} = 0$ at the optimum. Since the only benefit of cash is to provide insurance against the liquidity shock, it is intuitive that if the bank decides to be illiquid at date 1, it will hold zero cash. The bank's expected profit when choosing to be illiquid at date 1 is then:

$$\Pi^{illi} = \alpha\theta y_H + (1-\alpha)\,\ell - 1 + E$$

Finally, to determine the optimal cash policy of the bank, we have to compare Π^{li} and Π^{illi}. We see that the condition:

$$\Pi^{li} \geq \Pi^{illi}$$

[4] The superscript "*illi*" refers to illiquidity.

is equivalent to

$$(1-\alpha)\,\theta y_L-(1-\alpha)\,\ell\geq\max\left(\frac{1-E-\theta f^*}{1-\theta f^*},0\right)(\alpha\theta y_H+(1-\alpha)\,\theta y_L-1)\tag{6}$$

Note that the LHS of Inequality (6) is the expected loss in value due to early liquidation of the long-term asset while the RHS represents the cost of buying insurance against liquidity risk (i.e. holding cash) for the bank. Clearly, the bank chooses to be insured only if the insurance cost is lower than the loss in the value. Inequality (6) results in a condition on the bank's leverage as follows:

$$E\geq(1-\theta f^*)\,\frac{\alpha\theta y_H+(1-\alpha)\,\ell-1}{\alpha\theta y_H+(1-\alpha)\,\theta y_L-1}=E^*\tag{7}$$

The following proposition summarizes the characterization of the bank's optimal cash holding policy:

Proposition 1. *When the bank is undercapitalized (i.e. $E<E^*$), it chooses to be illiquid and holds zero cash. The bank chooses to be liquid only when it is well capitalized (i.e. $E\geq E^*$). In that case, the bank holds an amount of cash equal to* $\max\left(\frac{1-E-\theta f^*}{1-\theta f^*},0\right)$ *and the liquidity coverage ratio (i.e. $\frac{c}{D}$) is increasing with the leverage.*

We graphically represent in Figure 3 the cash holding policy characterized in Proposition 1. We first observe that the bank holds sufficient cash to be insured against the liquidity shock if and only if the bank's capital ratio is high enough. This result is due to the fact that when the bank's capital ratio decreases, the bank holds more debts, which exposes it to a higher liquidity shock. This higher exposure in turn leads to a higher cost of insurance. We see clearly in Inequality (7) that the insurance cost is decreasing with the bank's capital ratio E. When this ratio is too low, buying insurance against the liquidity shock becomes too costly, which induces the bank to gamble. The second conclusion obtained in Proposition 1 concerns the increasing relationship between the liquidity coverage ratio and the leverage of the bank when it is well capitalized. The intuition behind it is straightforward. Once the bank chooses to be liquid, the amount of cash it holds is increasing with its exposure to liquidity risk.

Proposition 1 brings out the positive impact that a restriction on the bank's leverage can have on its incentives to manage its liquidity. In the present model, capital requirements can perfectly do the job of improving the management of liquidity risk by banks. As noted in the introduction, although we do not claim that this is a general result, this insight is itself interesting in the sense that it shows that any proposal concerning a liquidity requirement needs to be jointly considered with the capital regulation in order to avoid overregulation. Another interesting insight derived from Proposition 1 pertains to the impact of a decrease in the likelihood of the liquidity shock on the capital ratio threshold:

Corollary 1. *The capital ratio threshold $E^*$is decreasing with the probability* $(1-\alpha)$ *that the liquidity shock happens.*

Corrolary 1 states that the capital ratio threshold increases when the likelihood of the shock decreases. Put differently, the capital ratio threshold is higher for the liquidity risk that has smaller probability of occurrence. Corollary 1 thus implies that it is much more difficult to induce banks to properly manage the tail risk.

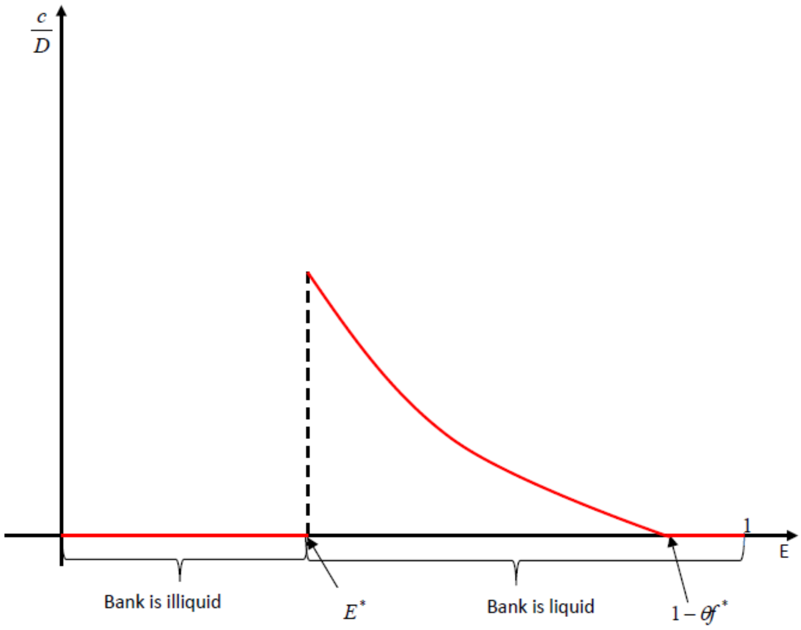


Figure 3: The bank's optimal cash holding policy

5 Multiple Banks Setting with Asset Sales

In the previous setting, we assume away the possibility that a secondary market for long-term assets is opened at date 1, which allows banks to sell them when in need of liquidity. In this section, we examine the consequences of permitting the sales of long-term assets.

5.1 Environment

We consider a model with three banks, called A, B and C. Bank i ($i = A, B, C$) has an amount of equity equal to E_i. Each banks has access to the same investment technologies and is subject to the same moral hazard problem as described in the previous section. We assume that the liquidity shock represents a common exposure of three banks: at date 1, the realization of $\tilde{y}$ is the same to all of them. The difference with the previous setup is that to pay back their short-term debt, banks now have three options instead of two:

(i) The amount of cash held from date 0

(ii) The new debt issued against the date 2 - cash flow generated by the project

(iii) The proceeds from selling their long-term assets.

With regard to the secondary market where banks can sell their asset, we assume, in accordance with Assumption 3 about the specificity of the banks' asset, that potential purchasers of a bank's long-term asset are other banks. Hence, in the present paper, we distinguish between asset sale and asset liquidation. Asset sale corresponds to the transfer of the asset from one specialist to the other who has the same ability to redeploy it. As to asset liquidation, it is equivalent to the transfer of the asset to a non-specialist who can extract a much lower surplus from the assets than specialist.

In order to characterize the equilibria of the present economy, we proceed as follows: we first examine the market for asset sales and analyze how the market price is determined. Then we study the banks' incentives for liquidity holdings. Finally, we investigate the existence and the main features of different rational expectation equilibria.

5.2 Borrowing Capacity

As in the basic model, if the high state is realized, all banks can roll over their debt. If the low state is realized, the maximum borrowing capacity (per unit of long-term asset) for each bank is θf^*.

5.3 Market for Asset Sales

We now analyze the secondary market of long-term assets. For this purpose, we introduce some additional notations as follows:

- ρ_i denotes the bank i's liquidity demand (per unit of long-term asset) at date 1:

$$\rho_i = \frac{D_i - c_i}{1 - c_i} \text{ for } i = A, B, C$$

- p is the unit price of the long-term assets.
- β_i $(i = A, B, C)$ indicates the fraction of long-term assets sold by bank i to cover its liquidity need.
- γ_i $(i = A, B, C)$ is the volume of long-term assets acquired by bank i.

A. *Demand and Supply of Long-Term Assets*

We start with the determination of the individual suppy and demand functions. Since the maximum funding capacity (per unit of long-term asset) for each bank is θf^*, banks who have to sell their long-term assets are the ones with ρ exceeding θf^*. In contrast, banks that have ρ lower than or equal to θf^* are in excess of liquidity and thus, can buy assets.

The fraction of asset β_i sold by each bank i $(i = A, B, C)$ with ρ_i greater than θf^* is determined as follows:

$$\beta_i (1 - c_i) p + (1 - c_i)(1 - \beta_i)\theta f^* \geq D_i - c_i \tag{8}$$

In Inequality (8), the LHS is the total liquidity bank i could raise. It is the sum of respectively the proceeds from selling a fraction β_i of long-term asset and the liquidity obtained by issuing new debt against the remaining fraction $1 - \beta_i$. After simplification, we get:

$$\beta_i = \min\left(1, \frac{\rho_i - \theta f^*}{p - \theta f^*}\right) \tag{9}$$

Observe that funding liquidity expands with asset sales if and only if the unit price p is greater than θf^*. We assume for now $p \geq \theta f^*$ and will show later that it is indeed the case. The extent of asset sales is decreasing with the asset's price. A bank i will have to sell all of its existing investment when the price p falls below its liquidity demand ρ_i.

With regard to the asset demand γ_i of bank i who has excess liquidity, note that no bank would acquire assets at a price higher than their expected payoff. Hence, if $p > \theta y_L$, γ_i should be equal to zero. If $\theta f^* < p < \theta y_L$, γ_i is determined as follows:

$$(1 - c_i + \gamma_i)\,\theta f^* - (D_i - c_i) \geq \gamma_i p \tag{10}$$

The LHS of Inequality (10) is the total liquidity available to bank i for asset purchase. It consists of its spare debt capacity from existing assets, $(1 - c_i)\,\theta f^* - (D_i - c_i)$, plus the liquidity that can be raised against assets to be acquired, $\gamma_i \theta f^*$. After some arrangements, we have:

$$\gamma_i = (1 - c_i)\,\frac{\theta f^* - \rho_i}{p - \theta f^*}$$

Notice that if $p = \theta f^*$, the liquidity raised against assets to be acquired is sufficient to pay for the assets, which implies that the demand for the assets is infinitely high. To summarize, the long-term asset's demand function of bank i who has ρ_i lower than or equal to θf^* is as follows:

$$\gamma_i(\rho_i, p) = \begin{cases} 0 & \text{if} \quad p > \theta y_L \\ (1 - c_i)\,\frac{\theta f^* - \rho_i}{p - \theta f^*} & \text{if} \quad \theta f^* < p < \theta y_L \\ \text{any value between } 0 \text{ and } (1 - c_i)\,\frac{\theta f^* - \rho_i}{p - \theta f^*} & \text{if} \quad p = \theta y_L \\ \infty & \text{if} \quad p = \theta f^* \end{cases} \tag{11}$$

B. *Unit Price of Long-Term Assets*

Now, we turn to investigate the unit price of long-term assets. Because of limited market participation, the price is determined by the amount of liquidity available in the market for asset purchases, which in turn depends on the banks' cash holding decision at date 0. In this section, we characterize the equilibrium price for all possible distributions of liquidity in the banking system at date 1. In the following, i, j and k can be either A, B or C.

