Bank Information Sharing and Liquidity Risk[*]

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Abstract

This paper proposes a novel rationale for the existence of bank information sharing schemes. We suggest that banks can voluntarily disclose borrowers' credit history in order to maintain asset market liquidity. By entering an information sharing scheme, banks will face less adverse selection when selling their loans in secondary markets. This reduces the cost of asset liquidation in case of liquidity shocks. The benefit, however, has to be weighed against higher competition and lower profitability in prime loan markets. Information sharing can arise endogenously as banks trade-off between asset liquidity and rent extraction. Different from the previous literature, we allow for borrower's non-verifiable credit history, and show that banks still have incentives to truthfully disclose such information in competitive credit markets.

JEL Classification: G21.

Keywords: Information Sharing, Funding Liquidity Risk, Market Liquidity, Adverse Selection in Secondary Market.

1 Introduction

One of the reasons for the existence of banks is their liquidity transformation service provided by borrowing short-term and lending long-term. The funding liquidity risk is a natural by–product of the banks' *raison d'être* (Diamond and Dybvig, 1983). This paper argues that such funding risk can be at the root of the existence of information sharing agreements among banks. The need for information sharing arises because banks in need of liquidity may have to sell their assets in secondary markets. Information asymmetry in such markets can make the cost of asset liquidation particularly high. Information sharing allows banks to reduce adverse selection in secondary loan markets, which in turn reduces the damage of premature liquidation.[1]

The benefit of information sharing, however, has to be traded off with its potential cost. Letting other banks know the credit worthiness of its own borrowers, an incumbent bank sacrifices its market power. Likely its competitors will forcefully compete for the good borrowers. The intensified competition will reduce the incumbent bank's profitability. Our paper provides a throughout analysis of this trade-off.

Our theory of bank information sharing is motivated by the features of consumer credit markets in the US. These markets are competitive and contestable. At the same time, banks are able to securitize and re-sell the loans originated in these markets. We argue that the two features are linked, and both related to credit information sharing. On the one hand, the shared information on a borrower's credit history—typically summarized by a FICO score—reduces significantly the asymmetry information about the borrower's creditworthiness. This enables banks to compete for the borrower with which they have no previous lending relationship. On the other hand, the resulting loan is more marketable because the information contained in the FICO score signals the quality of the loan when it is on sale, so that potential buyers of the loan do not fear the winners' curse. In sum, the shared credit history both intensifies primary credit market competition and promotes secondary market liquidity.

The observation that the information sharing has helped to promote securitization in the US has inspired European regulators. In their effort to revive the securitization market in the post-crisis Europe, the European Central Bank and the Bank of England have jointly

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[1]A similar argument can be made for collateralized borrowing and securitization, where the reduced adverse selection will lead to lower haircut and higher prices for securitized assets.

pointed out that "credit registers could also improve the availability quality of information that could, in principle, also benefit securitization markets by allowing investors to build more accurate models of default and recovery rates" (BoE and ECB, 2014). Our paper provides theoretical supports that credit information sharing schemes can indeed promote asset marketability by reducing information asymmetry, and such schemes are sustainable as it can be in banks' own interests to share the information.

We develop a simple model to analyze the trade-off that information sharing entails. We consider an economy made of two banks, one borrower, many depositors and asset buyers. One of the banks is a relationship bank that has a long-standing lending relationship with the borrower. The borrower can be safe or risky, and both types have projects of positive NPVs. However, a safe borrower's project will surely succeed while a risky borrower's does so only with a certain probability. The relationship bank knows both the credit worthiness (i.e., the type) and the credit history (i.e., any past default) of the borrower's. While the information on borrower credit worthiness cannot be communicated, the credit history can be shared. The second bank is a distant bank which has neither lending relationship with the borrower nor any information about the borrower. The distant bank can, however, compete for the borrower by offering competitive loan rates. It can still lose from lending if not pricing the loan correctly.

The relationship bank is subject to a liquidity risk, which we model as a possibility of a bank run. When the liquidity need arises, the relationship bank can sell in a secondary market the loan that it granted to the borrower. Since the quality of the loan is unknown to outsiders, the secondary market for asset is characterized by adverse selection. Even holding a safe loan, the relationship bank can incur the risk of bankruptcy by selling it at a discount. Ex ante, the relationship bank voluntarily shares the borrower's credit history when the benefit, represented by the higher asset liquidity, outweighs the cost, given by the lower rents.