(i) $\rho_i \leq \rho_j \leq \rho_k \leq \theta f^*$: All banks are liquid and can roll over their debt. Therefore, there is no asset trading at date 1.

(ii) $\rho_i \leq \theta f^* < \rho_j \leq \rho_k$: Bank i is liquid while banks j and k have to sell some of their long-term assets if the low state is realized. From the individual demand and supply functions characterized in respectively (11) and (9), we can compute the total supply $S(p)$ as well as the total demand $D(p)$ as follows:

$$S(p) = (1 - c_j)\min\left(1, \frac{\rho_j - \theta f^*}{p - \theta f^*}\right) + (1 - c_k)\min\left(1, \frac{\rho_k - \theta f^*}{p - \theta f^*}\right)$$

and

$$D(p) = (1 - c_i)\,\frac{\theta f^* - \rho_i}{p - \theta f^*} \text{ for } p < \theta y_L$$

The equilibrium price satisfies the market-clearing condition:

$$ED(p) \equiv D(p) - S(p) = 0$$

If excess demand $ED(p)$ is positive for all $p < \theta y_L$, then in the equilibrium $p = \theta y_L$. There exists three subcases:

(ii.1) $\theta f^* \leq p \leq \rho_j \leq \rho_k$: We have:

$$\min\left(1, \frac{\rho_j - \theta f^*}{p - \theta f^*}\right) = 1 \text{ and } \min\left(1, \frac{\rho_k - \theta f^*}{p - \theta f^*}\right) = 1$$

This case corresponds to the situation where both banks j and k cannot raise enough liquidity even when they sell all of their long-term assets. They are thus closed at date 1. Their assets are sold by their debtholders to bank i. The excess demand $ED(p)$ is then computed as follows:

$$ED(p) = (1 - c_i) \frac{\theta f^* - \rho_i}{p - \theta f^*} - (1 - c_j + 1 - c_k)$$

Hence, the condition that excess demand be zero leads to the following relationship:

$$p = \theta f^* + \frac{(1 - c_i)(\theta f^* - \rho_i)}{2 - c_j - c_k} \tag{12}$$

which implies that the equilibrium price is given by:

$$p = \min\left(\theta f^* + \frac{(1 - c_i)(\theta f^* - \rho_i)}{2 - c_j - c_k}, \theta y_L\right) \tag{13}$$

(ii.2) $\theta f^* < \rho_j < p \leq \rho_k$: In this case, bank j can overcome the liquidity shock after selling a fraction β_j, which is strictly less than 1, of its long-term assets. However, bank k has to sell all of its long-term assets and thus, will be closed at date 1 following the occurrence of the liquidity shock. Zero excess demand is equivalent to:

$$ED(p) = \frac{(1 - c_i)(\theta f^* - \rho_i)}{p - \theta f^*} - \left(\frac{(1 - c_j)(\rho_j - \theta f^*)}{p - \theta f^*} + (1 - c_k)\right) = 0$$

Therefore, the equilibrium price is represented as follows:

$$p = \min\left(\theta f^* + \frac{(\theta f^* - \rho_i)(1 - c_i) - (\rho_j - \theta f^*)(1 - c_j)}{1 - c_k}, \theta y_L\right) \tag{14}$$

(ii.3) $\theta f^* < \rho_j \leq \rho_k < p$: Both banks j and k survive the liquidity shock and continue to operate at date 1. The excess demand function is given by:

$$ED(p) = \frac{(1 - c_i)(\theta f^* - \rho_i)}{p - \theta f^*} - \frac{(1 - c_j)(\rho_j - \theta f^*) + (1 - c_k)(\rho_k - \theta f^*)}{p - \theta f^*}$$

We see that in this case, the equilibrium price is determined by a positive excess demand condition as follows:

$$\frac{(1 - c_i)(\theta f^* - \rho_i)}{p - \theta f^*} \geq \frac{(1 - c_j)(\rho_j - \theta f^*) + (1 - c_k)(\rho_k - \theta f^*)}{p - \theta f^*} \tag{15}$$

which means that in the equilibrium the price p equals to θy_L.

(iii) $\rho_i \leq \rho_j \leq \theta f^* < \rho_k$: Two banks i and j are liquid whereas bank k is illiquid and must sell its long-term assets at date 1 if the low state is realized. Similarly to Case (ii), the total supply and total demand are computed as follows:

$$S(p) = (1 - c_k) \min\left(1, \frac{\rho_k - \theta f^*}{p - \theta f^*}\right)$$

$$D(p) = (1 - c_i) \frac{\theta f^* - \rho_i}{p - \theta f^*} + (1 - c_j) \frac{\theta f^* - \rho_j}{p - \theta f^*} \text{ for } p < \theta y_L$$

There are two subcases:

(iii.1) $\theta f^* < p \leq \rho_k$: Bank k has to sell all of its long-term assets and then, is closed at date 1. The equilibrium price is given by:

$$p = \min\left(\theta f^* + \frac{(\theta f^* - \rho_i)(1 - c_i) + (\theta f^* - \rho_j)(1 - c_j)}{1 - c_k}, \theta y_L\right) \tag{16}$$

(iii.2) $\theta f^* < \rho_k < p$: Bank k survives the liquidity shock. As in Case (ii.3), the price is at it frictionless value of θy_L and the positive excess demand condition is as follows:

$$\frac{(1 - c_i)(\theta f^* - \rho_i) + (1 - c_j)(\theta f^* - \rho_j)}{p - \theta f^*} \geq \frac{(1 - c_k)(\rho_k - \theta f^*)}{p - \theta f^*} \tag{17}$$

(iv) $\theta f^* < \rho_i \leq \rho_j \leq \rho_k$: All banks are illiquid. Therefore, at date 1, no bank is able to absorb the assets put for sales, which implies that long-term assets are transfered to non-specialists. In other words, at date 1, all banks are closed and their investment is liquidated at the value ℓ.

From these representations of the equilibrium price, several observations can be made. First, we see that the price of long-term assets never falls below θf^*(as remarked in (9)). This is because the buyer banks can always raise θf^* of liquidity against each additional unit of asset they purchase. Typically, the equilibrium price is determined by the sum of θf^* and an other term that captures the effect of spare liquidity in the system. Let us look at, for instance, Equation (12) in Case (ii.1). The spare liquidity in the system is represented by $(1 - c_i)(\theta f^* - \rho_i)$, the excess liquidity held by bank i. Whether or not the price deviates from the asset's value depends on the magnitude of this spare liquidity. If it is high enough, the RHS of Equation (12) is then greater than θy_L, which implies that excess demand is positive for all $p < \theta y_L$. Hence, the price equals to the asset's expected payoff, i.e. $p = \theta y_L$ (see (13)). In the other case, the price is strictly less than the asset's value, which reflects a fire-sale discount. The same observations apply to Cases (ii.2) and (iii.1). For Cases (ii.3) and (iii.2), we see that the market should either be in excess demand or in excess supply. If the market is in excess supply, then the price should equal θf^*, which violates the conditions characterizing these two cases. Therefore, Cases (ii.3) and (iii.2) only occur when the spare liquidity in the system is high enough so that the equilibrium price is at its frictionless value of θy_L. These observations lead to the following lemma:

Lemma 2. *The equilibrium price of long-term assets is represented by either (13) or (14) or (15) or (16) or (17), depending on the distribution of liquidity in the banking system. It has the following properties:*

1. *It is increasing in the funding liquidity of the long-term asset.*

2. *It is lower than the asset's value when the spare liquidity in the banking system is low.*

5.4 Banks' Incentives for Liquidity Holdings

Given the above representations of the asset price, we are now equipped to analyse the banks' cash holding decisions. From a date 0 perspective, each bank must choose how much cash it holds on its balance sheet. Its decision is affected by its expectation about the price of long-term assets at date 1, which depends on other banks' decisions. To gain insights on the banks' incentives to hold cash, let us first formulate in this section the program that determines a bank i's cash holding decision given the choices of two other banks j and k. Then, we will examine how the possibility of acquiring asset cheap affects the banks' motivation for holding cash.

A. *The Banks' Optimization Problem*

We first write the program that determines the amount of cash bank i will hold if choosing to be liquid. Notice that its decision will depend on whether or not it has the opportunity to purchase some long-term assets at date 1. Hence, we distinguish between two situations: with and without asset trading at date 1:

When two other banks j and k also choose to be liquid, no long-term assets are put for sale at date 1 by other banks. Bank i's expected profit if choosing to be liquid is then computed as follows[5]:

$$\Pi_i^{li\_ntr} = \underset{c_i \in [0,1]}{Max} \left\{ \alpha\theta \left[(1-c_i)\, y_H - \frac{D_i - c_i}{\theta} \right] + (1-\alpha)\,\theta \left[(1-c_i)\, y_L - \frac{D_i - c_i}{\theta} \right] \right\}$$

subject to the break-even condition of short-term investors:

$$\alpha D_i + (1-\alpha)\, D_i = 1 - E_i \tag{18}$$

and the liquidity constraint:

$$\rho_i \leq \theta f^* \tag{19}$$

After subtracting the break-even condition from the objective function, we can rewrite the above program as follows:

Program $\wp^{li\_ntr}$

$$\Pi_i^{li\_ntr} = \underset{c_i \in [0,1]}{Max} \left\{ \alpha\theta y_H + (1-\ \alpha)\,\theta y_L - 1 + E_i - c_i \left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1 \right) \right\} \tag{20}$$

[5]In what follows, the superscripts "li_ntr", "li_tr", "$illi\_ntr$" and "$illi\_tr$" refers respectively to "liquidity and no trading", "liquidity and trading", "illiquidity and no trading" and "illiquidity and trading".

subject to

$$c_i \geq \frac{1 - E_i - \theta f^*}{1 - \theta f^*}$$

When at least one of the two other banks chooses to be illiquid, some assets will be put for sale at date 1 in the secondary market, which gives bank i the occasion to buy some assets if it has available liquidity. Given that bank i's demand for the asset is represented by γ_i defined by (11), bank i's expected profit if choosing to be liquid is thus:

$$\Pi_i^{li\_tr} = \underset{c_i \in [0,1]}{Max} \left\{ \alpha\theta \left[(1 - c_i)\, y_H - \frac{D_i - c_i}{\theta} \right] + (1 - \alpha)\,\theta \left[(1 - c_i + \gamma_i)\, y_L - \frac{D_i - c_i + p\gamma_i}{\theta} \right] \right\} \tag{21}$$

subject to the two same constraints (18) and (19).