The analysis unfolds in three steps. First, we provide an existence result, pinning down the conditions under which information sharing can sufficiently boost the liquidity value and saves the relationship bank from illiquidity. This result is non-trivial because information sharing has two countervailing effects. On the one hand, observing a good credit history, asset buyers are willing to pay more for the loan on sale since it is more likely to be originated by the safe borrower. On the other hand, the distant bank competes

more aggressively for this loan for exactly the same reason. This drives down the loan rate charged by the relationship bank and reduces the face value of the loan. We show that the first effect always dominates.

Second, we look at the equilibrium and characterize the conditions when the relationship bank voluntarily shares information. These conditions coincide with the aforementioned existence conditions if the relationship bank's funding liquidity risk is sufficiently high. Otherwise, the parameter constellation in which the relationship bank chooses to share information is smaller than the existence region. When funding risk is relatively low, the expected benefit of increased liquidation value are outweighed by the reduction in expected profits due to intensified competition.

Lastly, we relax the common assumption in the literature that the credit history, once shared, is verifiable. Such assumption is restrictive from a theoretical point of view.[2] In the context of our model, the relationship bank will have incentives to mis-report its borrower's credit history in order to gain more from the loan sale. Therefore, we allow for the possibility that the relationship bank can manipulate credit reporting and overstates past loan performances. We show that the relationship bank has an incentive to truthfully disclose its borrower's credit history because, by doing so, it captures the borrower who has defaulted in the past but nevertheless has a positive NPV project.[3] It turns out that such incentive exists in a parameter constellation narrower than the one in which information sharing is chosen in equilibrium under the assumption of verifiable credit history. In particular, the relationship bank has incentives to truthfully communicate the borrower' credit history when the credit market is competitive. In fact, a necessary and sufficient condition for information sharing to be sustained as a truth-telling equilibrium is that the relationship bank can increase the loan rate charged on borrower with default credit history.

Our model comes with two qualifications. First, historically, governments' goal in

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[2]The assumption is also questionable from a practical point of view. Giannetti et al. (2015) show that banks manipulated their internal credit ratings of their borrowers before reporting to Argentinian credit registry. On a more casual level, information manipulation can take place in the form of 'zombie' lending, like it occurred in Japan with the ever-greening phenomenon or in Spain where banks kept on lending to real estate firms likely to be in distress after housing market crash.

[3]One established way to sustain truth telling would be to employ a dynamic setting where the bank has some reputation at stake. We instead show that truth telling can be a perfect Bayesian equilibrium even in a static setup.

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creating public credit registries has been to improve SMEs' access to financing in primary loan markets. Our theory shows that an overlooked benefit of information sharing is the development of secondary markets for loans. Second, it is not our intention to claim that information sharing is the main reason for the explosion of the markets for asset-backed securities. It is ultimately an empirical question to what extent information sharing had fuelled such markets expansion.

The conjecture that information sharing is driven by market liquidity is novel and complementary to existing rationales. Previous literature has mostly explained the existence of information sharing by focusing on the prime loan market. In their seminal paper, Pagano and Jappelli (1993) rationalize information sharing as a mechanism to reduce adverse selection. Sharing ex-ante more accurate information about borrowers reduces their riskiness and increases banks' expected profits. Similarly, information sharing can mitigate moral hazard problems (Padilla and Pagano, 1997 and 2000). We see information sharing as stemming also from frictions on the secondary market for loan sale instead of only on the prime loan market. The two explanations are in principle not mutually exclusive.

Another strand of the literature argues that information sharing allows the relationship bank to extract more monopolistic rent. When competition for borrowers occurs in two periods, inviting the competitor to enter in the second period by sharing information actually dampens the competition in the first period (Bouckaert and Degryse, 2004; Gehrig and Stenbacka 2007). Sharing information about the past defaulted borrowers deters the entry of competitor, which allows the incumbent bank to capture those unlucky but still good borrowers (Bouckaert and Degryse, 2004). This mechanism is also present in our model, and it is related to our analysis with unverifiable credit history.

Finally, another stream of literature focuses on the link between information sharing and other banking activities. The goal is to show how information sharing affects other dimensions of bank lending decisions, more than to provide a rationale of why banks voluntary share credit information. For example, information sharing can complement collateral requirement since the bank is able to charge high collateral requirement only after the high risk borrowers are identified via information sharing (Karapetyan and Stacescu 2014b). Information sharing can also induce information acquisition. After hard information is communicated, collecting soft information becomes a more urgent task for the bank to boost its profits (Karapetyan and Stacescu 2014a).

Our theoretical exposition also opens road for future empirical research. The model implies that information sharing will facilitate banks' liquidity management and loan securitization. The model also suggests that information sharing system can be more easily established in countries with competitive banking sector, and in credit market segments where competition is strong. These empirical predictions would complement the existing empirical literature which has mostly focused on the impact of information sharing on bank risks and firms' access to bank financing. Among the many, Doblas-Madrid and Minetti (2013) provide evidence that information sharing reduces contract delinquencies. Houston et al. (2010) find that information sharing is correlated with lower bank insolvency risk and likelihood of financial crisis. Brown et al. (2009) show that information sharing improves credit availability and lower cost of credit to firms in transition countries.

The remainder of this paper is organized as follows. In the next section we present the model. In Section 3 we show under which conditions information sharing arises endogenously both when borrower's credit history is verifiable (Section 3.1) and when it is not verifiable (Section 3.2). Section 4 analyzes several robustness. Section 5 discusses the model's welfare and policy implications. Section 6 concludes.

2 The Model

We consider a three-period economy with timing $t = 0, 1, 2, 3$. The agents in the economy consist of two banks (a relationship bank and a distant bank), one borrower and many depositors as well as asset buyers. All agents are risk neutral. The gross return of the risk-free asset is indicated as r_0.

We assume that a bank has one loan on its balance sheet. The loan requires 1 unit of initial funding, and its returns depend on the type of the borrower. The borrower can be either safe (H-type) or risky (L-type). The ex-ante probability of the safe type $\Pr(H)$ is equal to α, and for the risky type $\Pr(L)$ is equal to $1 - \alpha$. A safe borrower generates a payoff $R > r_0$ with certainty, and a risky borrower generates a payoff that depends on a state $s = \{G, B\}$. In the good state G, the payoff is the same as a safe borrower R, but in the bad state B the payoff is 0. The ex-ante probabilities of the two states are $\Pr(G) = \pi$ and $\Pr(B) = 1 - \pi$, respectively. We assume both types of loans have positive NPVs, that

is, $\pi R > r_0$.[4]

The relationship bank has an ongoing lending relationship with the borrower. It privately observes both the credit worthiness (i.e., the type) and the repayment history of the borrower. While the former is assumed to be soft information and cannot be communicated to the others, the latter is assumed to be hard information that can be shared with third parties. We model the decision to share hard information as a unilateral decision made by the relationship bank in $t = 0$. If the relationship bank chooses to share the credit history of its borrower, it makes an announcement in $t = 1$ about whether the borrower had defaulted or not. We label a credit history without previous defaults by $\overline{D}$ and a credit history with defaults by D. A safe borrower has a credit history $\overline{D}$ with probability 1, and a risky borrower has a credit history $\overline{D}$ with probability π_0 and a credit history D with probability $1 - \pi_0$.[5] Notice that we assume that the occurrence of state s, which captures an aggregate risk, is independent from the default history of the borrower, which nails down an idiosyncratic risk of the borrower.[6]

The distant bank has no lending relationship with the borrower and observes no information about the borrower's type. It does not know the credit history either, unless the relationship bank shares such information. The distant bank can compete in $t = 2$ for the borrower by offering lower loan rates, but to initiate the new lending relationship it bears a fixed cost c. Such a cost instead represents a sunk cost for the relationship bank.[7] The bank that wins loan market competition will be financed solely by deposits and set the rate of deposits. In a competitive deposit market, the depositors demand to earn the risk-free rate r_0 in expectation. We abstract from risk-shifting induced by limited liability and assume perfect market discipline so that deposit rates are determined based on the

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[4]One potential interpretation is to consider the H-type being prime mortgage borrowers, and L-type being subprime borrowers. While both can pay back their loans in a housing boom, the subprime borrowers will default once housing price drops. However, the probability of a housing market boom is sufficiently large that it is still profitable to lend to both types.