The first bracket in (21) is simply what bank i gets in case of success at date 2 if a high state is realized at date 1. It is the difference between the project's cash flow and the face value of the new debt issued at date 1. The second bracket is the bank's profit following the realization of the low state. Note that at date 1, when the low state is observed, bank i purchases a volume γ_i of assets put for sales by illiquid banks, which means that it holds $1 - c_i + \gamma_i$ units of long-term assets after the trade. To pay for the transaction, bank i has to borrow an additional amount equal to $p\gamma_i$, which explains why the face value of its new debts is now $\frac{D_i - c_i + p\gamma_i}{\theta}$. After some rearrangements, we get:

Program $\wp^{li\_tr}$

$$\Pi_i^{li\_tr} = \underset{c_i \in [0,1]}{Max} \left\{ \begin{array}{c} \alpha\theta y_H + (1 - \alpha)\,\theta y_L - 1 + E_i - c_i\,(\alpha\theta y_H + (1 - \alpha)\,\theta y_L - 1) \\ + (1 - \alpha)\,\gamma_i\,(\theta y_L - p) \end{array} \right\} \tag{22}$$

subject to

$$c_i \geq \frac{1 - E_i - \theta f^*}{1 - \theta f^*} \tag{23}$$

As for the program determining the amount of cash bank i holds if choosing to be illiquid, there are also two cases:

If both other banks choose to be illiquid, bank i will have to liquidate its long-term investment when the low state is realized at date 1. Its expected profit is then:

$$\Pi_i^{illi\_ntr} = \underset{c_i \in [0,1]}{Max} \left\{ \alpha\theta \left[(1 - c_i)\, y_H - \frac{D_i - c_i}{\theta} \right] \right\}$$

subject to the break-even condition of short-term investors:

$$\alpha D + (1 - \alpha)\,(c_i + (1 - c_i)\,\ell) = 1 - E_i \tag{24}$$

and the illiquidity condition:

$$c_i < \frac{1 - E_i - \theta f^*}{1 - \theta f^*} \tag{25}$$

Again, subtracting (24) from the objective function, the above program becomes:

Program $\wp^{illi\_ntr}$

$$\Pi_i^{illi\_ntr} = \underset{c_i \in [0,1]}{Max} \; \{\alpha\theta y_H + (1 - \alpha)\,\ell - 1 + E - c\,(\alpha\theta y_H + (1 - \alpha)\,\ell - 1)\} \tag{26}$$

subject to:

$$c_i < \frac{1 - E_i - \theta f^*}{1 - \theta f^*}$$

If at least one of two other banks choose to be liquid, bank i can sell its long-term assets to liquid banks in the secondary market when it needs liquidity. Its expected profit is now computed as follows:

$$\Pi_i^{illi\_tr} = \underset{c_i \in [0,1]}{Max} \left\{ \begin{array}{c} \alpha\theta \left[(1 - c_i) \, y_H - \frac{D_i - c_i}{\theta} \right] \\ + (1 - \alpha) \max \left(\theta \left[(1 - c_i) (1 - \beta_i) \, y_L - \frac{D_i - c_i - (1 - c_i)\beta_i p}{\theta} \right], 0 \right) \end{array} \right\} \quad (27)$$

where β_i is defined by (9) and subject to

$$\alpha D_i + (1 - \alpha) \min \left[D_i, (c_i + (1 - c_i) \, \beta_i p + (1 - c_i) (1 - \beta_i) \, \theta f^* \right] = 1 - E_i$$

and

$$c_i < \frac{1 - E_i - \theta f^*}{1 - \theta f^*}$$

The second term in (27) is the bank i's expected profit following the realization of the low state. Note that if bank i is illiquid, at date 1 in the low state, it will sell a fraction β_i of its long-term assets and ends up with the remaining fraction $1 - \beta_i$. When this term takes a strictly positive value, only a part of the investment is sold and thus, bank i survives the liquidity shock. Otherwise, bank i has to sell all of its investment and its expected profit is equal, by limited liability, to zero. After simplifications, we obtain:

Program $\wp^{illi\_tr}$

$$\Pi_i^{illi\_tr} = \underset{c_i \in [0,1]}{Max} \left\{ \begin{array}{c} \alpha\theta y_H + (1 - \alpha) \, \theta y_L - 1 + E_i - c_i \left(\alpha\theta y_H + (1 - \alpha) \, \theta y_L - 1 \right) \\ - (1 - \alpha) (1 - c_i) \, \beta_i \left(\theta y_L - p \right) \end{array} \right\} \quad (28)$$

subject to

$$c_i < \frac{1 - E_i - \theta f^*}{1 - \theta f^*}$$

B. *Speculative Motive of Holding Cash*

To shed light on the banks' motivation for holding cash that stems from the opportunity to acquire other banks' asset, we compare two Programs $\wp^{li\_ntr}$ and $\wp^{li\_tr}$. We see that the possibility of acquiring other banks' assets at date 1, which happens with probability $(1 - \alpha)$, generates an additional profit of $\gamma_i \left(\theta y_L - p \right)$ to the liquid bank i. We refer to it as trading profit and denote it by TP_i. The first remark is that TP is strictly positive if and only if $p < \theta y_L$. In other words, the option to buy assets affects the banks' cash holding incentives only when assets are traded at fire-sale price. As a mean to analyse the effects of one additional unit of cash held by bank i on its trading profit, we compute the first derivative of TP_i with respect to c_i as follows:

$$\frac{dTP_i}{dc_i} = \left(\theta y_L - p \right) \frac{d\gamma_i}{dc_i} - \gamma_i \frac{dp}{dc_i} \quad (29)$$

As long as $p < \theta y_L$, we have:

$$\frac{d\gamma_i}{dc_i} = \frac{1-\theta f^*}{p-\theta f^*} - \frac{\gamma_i}{p-\theta f^*}\frac{dp}{dc_i}$$

which implies that (29) yields:

$$\frac{dTP_i}{dc_i} = (\theta y_L - p)\frac{1-\theta f^*}{p-\theta f^*} - \left[(\theta y_L - p)\frac{\gamma_i}{p-\theta f^*} + \gamma_i\right]\frac{dp}{dc_i} \tag{30}$$

Observe that one additional unit of cash held by bank i has two effects on the trading profit. The positive effect, captured by the first term in (30), is the impact on the bank i's excess liquidity available to acquire the asset: one more unit of cash increases the excess liquidity by $1-\theta f^*$, which allows bank i to buy more assets for a given price p. However, one additional unit of cash held by bank i will also increase the asset price, which decreases the volume of assets bank i can buy for a given level of excess liquidity as well as the profit per unit of asset acquired. The optimal amount of cash the liquid bank i holds, when it expects to have the opportunity to purchase some assets at date 1, is then determined by the following FOC:

$$\begin{cases} -\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right) + (1-\alpha)\frac{dTP_i}{dc_i} + \lambda = 0 \\ \lambda\left(c_i - \frac{1-E_i-\theta f^*}{1-\theta f^*}\right) = 0 \text{ and } \lambda \geq 0 \end{cases} \tag{31}$$

where λ is the Lagrange multiplier associated with the liquidity constraint (23). We state in the following proposition the amount of cash bank i holds when it chooses to be liquid. For this purpose, we define the following variable δ:

$$\delta = \frac{(1-\alpha)\,(1-\theta f^*)\,(\theta y_L - \theta f^*)}{\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1 + (1-\alpha)\,(1-\theta f^*)} \tag{32}$$

Proposition 2. *In a model with three banks, if a bank i chooses to be liquid, its cash holdings are as follows:*

1. *Given that two other banks choose to be liquid:*

$$c_i^{li\_ntr} = \frac{1-E_i-\theta f^*}{1-\theta f^*}$$

2. *Given that at least one of two other banks chooses to be illiquid:*

a) *If both banks j and k choose to be illiquid or as long as $p = \theta y_L$:*

$$c_i^{li\_tr} = \frac{1-E_i-\theta f^*}{1-\theta f^*}$$

b) *In the other case, i.e. among two other banks, one bank chooses to be liquid, say bank j, one bank chooses to be illiquid and is closed, say bank k:*

$$c_i^{li\_tr} = \max\left[\frac{1-E_i-\theta f^*}{1-\theta f^*}, \frac{1-E_i-\theta f^*}{1-\theta f^*} + \frac{\sqrt{\delta\,(1-c_k)\,\varepsilon_j} - \varepsilon_j}{1-\theta f^*}\right]$$

where ε_j is the excess liquidity held by bank j, i.e.

$$\varepsilon_j = (1-c_j)\,(\theta f^* - \rho_j)$$

Proof. Appendix □

Some more intuitions underlying Proposition 2 are worth providing here.

First, as we already noted, the possibility of acquiring assets provides banks with an additional reason to hold cash only when assets are traded at fire-sale price. If banks expect that $p = \theta y_L$, the trading profit is zero and banks hold cash only for insuring themself against the liquidity shock, which explains why the liquid bank i just holds the minimum amount of cash, i.e. $c_i = \frac{1-E_i-\theta f^*}{1-\theta f^*}$, when it expects $p = \theta y_L$.

Second, even when the trading profit is strictly positive, bank i still decides to hold a minimum amount of cash if it expects all other banks to be illiquid. The reason is that when bank i is the only potential buyer of assets, one additional unit of cash held by bank i will have a strong effect on the price, which compensates or even outweigh the effect on its excess liquidity. Hence, its trading profit is decreasing with its cash holdings.

Third, when the liquid bank i expects to have competitors in the secondary market of the asset, the effect of its cash holding on the price is weaker. As a consequence, bank i may hold more than the minimum required amount if its competitors' cash holdings are low.

5.5 Equilibria

We study now the existence and the main features of rational expectation equilibria. We focus on pure strategy equilibria, which can be one of the following types: (i) all three banks are liquid; (ii) one bank is liquid and two banks are illiquid; (iii) two banks are liquid and one bank is illiquid; (iv) all three banks are illiquid. We will characterize the conditions on the banks' leverage under which each of those equilibria exists as well as their properties.

Equilibrium definition: A quadruple $(c_A^*, c_B^*, c_C^*, p^*)$ is a rational expection equilibrium if and only if:

(1) c_i^* is the optimal cash holdings of bank i $(i = A, B, C)$ given p^*

(2) p^* is the equilibrium price induced by the choices (c_A^*, c_B^*, c_C^*)

Before proceeding with the characterization, we define the following thresholds:

$$E_1 = (1-\theta f^*)\,\frac{\alpha\theta y_H + (1-\alpha)\,\theta f^* - 1}{\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1}$$

$$E_2 = E_1 - (1-\theta f^*)\,\frac{\delta}{2}\left(\frac{1}{1-\theta f^*} - \frac{1-\alpha}{\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1}\right)$$

$$E_3 = E_1 - (1-\theta f^*)\,\frac{\frac{1-\alpha}{4}\left(\theta y_L - \theta f^* - \frac{\delta}{2}\right)}{\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1}$$

$$E_4 = E_1 - (1-\theta f^*)\,\frac{\frac{1-\alpha}{2}\left(\theta y_L - \theta f^*\right)}{\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1}$$

$$E_5 = (1 - \theta f^*) \frac{\alpha \theta y_H + (1 - \alpha) \ell - 1 - 2(1 - \alpha)(\theta y_L - \theta f^*)}{\alpha \theta y_H + (1 - \alpha) \theta y_L - 1}$$

where δ is defined by Expression (32). It is easily to check that $E_1 > E_2 > E_3 > E_4 > E_5$.

We begin by stating the characteristics of two extreme equilibria where all banks choose either to be liquid or to be illiquid.

Proposition 3. *In a model with three banks:*

(a) *An equilibrium where all banks choose to be liquid exists if and only if*

$$E_i \geq E_1 \textit{ for all } i = A, B, C$$

In this equilibrium, each bank holds an amount of cash equal to

$$c_i^* = \frac{1 - E_i - \theta f^*}{1 - \theta f^*} \textit{ for all } i$$

(b) *An equilibrium where all banks choose to be illiquid exists if and only if*

$$E_i < E_5 \textit{ for all } i = A, B, C$$

In this equilibrium, each bank holds a zero amount of cash, i.e. $c_i^* = 0$ *for all* i*. All banks will be closed at date* 1 *if the low state is realized.*

Proof. Appendix □

To gain some intuition on the construction of those equilibria, consider for instance the equilibrium where all banks choose to be liquid. From Program $\wp^{li\_ntr}$, we see clearly that the optimal amount of cash each bank i holds if choosing to be liquid given that two other banks also choose to be liquid is equal to $\frac{1-E_i-\theta f^*}{1-\theta f^*}$. Next, we have to make sure that no bank has incentives to deviate. Evidently no bank should deviate by holding more than $\frac{1-E_i-\theta f^*}{1-\theta f^*}$. If a bank i deviates by holding less, i.e. by choosing to be illiquid, we could show that its expected profit is as follows:[6]

$$\Pi_i^{de} = \alpha \theta y_H + (1 - \alpha) \theta f^* - 1 + E_i$$

Therefore, to ensure no deviation, the following condition must be satisfied for all i:

$$\alpha \theta y_H + (1 - \alpha) \theta y_L - 1 + E_i - \frac{1 - E_i - \theta f^*}{1 - \theta f^*} (\alpha \theta y_H + (1 - \alpha) \theta y_L - 1) \geq \alpha \theta y_H + (1 - \alpha) \theta f^* - 1 + E_i$$

which yields

$$E_i \geq E_1 \text{ for all } i$$

Now, we turn to the equilibrium where one bank is liquid and two banks are illiquid.

[6] The superscript "de" refers to deviation.

Proposition 4. *In a model with three banks, an equilibrium where one bank chooses to be liquid and two banks choose to be illiquid exists if and only if in the banking system, one bank has capital ratio greater than E_5 while two other banks have capital ratio lower than E_4. In this equilibrium, at date 0, the liquid bank holds an amount of cash:*

$$c^* = \frac{1 - E - \theta f^*}{1 - \theta f^*}$$

whereas each illiquid bank holds zero cash. At date 1, two illiquid banks are closed if the low state is realized. Their long-term assets are sold to the liquid bank at a fire-sale price:

$$p^* = \theta f^* < \theta y_L$$

Proof. Appendix □

The construction of this equilibrium is similar to that of two equilibria characterized in Proposition 3. We first determine the amount of cash each bank holds in the equilibrium. Then we establish the conditions under which no bank has incentives to deviate. As shown in Part 2(a) of Proposition 2, when two other banks choose to be illiquid, the bank who chooses to be liquid will hold $\frac{1-E-\theta f^*}{1-\theta f^*}$ as cash. Concerning the banks that choose to be illiquid, their cash holdings are determined by Program $\wp^{illi\_tr}$. Thanks to the observation that the liquid bank holds an amount of cash that is just sufficient to overcome the liquidity shock, we know that the liquidity available in the market for asset purchases will be the one raised against the assets to be acquired. As a result, the asset price equals θf^*, which implies that illiquid banks must sell their entire investment. Replace $p = \theta f^*$ and $\beta = 1$ in the program $\wp^{illi\_tr}$, we see that the amount of cash illiquid banks carry on their balance sheet is zero.

With regard to the no-deviation conditions, we could prove that if the liquid bank i deviates, its expected profit is

$$\Pi_i^{de} = \alpha\theta y_H + (1-\alpha)\ell - 1 + E$$

For the illiquid banks j and k, their expected profit in case of deviation is, for $m = j, k$

$$\begin{aligned} \Pi_m^{de} &= \alpha\theta y_H + (1-\alpha)\theta y_L - 1 + E_m - \frac{1 - E_m - \theta f^*}{1 - \theta f^*}(\alpha\theta y_H + (1-\alpha)\theta y_L - 1) \\ &\quad + \frac{1}{2}(1-\alpha)(\theta y_L - \theta f^*) \end{aligned}$$

To guarantee that no banks deviate, the banks' leverage must satisfy the two following conditions:

$$E_i \geq E_5$$

and

$$E_j \leq E_4 \text{ and } E_k \leq E_4$$

The final pure strategy equilibrium we would like to characterize is the one where two banks choose to be liquid and one bank chooses to be illiquid. There exists two equilibria of this type.

Proposition 5. *In a model with three banks, there exists two equilibria where two banks choose to be liquid and one bank chooses to be illiquid. They differ in the amount of cash held by liquid banks.*

(a) *In one equilibrium, at date* 0*, each liquid bank holds an amount of cash:*

$$c^* = \frac{1 - E - \theta f^* + \frac{\delta}{4}}{1 - \theta f^*}$$

and the illiquid bank holds zero cash. At date 1*, the illiquid bank is closed if the low state is realized. Its long-term assets are sold by its debtholders to two liquid banks at a fire-sale price:*

$$p^* = \theta f^* + \frac{\delta}{2} < \theta y_L$$

This equilibrium exists if and only if in the banking system, two banks have capital ratio greater than E_3 while the other bank has capital ratio lower than E_2.

(b) *In the other equilibrium, at date* 0*, each liquid bank holds an amount of cash:*

$$c^* = \frac{1 - E - \theta f^*}{1 - \theta f^*}$$

and the illiquid bank holds zero cash. At date 1*, the illiquid bank is closed if the low state is realized. Its long-term assets are sold by its debtholders to two liquid banks at a fire-sale price:*

$$p^* = \theta f^* < \theta y_L$$

This equilibrium exists if and only if in the banking system, two banks have capital ratio greater than E_4 while the other bank has capital ratio lower than E_1.

Proof. Appendix □

To construct those equilibria, we start with the observation that an equilibrium where $p = \theta y_L$ cannot exist. Indeed, if a liquid bank expects that $p = \theta y_L$, it will hold an amount of cash that is just sufficient to overcome the liquidity shock, i.e. $\rho = \theta f^*$, which implies that $(\theta f^* - \rho)(1 - c) = 0$. Accordingly, the excess demand condition (17) cannot be satisfied, or put differently no equilibrium where $p = \theta y_L$ exists. Therefore, in the equilibrium where two banks are liquid and one bank is illiquid, the equilibrium price should be represented by (16). Due to Part 2(b) of Proposition 2, we know that the optimal cash holdings of two banks i and j who choose to be liquid in the equilibrium are determined by the following system:

$$\begin{cases} \varepsilon_i = \max\left(0, \sqrt{\delta(1 - c_k)\varepsilon_j} - \varepsilon_j\right) \\ \varepsilon_j = \max\left(0, \sqrt{\delta(1 - c_k)\varepsilon_i} - \varepsilon_i\right) \end{cases} \tag{33}$$

where ε_m, the excess liquidity held by bank $m, m = i, j$, is defined by:

$$\varepsilon_m = (\theta f^* - \rho_m)(1 - c_m)$$

Note that System (33) has two solutions:

$$\varepsilon_i = \varepsilon_j = 0 \text{ or } \varepsilon_i = \varepsilon_j = \frac{\delta(1 - c_k)}{4}$$