[5]This is equivalent to assume a preliminary (i.e., in $t = -1$) round of lending between the relationship landing and the borrower. In this preliminary lending, the safe borrower would generate no default history, and the risky borrower would default with a probability $1 - \pi_0$.

[6]To follow our analogy with the subprime morgage market, we assume that the probability of a housing market boom π is independent of the borrower's repayment record about the morgage on his home.

[7]The fixed cost c can be interpreted as the cost that the distant bank has to pay to establish new branches, to hire and train new staffs, etc. Alternatively, it can represent the borrower's switching cost that is paid by the distant bank.

bank's riskiness. Depositors are assumed to have the same information about the borrower as the distant bank—observing the default history of the borrower only if the relationship bank shares it.

To capture the funding liquidity risk, we assume that the relationship bank faces a run in $t = 3$ with a probability equal to ρ.[8] We interpret also the risk of a run as an idiosyncratic risk at the bank-level and so it is assumed to be independent of the state s. When the run happens, all depositors withdraw their funds, and the relationship bank has to raise liquidity to meet the depositors' withdrawals.[9] We assume that physical liquidation of the bank's loan is not feasible, and only financial liquidation—a loan sale to asset buyers on a secondary market—is possible. We assume that the secondary loan market is competitive, and risk neutral asset buyers only require to break even in expectation. We finally assume that the loan is indivisible and the bank has to sell it as a whole. The state s realizes in $t = 3$ before the possible run, and it becomes public information. Asset buyers observe the state, but are uninformed of the credit worthiness of the relationship borrower's. They can nevertheless condition their bids on the borrower's credit history if the relationship bank shares such information.

It is the relationship bank's private information whether it faces a run or not. Therefore, a loan can be on sale for two reasons: either due to funding liquidity needs, in which case an H-type loan can be on sale, or due to a strategic sale for arbitrage, in which case only an L-type loan will be on sale. The possibility of a strategic asset sale leads to adverse selection in the secondary asset market. H-type loans will be underpriced in an asset sale and even a solvent relationship bank that owns an H-type loan can fail due to illiquidity. In the case of a bank failure, we assume that bankruptcy costs results in zero salvage value. Such liquidity risk and costly liquidation gives the relationship bank the incentive to disclose the credit history of its borrower, since the shared information reduces adverse selection and boost asset liquidity.

The sequence of events is summarized in Figure 1. The timing captures the fact that information sharing is a long-term decision (commitment), while competition in the loan market and the liquidity risk faced by the bank are shorter-term concerns.

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[8]While the relationship bank faces the liquidity risk that the distant bank does not face, the relationship bank has an extra tool (information sharing decision) to manage that risk. Our set up is symmetric in this respect.

[9]This is also a feature of global-games-based bank run games with private signal.

[Put Figure 1 here]

3 Equilibrium Information Sharing

We consider two alternative assumptions on the verifiability of the shared information. In section 3.1, we assume that the borrower's credit history, once shared, is verifiable. The relationship bank cannot manipulate such information. We relax this assumption in section 3.2, by allowing the relationship bank to overstate the past loan performance—to claim the borrower having no default history when the borrower actually has.

We solve the decentralized solution by backward induction. We proceed as follows: i) determine the prices at which loans are traded in the secondary market; ii) compute the deposit rates at which depositors supply their funds to the bank; iii) determine the loan rates at which the bank offers credit to the borrower; iv) decide if the relationship bank wants to share the information on the borrower's credit history or not. When the shared information is not verifiable, we also analyze the conditions under which the relationship bank has the incentive to truthfully report.

3.1 Verifiable Credit History

3.1.1 Secondary-market Loan Prices

We start with determining secondary-market loan prices for given loan rates.[10] Depending on whether banks share information or not, the game has different information structures. Asset prices, loan rates, and deposit rates cannot be conditional on the borrower's credit history without information sharing, but can depend on the credit history if the information is shared. We indicate with P_i^s the asset price in State $s \in \{G, B\}$ and under an information-sharing regime $i \in \{N, S\}$, where N denotes no information sharing scheme in place, and S refers to the presence of shared credit history. The asset prices vary across states, because asset buyers observe State s.