As the illiquid bank k will choose to hold zero cash, i.e. $c_k = 0$, we obtain that in the equilibrium, two liquid banks either hold:

$$c_i^* = c_j^* = \frac{1 - E - \theta f^* + \frac{\delta}{4}}{1 - \theta f^*}$$

or hold:

$$c_i^* = c_j^* = \frac{1 - E - \theta f^*}{1 - \theta f^*}$$

Regarding the no-deviation conditions, we find that for the first equilibrium, liquid banks m, $m = i, j$, never deviate if and only if:

$$\begin{aligned} & E_m + \frac{E_m - \frac{\delta}{4}}{1 - \theta f^*}(\alpha\theta y_H + (1 - \alpha)\theta y_L - 1) + \frac{1 - \alpha}{2}\left(\theta y_L - \theta f^* - \frac{\delta}{2}\right) \\ \geq \;\; & \alpha\theta y_H + (1 - \alpha)\theta f^* - 1 + E_m + (1 - \alpha)\frac{\delta}{8} \end{aligned}$$

which is equivalent to

$$E_m \geq E_3 \text{ for } m = i, j$$

As of illiquid bank k, to ensure that it will not deviate, the following condition must be satisfied:

$$\alpha\theta y_H + (1 - \alpha)\theta f^* - 1 + E_k + (1 - \alpha)\frac{\delta}{2} \geq E_k + \frac{E_k + \frac{\delta}{2}}{1 - \theta f^*}(\alpha\theta y_H + (1 - \alpha)\theta y_L - 1)$$

which yields

$$E_k \leq E_2$$

For the second equilibrium, equivalent conditions are respectively

$$\begin{aligned} & E_m + \frac{E_m}{1 - \theta f^*}(\alpha\theta y_H + (1 - \alpha)\theta y_L - 1) + (1 - \alpha)\frac{1}{2}(\theta y_L - \theta f^*) \\ \geq \;\; & \alpha\theta y_H + (1 - \alpha)\theta f^* - 1 + E_m \end{aligned}$$

and

$$\alpha\theta y_H + (1 - \alpha)\theta f^* - 1 + E_k \geq E_k + \frac{E_k}{1 - \theta f^*}(\alpha\theta y_H + (1 - \alpha)\theta y_L - 1)$$

After simplifications, we get

$$E_m \geq E_4 \text{ for } m = i, j$$

and

$$E_k \leq E_1$$

Figure 4 summarises the equilibria in these above propositions. Notice that the condition on banks' leverage characterized in these propositions cover all possible distribution of leverage in a banking system consisting of three banks. In other words, any possible

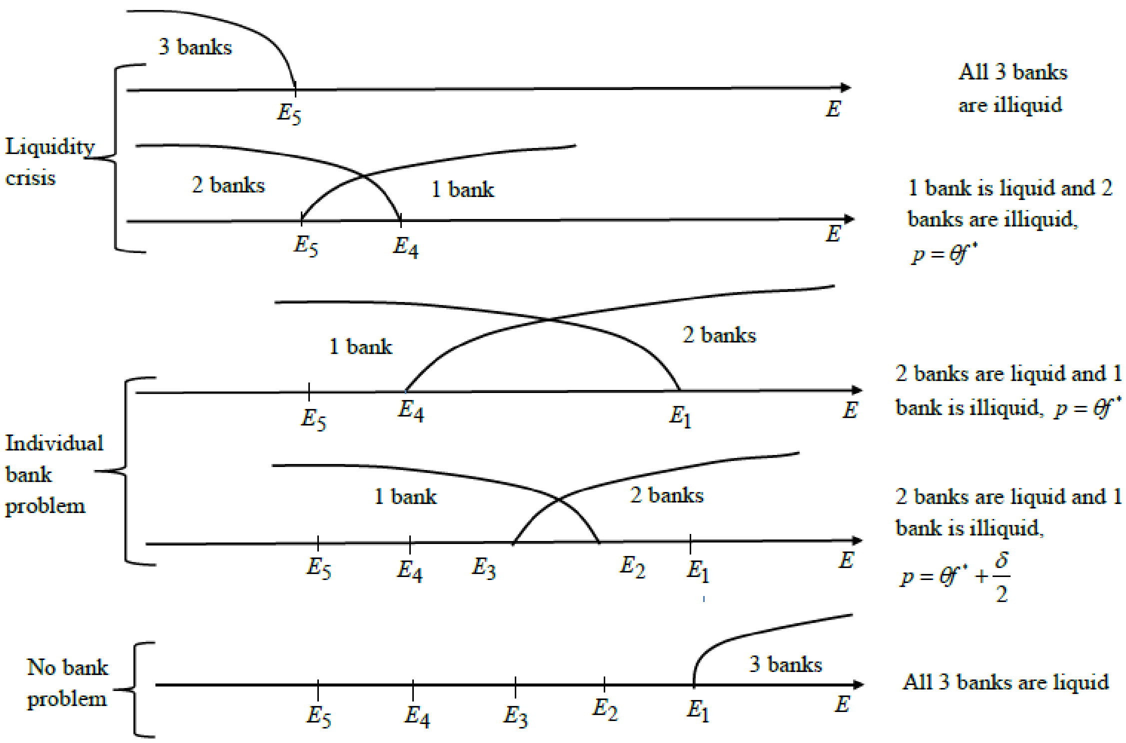


Figure 4: Summary of Equilibria

distribution of leverage in a system of three banks corresponds to one of the 4 above-mentioned pure strategy equilibria that differ in the number of banks that are in trouble following a liquidity shock[7]. The most interesting observation derived from Figure 4 is that a more leveraged banking system is more vulnerable to a liquidity shock. Especially, a system composed of banks that operate with very high leverage could entirely collapse following the materialization of a liquidity shock. If we define a liquidity crisis being the situation where a large number of banks are closed as a consequence of a liquidity shock, we could interpret the two equilibria characterized in Part (b) of Proposition 3 and in Proposition 4 as the cases where a liquidity crisis occurs. The equilibria characterized in Proposition 5 can be seen as the case where there is individual bank problem in the system. A totally healthy banking system corresponds to the equilibrium described in Part (a) of Proposition 3.

5.6 Discussion

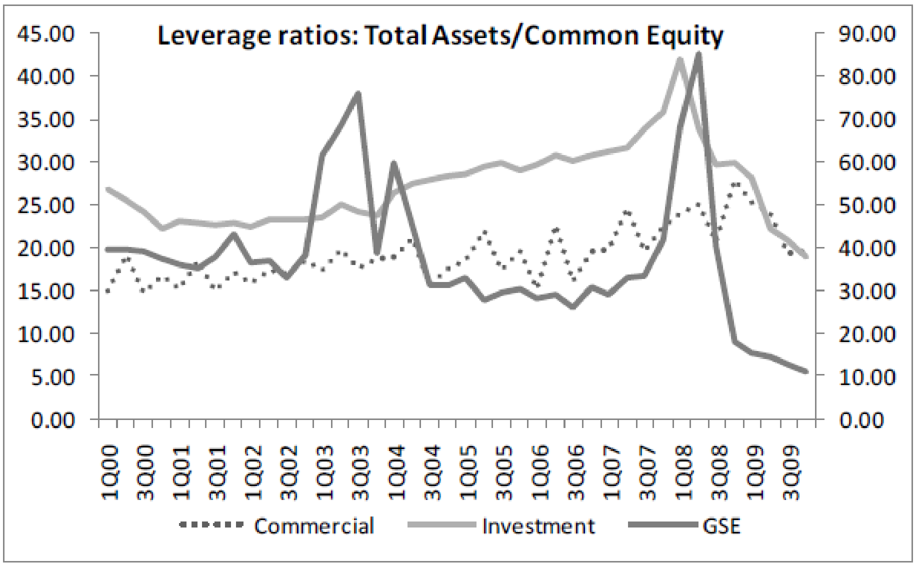
In this section, we first discuss the analogy of our predicted pattern of liquidity crises with the recent financial crisis of 2007-2009. Then we discuss the role of public liquidity support.

A. *Leverage and Liquidity Crises*

[7]In the situation characterized in Part (a) of Proposition 5, there exists 2 equilibria. However, in both equilibria, the number of banks that fail is the same.

Although the present setup is highly stylized and abstracts from many institutional aspects of financial system, it does provide interesting insights into the unfolding of the recent crisis. As noted above, our model predicts that a banking system where banks are highly leveraged can be prone to liquidity crises. The pattern of the crises is typical: High leverage results in low ex-ante liquidity holdings of banks. Then, when a liquidity shock is realized, many banks have trouble in honoring their debt obligations and the banks' assets are sold only at fire-sale prices, which causes the failure of banks that are in need of liquidity. This pattern is consistent with what was observed during the 2007-2009 crisis.

Figure 6b – Leverage Ratios – Total Assets/Common Equity for large US financial firms



Source: Balance Sheets of all banks, from Bloomberg, SEC filings, Annual reports
Note: Commercial/Investment Bank Ratios are on the primary axis and GSE ratios on the secondary axis.

Figure 5: Evidence on banks' leverage (Source: Figure 6b in Acharya et al. 2012

Indeed, many analyses show that during the period running up to the crisis, the banking system was characterized by both high leverage and low liquidity holdings. Acharya et al. (2012) report that in the period of 2000 - 2007, bank capital structures were looking increasingly levered. Figure 5 that is taken from their study represents the leverage ratios of the 25 large US financial firms. We observe that the total asset/common equity ratio of both commercial and investment banks increased significantly from 2000 to 2007. Precisely, for commercial banks, it grew from 15.0 in the first quarter of 2000 to 22.51 in the fourth quarter of 2007 whereas for investment banks, it raised from 26.90 to 35.85. Furthermore, Acharya et al. (2012) also pointed out that the increase in the debt financing was primarily realized via the use of short-term debt, which is compatible with our assumption about the dominance of short-term component in banks' leverage.

Regarding the banks' liquidity holdings, Figure 6, taken from Berrospide 2013, show that from 2005 to 2008, the liquid assets/total assets ratio for U.S. commercial banks decreases constantly from around 23% to around 18%. This combination of high short-term

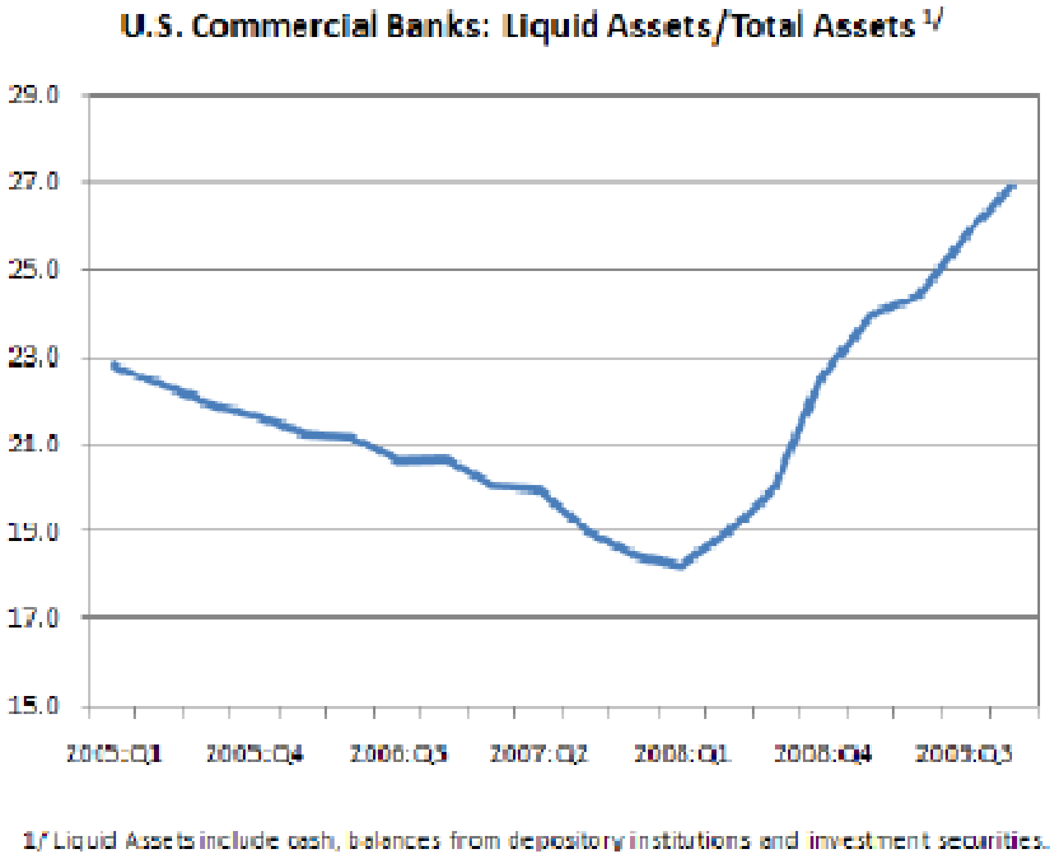


Figure 6: Evidence on U.S. commercial banks' liquidity holdings (source: Figure 1 in Berrospide 2013

leverage and low liquid asset holdings were source of many serious liquidity problem experienced by banks following the revelation of the increase in subprime mortgage defaults. Many banks were unable to roll-over their asset-backed commercial papers. The market prices of various banks' assets were significantly below what plausible fundamentals would suggest they should be.

B. *Public Liquidity Support*

This paper highlights that highly leveraged banks have little incentives to prudently manage their liquidity risk, which make them vulnerable to shocks that tend to cripple their funding capacity. Can public liquidity support be solution in these situations? We have to distinguish between two aspects: one is its ability to avoid the liquidity crises and the second is its impact on the banks' ex-ante incentives to manage their liquidity risk.

Note first that public intervention via lender of last resort cannot help to solve the liquidity problem banks face at date 1. Since banks' borrowing capacity is saturated, lending more to the banks will violate the incentive compatibility condition, which induces banks to switch to bad assets. Other forms of public support such as providing liquidity in exchange of ownership or by acquiring the banks' long-term assets can prevent the liquidity crises from happening in this simple setup. However, in more elaborate setups where there is uncertainty about the banks' solvency and/or adverse selection problem concerning the banks' assets., the design of public intervention is a complex issue. Moreover, if banks anticipate that they will receive liquidity support at date 1, their incentives to hold liquid assets to self-insure at date 0 will be destroyed. In other words, although public liquidity support may, in some situations, help to mitigate the liquidity problem, it is detrimental to the banks' ex-ante incentives.

6 Conclusion

This paper develops a model of banks' liquidity management that endogenizes the magnitude of the banks' exposure to the liquidity shock. The model helps to clarify the linkage between the banks' leverage and their incentives to manage their liquidity. We find that an individual bank tends to prudently manage its liquidity risk only if it is well capitalized. We also show that a banking system that is composed of highly leveraged bank can be prone to liquidity crises. In our model, the crises have pattern that is analogous with the recent crisis.

We believe that the present framework provides a useful workhorse for future research that helps to deepen our understanding of the impact of banks' leverage on their incentives for liquidity management. One promising extension would be to endogenise the banks' choice of leverage. This would allow to analyse the effects of banks' capital on their effort in reducing the likelihood of a liquidity shock. Another interesting extension would be to take into account the role of long-term debt and ask whether holding liquid assets and funding by long-term debt are perfect substitute from a liquidity risk perspective.

Appendix

A Proof of Proposition 2

Part 1: From Program $\wp^{li\_ntr}$,it is evident that $c_i^{li\_ntr} = \frac{1-E_i-\theta f^*}{1-\theta f^*}$.

Part 2: If $p = \theta y_L$, we have $TP_i = 0$, which clearly impliest that $c_i^{li\_tr} = \frac{1-E_i-\theta f^*}{1-\theta f^*}$. When p is strictly less than θy_L, we just need to compute the first derivative of TP_i with respect to c_i for different representations of the equilibrium price p given in Section 5.3.

(+) If $p = \theta f^* + \frac{1-c_i}{2-c_j-c_k}(\theta f^* - \rho_i)$, we have:

$$\frac{dp}{dc_i} = \frac{1-\theta f^*}{2-c_j-c_k}$$

Hence, we get:

$$\frac{dTP_i}{dc_i} = (\theta y_L - p)\frac{1-\theta f^*}{p-\theta f^*} - \gamma_i\left[\frac{\theta y_L - p}{p-\theta f^*} + 1\right]\frac{1-\theta f^*}{2-c_j-c_k}$$

or

$$\frac{dTP_i}{dc_i} = (\theta y_L - p)\frac{1-\theta f^*}{p-\theta f^*} - \gamma_i\frac{\theta y_L - \theta f^*}{p-\theta f^*}\frac{1-\theta f^*}{2-c_j-c_k} \tag{34}$$

Note that when $p = \theta f^* + \frac{1-c_i}{2-c_j-c_k}(\theta f^* - \rho_i)$, we can compute γ_i as follows:

$$\gamma_i = \frac{(1-c_i)(\theta f^* - \rho_i)}{p - \theta f^*} = 2 - c_j - c_k \tag{35}$$

Substitue (35) into (34), we get:

$$\frac{dTP_i}{dc_i} = (\theta y_L - p)\frac{1-\theta f^*}{p-\theta f^*} - (2-c_j-c_k)\frac{\theta y_L - \theta f^*}{p - \theta f^*}\frac{1-\theta f^*}{2-c_j-c_k}$$

or

$$\frac{dTP_i}{dc_i} = -(1-\theta f^*) < 0 \tag{36}$$

(+) If $p = \theta f^* + (\theta f^* - \rho_i)\frac{1-c_i}{1-c_k} - (\rho_j - \theta f^*)\frac{1-c_j}{1-c_k}$, we obtain:

$$\frac{dp}{dc_i} = \frac{1-\theta f^*}{1-c_k}$$

Therefore,

$$\frac{dTP_i}{dc_i} = (\theta y_L - p)\frac{1-\theta f^*}{p-\theta f^*} - \gamma_i\left[\frac{\theta y_L - p}{p-\theta f^*} + 1\right]\frac{1-\theta f^*}{1-c_k}$$

After some similar arrangements as above, we get:

$$\frac{dTP_i}{dc_i} = -(1-\theta f^*) - \frac{1-\theta f^*}{p-\theta f^*}(\theta y_L - \theta f^*)\frac{(\rho_j - \theta f^*)(1-c_j)}{(\theta f^* - \rho_i)(1-c_i) - (\rho_j - \theta f^*)(1-c_j)} < 0$$

(+) If $p = \theta f^* + (\theta f^* - \rho_i)\frac{1-c_i}{1-c_k} + (\theta f^* - \rho_j)\frac{1-c_j}{1-c_k}$, similarly to the previous cases, after some computations, we obtain:

$$\frac{dTP_i}{dc_i} = -(1-\theta f^*) + (1-\theta f^*)\frac{\theta y_L - \theta f^*}{p-\theta f^*}\frac{(\theta f^* - \rho_j)(1-c_j)}{(\theta f^* - \rho_i)(1-c_i) + (\theta f^* - \rho_j)(1-c_j)}$$

Hence, the FOC (31) implies that:

$$\begin{aligned} & (1-\alpha)(1-\theta f^*)\frac{\theta y_L - \theta f^*}{p-\theta f^*}\frac{(\theta f^* - \rho_j)(1-c_j)}{(\theta f^* - \rho_i)(1-c_i) + (\theta f^* - \rho_j)(1-c_j)} \\ \leq \quad & \alpha\theta y_H + (1-\alpha)\theta y_L - 1 + (1-\alpha)(1-\theta f^*) \end{aligned}$$

or

$$\delta\frac{(\theta f^* - \rho_j)(1-c_j)(1-c_k)}{\left[(\theta f^* - \rho_i)(1-c_i) + (\theta f^* - \rho_j)(1-c_j)\right]^2} \leq 1$$

which is equivalent to:

$$(\theta f^* - \rho_i)(1-c_i) \geq \sqrt{\delta(1-c_k)\varepsilon_j} - \varepsilon_j \tag{37}$$

Therefore, if $\sqrt{\delta(1-c_k)\varepsilon_j} - \varepsilon_j \geq 0$, we have:

$$c_i = \frac{1-E_i-\theta f^*}{1-\theta f^*} + \frac{\sqrt{\delta(1-c_k)\varepsilon_j} - \varepsilon_j}{1-\theta f^*}$$

Otherwise, the liquidity constraint is binding, i.e.