We first examine the asset prices under no information sharing. In this case, a unified rate R_N would prevail for all types of borrowers, because loan rates cannot be conditional

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[10]For the rest of the paper, we refer to the prices at which the loan is traded in the secondary market succinctly as 'asset prices'.

on the credit history. When the state is good, both types of borrower will generate income R and repay the loan in full. As asset buyers receive a zero profit in a competitive market, we have

$$P_N^G = R_N \tag{1}$$

independently of the borrower's type. When the state is bad, the L-type borrower will generate a zero payoff. But asset buyers cannot update their prior beliefs since the relationship bank does not share any information on borrower's credit history. For any positive price, an L-type loan will be on sale even if the relationship bank faces no bank run. Due to the possible presence of an L-type loan, an H-type loan will be sold at a discount. Consequently, the relationship bank will put an H-type loan on sale only if it faces the liquidity need of deposit withdrawals. The market is characterized by adverse selection. The price P_N^B is determined by the following break-even condition of asset buyers

$$\Pr(L)(0 - P_N^B) + \Pr(H)\Pr(\text{run})(R_N - P_N^B) = 0 \,,$$

which implies

$$P_N^B = \frac{\alpha\rho}{(1-\alpha) + \alpha\rho} R_N. \tag{2}$$

It follows immediately that the H-type loan is under-priced (sold for a price lower than its fundamental value R_N) because of adverse selection in the secondary asset market.

With information sharing, asset prices can be conditional on the borrower's credit history D or $\overline{D}$ too. If the state is good, no loan will default, and the asset price equal to the face value of the loan. We have

$$P_S^G(D) = R_S(D) \tag{3}$$

and

$$P_S^G(\overline{D}) = R_S(\overline{D}), \tag{4}$$

where $R_S(D)$ and $R_S(\overline{D})$ denote the loan rates for a borrower with and without default history, respectively.[11] Notice that, even in the good state, asset prices can differ because the loan rates vary with the borrower's credit history.

The shared credit history also affects asset prices in the bad state. When the relationship bank discloses a previous default, the borrower is perceived as an L-type for sure.

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[11]We will solve the loan rates R_N, $R_S(D)$ and $R_S(\overline{D})$ in subsection 3.1.3.

Therefore posterior beliefs are $\Pr(H \mid D) = 0$ and $\Pr(L \mid D) = 1$. Since an L-type borrower defaults in State B with certainty, we have

$$P_S^B(D) = 0. \tag{5}$$

When the announced credit history is $\overline{D}$ (no previous default), the posterior beliefs, according to Bayesian rule, are

$$\Pr(H \mid \overline{D}) = \frac{\Pr(H, \overline{D})}{\Pr(\overline{D})} = \frac{\alpha}{\alpha + (1 - \alpha)\pi_0} > \alpha$$

and

$$\Pr(L \mid \overline{D}) = \frac{\Pr(L, \overline{D})}{\Pr(\overline{D})} = \frac{(1 - \alpha)\pi_0}{\alpha + (1 - \alpha)\pi_0} < 1 - \alpha.$$

Intuitively, asset buyers use the credit history as an informative yet noisy signal of the borrower's type. A borrower with no previous default is more likely to be of an H-type, thus $\Pr(H \mid \overline{D}) > \alpha$.

Given the posterior beliefs, asset buyers anticipate that the relationship bank always sells an L-type loan and holds an H-type loan to maturity if no bank run occurs. Therefore, the price $P_S^B(\overline{D})$ they are willing to pay is given by the following break-even condition

$$\Pr(L \mid \overline{D})[0 - P_S^B(\overline{D})] + \Pr(H \mid \overline{D}) \Pr(\text{run})[R_S(\overline{D}) - P_S^B(\overline{D})] = 0,$$

which implies

$$P_S^B(\overline{D}) = \frac{\alpha \rho}{(1 - \alpha)\pi_0 + \alpha \rho} R_S(\overline{D}). \tag{6}$$

Comparing (2) with (6), one can see that, conditional on $\overline{D}$-history, the perceived chance that a loan is H-type is higher under information sharing. This is because an L-type borrower with a bad credit history D can no longer be pooled with an H-type in an asset sale. Information sharing, therefore, mitigates the adverse selection problem by 'purifying' the market. However, we cannot yet draw a conclusion on the relationship between the asset prices until we determine the equilibrium loan rates R_N and $R_S(\overline{D})$.