$$c_i = \frac{1-E_i-\theta f^*}{1-\theta f^*}$$

B Proof of Proposition 3

Part (a): As already shown in Proposition 2, given that two other banks choose to be liquid, if a bank i chooses to be liquid, it will hold $c_i = \frac{1-E_i-\theta f^*}{1-\theta f^*}$. Now we have to check that no bank has incentives to deviate. Clearly, given that other banks are liquid, no bank has incentives to deviate by holding more than $\frac{1-E_i-\theta f^*}{1-\theta f^*}$. Under which conditions they will not deviate by holding less? If a bank i deviates by choosing to be illiquid, its expected profit is as follows[8]:

$$\Pi_i^{de} = \Pi_i^{illi\_tr} = \underset{0 \leq c_i \leq 1}{Max} \left\{ \begin{array}{c} \alpha\theta y_H + (1-\alpha)\theta y_L - 1 + E_i - c_i\left(\alpha\theta y_H + (1-\alpha)\theta y_L - 1\right) \\ -(1-\alpha)(1-c_i)\beta_i(\theta y_L - p) \end{array} \right\}$$

subject to

$$c_i < \frac{1-E_i-\theta f^*}{1-\theta f^*}$$

Since two other banks hold an amount of cash that is just sufficient to overcome the liquidity shock, their spare debt capacity from existing assets is zero. Hence, the liquidity available for asset purchase comes from the liquidity raised against assets to be acquired, which implies that $p = \theta f^*$. Therefore, the bank i's expected profit in case of deviation is:

$$\Pi_i^{de} = \alpha\theta y_H + (1-\alpha)\theta f^* - 1 + E_i$$

Bank i will not deviate as long as the following condition is satisfied:

$$\alpha\theta y_H + (1-\alpha)\theta y_L - 1 + E_i - \frac{1-E_i-\theta f^*}{1-\theta f^*}\left(\alpha\theta y_H + (1-\alpha)\theta y_L - 1\right) \geq \alpha\theta y_H + (1-\alpha)\theta f^* - 1 + E_i$$

which is equivalent to

$$E_i \geq E_1$$

Part (b): In the equilibrium, the expected profit for each bank i is

$$\Pi_i^{illi\_ntr} = \alpha\theta y_H + (1-\alpha)\ell - 1 + E_i$$

If a bank i deviates by choosing to be liquid, its expected profit is computed as follows:

$$\Pi_i^{de} = \Pi_i^{li\_tr} = \underset{c_i}{Max} \left\{ \begin{array}{c} \alpha\theta y_H + (1-\alpha)\theta y_L - 1 + E_i - c_i\left(\alpha\theta y_H + (1-\alpha)\theta y_L - 1\right) \\ +(1-\alpha)\gamma_i(\theta y_L - p) \end{array} \right\} \tag{38}$$

subject to

$$c_i \geq \frac{1-E_i-\theta f^*}{1-\theta f^*} \tag{39}$$

As shown in Part (2) of Proposition 2, bank i will hold $c_i^{de} = \frac{1-E_i-\theta f^*}{1-\theta f^*}$, which implies $p = \theta f^*$. Hence,

$$\begin{aligned} \Pi_i^{de} &= \alpha\theta y_H + (1-\alpha)\theta y_L - 1 + E_i - \frac{1-E_i-\theta f^*}{1-\theta f^*}\left(\alpha\theta y_H + (1-\alpha)\theta y_L - 1\right) \\ &\quad + (1-\alpha)2(\theta y_L - \theta f^*) \end{aligned}$$

[8]The superscript "de" refers to deviation.

The condition for no deviation is the following

$$\alpha\theta y_H + (1-\alpha)\,\ell - 1 + E_i \geq \Pi_i^{de}$$

which yields:

$$E_i \leq E_5$$

C Proof of Proposition 4

We first determine the optimal amount of cash each bank holds in the equilibrium. Then, we characterize the conditions under which no bank has incentives to deviate.

- *Equilibrium cash holdings of the liquid bank:* As shown in part 2(a) of Proposition 2, if two other banks choose to be illiquid, a bank i who chooses to be liquid will hold $\frac{1-E_i-\theta f^*}{1-\theta f^*}$ as cash.

- *Equilibrium cash holdings of illiquid banks:* Consider a bank j who chooses to be illiquid in the equilibrium. Its cash holdings are determined by Program $\wp^{illi\_tr}$:

  $$\Pi_j^{illi\_tr} = \underset{c_j\in[0,1]}{Max}\left\{\begin{array}{c}\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1 + E_j - c_j\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right)\\ -(1-\alpha)\,(1-c_j)\,\beta_j\,(\theta y_L - p)\end{array}\right\}$$

  subject to

  $$c_j < \frac{1-E_j-\theta f^*}{1-\theta f^*}$$

  Since the liquid bank i just holds a minimum amount of cash, i.e. $\theta f^* - \rho_i = 0$, the liquidity available to buy assets comes from the liquidity raised against assets to be acquired. As a consequence, the equilibrium price equals θf^*, which implies that if bank j chooses to be illiquid, it will be closed at date 1. The program that determines its cash holdings can be rewritten as follows:

  $$\Pi_j^{illi\_tr} = \underset{c_j\in[0,1]}{Max}\left\{\begin{array}{c}\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1 + E_j - c_j\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right)\\ -(1-\alpha)\,(1-c_j)\,(\theta y_L - \theta f^*)\end{array}\right\}$$

  subject to

  $$c_j < \frac{1-E_j-\theta f^*}{1-\theta f^*}$$

  As the objective function is decreasing with c_j, in the equilibrium the illiquid banks hold zero cash.

To summarize, in the equilibrium, the liquid bank i holds an amount of cash equal to $\frac{1-E_i-\theta f^*}{1-\theta f^*}$ while two illiquid banks j and k hold zero cash. At date 1, if the low state is realized, two illiquid banks j and k sell all of their investment to bank i at price equal to θf^*. Bank i thus acquires two units of long-term assets. Its expected profit in the equilibrium is as follows:

$$\begin{aligned}\Pi_i^{li\_tr} &= \alpha\theta y_H + (1-\alpha)\,\theta y_L - 1 + E_i - \frac{1-E_i-\theta f^*}{1-\theta f^*}\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right)\\ &\quad +2\,(1-\alpha)\,(\theta y_L - \theta f^*)\end{aligned}$$

The expected profit of each illiquid bank in the equilibrium is:

$$\Pi_j^{illi\_tr} = \alpha\theta y_H + (1-\alpha)\,\theta f^* - 1 + E_j$$

and

$$\Pi_k^{illi\_tr} = \alpha\theta y_H + (1-\alpha)\,\theta f^* - 1 + E_k$$

- *No-deviation condition for the liquid bank i*: If bank i deviates by choosing to be illiquid, since two other banks are also illiquid, bank i's expected profit is defined by Program $\wp^{illi\_ntr}$. Clearly, if deviating, bank i will hold zero cash and has expected profit equal to:

  $$\Pi_i^{de} = \alpha\theta y_H + (1-\alpha)\,\ell - 1 + E_i$$

  Hence, bank i will never deviate if and only if the following condition is satisfied:

  $$\Pi_i^{li\_tr} \geq \Pi_i^{de}$$

  or equivalently

  $$E_i \geq E_5$$

- *No-deviation condition for illiquid banks j and k*: Consider for instance bank j: If it deviates by choosing to be liquid, its cash holdings are determined by Program $\wp^{li\_tr}$:

  $$\Pi_j^{li\_tr} = \underset{c_j\in[0,1]}{Max}\left\{\begin{array}{c}\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1 + E_j - c_j\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right)\\ +(1-\alpha)\,\gamma_j\left(\theta y_L - p\right)\end{array}\right\} \tag{40}$$

  subject to

  $$c_j \geq \frac{1-E_j-\theta f^*}{1-\theta f^*}$$

  Since the other liquid bank, bank i, just holds a minimum amount of cash, from the price representations (16) and (17), we have:

  $$p = \left\{\begin{array}{ll}\theta f^* + (1-c_j)\,(\theta f^* - \rho_j) & \text{if}\quad (1-c_j)\,(\theta f^*-\rho_j) < 1 - E_k - \theta f^*\\ \theta y_L & \text{if}\quad (1-c_j)\,(\theta f^*-\rho_j) \geq 1 - E_k - \theta f^*\end{array}\right.$$

  Clearly, bank j should never choose c_j such that $p = \theta y_L$. If $p = \theta f^* + (1-c_j)\,(\theta f^* - \rho_j)$, we compute the first derivative of the objective function in (40) as follows:

  $$\begin{aligned}\frac{\partial}{\partial c_j} &= -\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right) + \frac{(1-\alpha)\,(1-\theta f^*)\,(\theta y_L - p)}{p-\theta f^*}\\ &\quad -\frac{(1-\alpha)\,\gamma_j\,(1-\theta f^*)\,(\theta y_L - p)}{(p-\theta f^*)^2} - (1-\alpha)\,\gamma_j\,(1-\theta f^*)\end{aligned}$$

  After simplifications, we get

  $$\frac{\partial}{\partial c_j} = -\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right) - (1-\alpha)\,(1-\theta f^*) < 0$$

Thus, if deviating, bank j will hold $c_j = \frac{1-E_j-\theta f^*}{1-\theta f^*}$, which means $\theta f^* - \rho_j = 0$. Consequently, $p = \theta f^*$, $\gamma_j = \frac{1}{2}$ and the bank j's expected profit in case of deviation is:

$$\begin{aligned}\Pi_j^{de} &= \alpha\theta y_H + (1-\alpha)\theta y_L - 1 + E_j - \frac{1-E_j-\theta f^*}{1-\theta f^*}(\alpha\theta y_H + (1-\alpha)\theta y_L - 1) \\ &+ \frac{1}{2}(1-\alpha)(\theta y_L - \theta f^*)\end{aligned}$$

Therefore, the no-deviation condition for bank j is as follows:

$$\Pi_j^{illi\_tr} \geq \Pi_j^{de}$$

or

$$E_j \leq E_4$$

D Proof of Proposition 5

As above, the proof proceeds in two steps. First we determine the amount of cash each bank holds in the equilibrium. Then we characterize the condition under which no bank has incentives to deviate.

- *Equilibrium cash holdings of liquid banks*: Consider two banks i and j who choose to be liquid in the equilibrium. Since no equilibrium where $p = \theta y_L$ exists, as shown in Section 5.3, the equilibrium price is represented as follows:

$$p = \theta f^* + (\theta f^* - \rho_i)\frac{1-c_i}{1-c_k} + (\theta f^* - \rho_j)\frac{1-c_j}{1-c_k}$$

As established in Inequality (37), the cash holdings of bank i are determined as follows:

$$\varepsilon_i = \max\left(0, \sqrt{\delta(1-c_k)\varepsilon_j} - \varepsilon_j\right) \tag{41}$$

The same argument is applied to bank j, which means:

$$\varepsilon_j = \max(0, \sqrt{\delta(1-c_k)\varepsilon_i} - \varepsilon_i) \tag{42}$$

The system of two equations (41) to (42) has two solutions:

$$\varepsilon_i = \varepsilon_j = \frac{\delta(1-c_k)}{4}$$

or

$$\varepsilon_i = \varepsilon_j = 0$$

which implies:

$$c_i^* = c_j^* = \frac{1-E-\theta f^* + \frac{\delta}{4}}{1-\theta f^*}$$

or

$$c_i^* = c_j^* = \frac{1-E-\theta f^*}{1-\theta f^*}$$

- *Equilibrium cash holdings of illiquid bank*: Consider bank k who chooses to be illiquid in the equilibrium, its equilibrium cash holdings are determined by Program $\wp^{illi\_tr}$. As already noted, an equilibrium where $p = \theta y_L$ cannot exist. Hence, in the equilibrium, $\beta_k = 1$ and the bank k's problem becomes

$$\underset{c_k}{Max} \left\{ \begin{array}{c} \alpha\theta y_H + (1-\alpha)\,\theta y_L - 1 + E_k - c_k\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right) \\ -(1-\alpha)(1-c_k)(\theta y_L - p) \end{array} \right\} \tag{43}$$

subject to

$$c_i < \frac{1 - E_i - \theta f^*}{1 - \theta f^*}$$
$$\frac{\frac{1-E_k-(1-\alpha)(c_k+(1-c_k)p)}{\alpha} - c_k}{1 - c_k} > p$$

and

$$p = \theta f^* + \frac{\varepsilon_i + \varepsilon_j}{1 - c_k}$$

It is easily to check that the objective function of the above problem is decreasing with c_k. Hence, in the equilibrium, illiquid bank k holds zero cash.

In summary, there are two equilibria. In one equilibrium, each liquid bank i and j hold an amount of cash equal to $\frac{1-E-\theta f^*+\frac{\delta}{4}}{1-\theta f^*}$ and illiquid bank k holds zero cash. At date 1, if the low state is realized, bank k sells all of its investment to two liquid banks at price equal to $\theta f^* + \frac{\delta}{2}$. The expected profit of each bank is as follows:

$$\Pi_m^{li} = E_m + \frac{E_m - \frac{\delta}{4}}{1 - \theta f^*}\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right) + (1-\alpha)\,\frac{1}{2}\left(\theta y_L - \theta f^* - \frac{\delta}{2}\right) \text{ for } m = i, j$$

and

$$\Pi_k^{illi} = \alpha\theta y_H + (1-\alpha)\,\theta f^* - 1 + E_k + (1-\alpha)\,\frac{\delta}{2}$$

In the other equilibrium, each liquid bank i and j hold an amount of cash equal to $\frac{1-E-\theta f^*}{1-\theta f^*}$ and illiquid bank k holds zero cash. At date 1, if the low state is realized, bank k sells all of its investment to two liquid banks at price equal to θf^*. The expected profit of each bank is as follows:

$$\Pi_m^{li} = E_m + \frac{E_m}{1 - \theta f^*}\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right) + (1-\alpha)\,\frac{1}{2}\left(\theta y_L - \theta f^*\right) \text{ for } m = i, j$$

and

$$\Pi_k^{illi} = \alpha\theta y_H + (1-\alpha)\,\theta f^* - 1 + E_k$$

Now we turn to characterize the no-deviation conditions. First, we determine these conditions for the equilibrium described in Part (a) of the proposition.

- *Part (a): No-deviation condition for liquid banks:* Consider the liquid bank i. If it deviates by choosing to be illiquid, its expected profit is computed as follows:

$$\Pi_i^{de} = \underset{c_i\in[0,1]}{Max} \left\{ \begin{array}{c} \alpha\theta y_H + (1-\alpha)\,\theta y_L - 1 + E_i - c_i\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right) \\ -(1-\alpha)(1-c_i)\,\beta_i\,(\theta y_L - p) \end{array} \right\}$$

subject to

$$c_i < \frac{1 - E_i - \theta f^*}{1 - \theta f^*}$$

There are two possibilities: $\beta_i = 1$ or $\beta_i < 1$. If $\beta_i < 1$, the above program becomes

$$\Pi_i^{de} = \underset{c_i \in [0,1]}{Max} \left\{ \begin{array}{c} \alpha\theta y_H + (1-\alpha)\,\theta y_L - 1 + E_i - c_i\,(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1) \\ -(1-\alpha)\,(1-c_i)\,\beta_i\,(\theta y_L - p) \end{array} \right\}$$

subject to

$$c_i < \frac{1 - E_i - \theta f^*}{1 - \theta f^*}$$
$$\frac{1 - E_i - c_i}{1 - c_i} < p$$
$$p = \theta f^* + \frac{\delta}{4} - (\rho_i - \theta f^*)\,(1 - c_i)$$

By computing the FOC, we find that the objective function is increasing with c_i, which implies that Π_i^{de}, determined as above, is always lower than the bank i' expected profit in the equilibrium. If $\beta_i = 1$, the program determining the bank i's expected profit in case of deviation can be rewritten as follows:

$$\Pi_i^{de} = \underset{c_i \in [0,1]}{Max} \left\{ \begin{array}{c} \alpha\theta y_H + (1-\alpha)\,\theta y_L - 1 + E_i - c_i\,(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1) \\ -(1-\alpha)\,(1-c_i)\,(\theta y_L - p) \end{array} \right\}$$

subject to

$$c_i < \frac{1 - E_i - \theta f^*}{1 - \theta f^*}$$
$$\frac{1 - E_i - (1-\alpha)\,(c_i + (1-c_i)p)}{\alpha} - c_i > (1 - c_i)\,p$$

and

$$p = \theta f^* + \frac{\frac{\delta}{4}}{2 - c_i}$$

It is easily to check that the objective function is decreasing with c_i, which implies that:

$$\Pi_i^{de} = \alpha\theta y_H\,(1-\alpha)\,\theta f^* - 1 + E_i + (1-\alpha)\,\frac{\delta}{8}$$

Hence, bank i will not deviate if and only if

$$\begin{aligned} & E_i + \frac{E_i - \frac{\delta}{4}}{1 - \theta f^*}\,(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1) + (1-\alpha)\,\frac{1}{2}\left(\theta y_L - \theta f^* - \frac{\delta}{2}\right) \\ \geq\ & \alpha\theta y_H\,(1-\alpha)\,\theta f^* - 1 + E_i + (1-\alpha)\,\frac{\delta}{8} \end{aligned}$$

which yieds

$$E_i > E_3$$

The proof is similar to bank j, i.e.

$$E_j \geq E_3$$

- *Part (a): No-deviation condition for illiquid bank:* Note that illiquid bank k can deviate in two ways: the first option is that it deviates by choosing to be liquid. It can also deviate by holding more cash so that it is still illiquid but can survive the liquidity shock.

  - If bank k deviates by choosing to be liquid: given that two other banks are liquid, it is easily to see that bank k will hold an amount of cash equal to $\frac{1-E_i-\theta f^*}{1-\theta f^*}$ and its expected profit is as follows:

$$\Pi_k^{de} = E_k + \frac{E_k}{1-\theta f^*}\left(\alpha\theta y_H + (1-\alpha)\theta y_L - 1\right) \tag{44}$$

  - If bank k deviates by holding more cash so that it is still illiquid but can survice the liquidity shock: the cash holdings of bank k are determined by the following program:

$$\Pi_k^{de} = \underset{c_k}{Max}\left\{\begin{array}{c} \alpha\theta y_H + (1-\alpha)\theta y_L - 1 + E_k - c_k\left(\alpha\theta y_H + (1-\alpha)\theta y_L - 1\right) \\ -(1-\alpha)(1-c_k)\beta_k(\theta y_L - p) \end{array}\right\}$$

    subject to

$$c_k < \frac{1-E_k-\theta f^*}{1-\theta f^*} \tag{45}$$

$$p = \theta y_L$$

$$\frac{\frac{\delta}{2}}{p-\theta f^*} \geq \frac{(\rho_k - \theta f^*)(1-c_k)}{p-\theta f^*} \tag{46}$$

    Constraints (45) - (46) imply:

$$\frac{1-E_k-\theta f^*}{1-\theta f^*} > c_k \geq \frac{1-E_k-\theta f^*-\frac{\delta}{2}}{1-\theta f^*}$$

    Since the objective function is decreasing with c_k, we obtain that bank k will hold $\frac{1-E_k-\theta f^*-\frac{\delta}{2}}{1-\theta f^*}$ as cash and its expected profit is equal to:

$$\Pi_k^{de} = E_k + \frac{E_k+\frac{\delta}{2}}{1-\theta f^*}\left(\alpha\theta y_H + (1-\alpha)\theta y_L - 1\right) \tag{47}$$

    From (44) and (47), we see that bank k will never deviate if and only if:

$$\alpha\theta y_H + (1-\alpha)\theta f^* - 1 + E_k + (1-\alpha)\frac{\delta}{2} \geq E_k + \frac{E_k+\frac{\delta}{2}}{1-\theta f^*}\left(\alpha\theta y_H + (1-\alpha)\theta y_L - 1\right)$$

    which yields

$$E_k \leq E_2$$

Next, we determine the no-deviation conditions for the equilibrium described in Part (b) of the proposition.

- *Part (b): No-deviation condition for liquid banks*: Consider the liquid bank i: it can deviate by choosing to be illiquid. Note that among two other banks, one is illiquid and the other holds just a minimum required amount of liquidity to be liquid. Hence, if bank i deviates by choosing to be illiquid, it will be closed. Its expected profit is thus:

$$\Pi_i^{de} = \alpha\theta y_H + (1-\alpha)\,\theta f^* - 1 + E_i$$

Bank i will never deviate if and only if:

$$E_i + \frac{E_i}{1-\theta f^*}\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right) + (1-\alpha)\,\frac{1}{2}\left(\theta y_L - \theta f^*\right) \geq \alpha\theta y_H + (1-\alpha)\,\theta f^* - 1 + E_i$$

or equivalently

$$E_i \geq E_4$$

Similarly, liquid bank j will not deviate if and only if:

$$E_j \geq E_4$$

- *Part (b): No-deviation condition for illiquid bank*: bank k can only deviate by chossing to be liquid. Since two other banks are also liquid, the expected profit of bank k in case of deviation is as follows:

$$\Pi_k^{de} = E_k + \frac{E_k}{1-\theta f^*}\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right)$$

Hence, the no-deviation condition for bank k is

$$\alpha\theta y_H + (1-\alpha)\,\theta f^* - 1 + E_k \geq E_k + \frac{E_k}{1-\theta f^*}\left(\alpha\theta y_H + (1-\alpha)\,\theta y_L - 1\right)$$

which implies

$$E_k \leq E_1$$

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