3.1.2 Deposit Rates

We now determine equilibrium deposit rates r_i, for different information sharing schemes $i = \{N, S\}$, taking loan rates as given. We assume that depositors have the same information on the borrower's credit worthiness as the distant bank, and that deposits are fairly

priced for the risk perceived by the depositors. Therefore, the pricing of deposit rates can be conditional on the borrower's past credit history if the relationship bank shares the information. Having risk-sensitive deposit eliminates possible distortions due to limited liabilities, which avoids extra complications and sharpens the intuition of the model.[12]

We start with discussing the deposit rates charged to the relationship bank.[13] We assume that the relationship bank sets the deposit rate: it makes a take-it-or-leave-it offer to a large number of depositors who are price-takers in a competitive deposit market. The depositors form rational expectations about the bank's risk and require to break even in expectation. Since the relationship bank can be either risky or risk-free, the deposit rate varies accordingly. In particular, the deposit rate is endogenous to the secondary-market loan price. When the price is high, the relationship bank can survive a bank run if it happens, but when the asset price is low, the relationship bank can fail in such a run. In the latter case, the depositors will demand a premium for the liquidity risk.

First consider the situation where the relationship bank does not participate in the information sharing program. If the bank is risk-free, its deposit rate equals r_0. If the bank is risky, it will offer its depositors a risky rate $\widehat{r}_N$ which allows them to break even in expectation. To calculate $\widehat{r}_N$, note that the bank will never fail in the good state, regardless of its borrower's type, and regardless whether it experiencing a run or not.[14] When the state is bad, the relationship bank with an H-type loan will survive if it does not face a run.[15] But the bank will fail if it sells its asset in the bad state: this happens when the bank is forced into liquidation by runs or sells its L-type loan to arbitrage.[16] Given our assumption that the salvage value of the bank equals zero, the depositors will receive a zero payoff in those states. As the deposit rate is set before the realization of State s and

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[12]When the deposits are risk-insensitive and when the relationship bank can fail in runs if not engaged in information sharing, not to share information becomes a particular way of risk-taking. That is, the relationship bank takes on extra liquidity risk in order to remain an information monopoly. While our results will not qualitatively change under risk-insentive deposits, such an assumption makes the model less tractable and its intuition less clear.

[13]As it will be clear in the next section, it is the relationship bank that finances the loan. So the deposit rates charged on the relationship bank will be the deposit rates on the equilibrium path.

[14]This is guaranteed by the fact that $P_N^G = R_N$, and that the loan rate always exceeds the deposit rate in equilibrium.

[15]In this case, the relationship bank will hold the loan to the terminal date and receive R_N

[16]In both cases, the bank's liquidation value equals P_N^B.

that of (possible) bank runs, we then have the following break-even condition:

$$[\Pr(G) + \Pr(B)\Pr(H)\Pr(\text{no run})]\widehat{r}_N = r_0 \ ,$$

which implies

$$\widehat{r}_N = \frac{r_0}{\pi + \alpha(1-\pi)(1-\rho)} > r_0.$$

Lemma 1 characterizes the equilibrium deposit rate r_N and how it depends on the secondary-market loan price.

Lemma 1 *When the relationship bank does not share information, it pays an equilibrium deposit rate r_N such that, (i) $r_N = r_0$ if and only if $P_N^B \geq r_0$, and (ii) $r_N = \widehat{r}_N$ if and only if $P_N^B < r_0$.*

We first prove the 'if' (or sufficient) condition in Lemma 1. To prove it for claim (i), we consider two mutually exclusive cases. First, if the asset price is such that $P_N^B \geq \widehat{r}_N$, the bank's liquidation value will be greater than its liabilities. Thus, the deposit is safe and the deposit rate equals r_0. Second, if the asset price is such that $\widehat{r}_N > P_N^B \geq r_0$, the depositor will be able to break even, either when the offered deposit rate is r_0 or $\widehat{r}_N$. In the former case, the deposit is risk-free; in the latter case, the deposit is risky but the depositors are sufficiently compensated for the risk they bear. The latter case, however, cannot be an equilibrium, because the relationship bank will optimally choose deposit rate equal to r_0 to benefit from a lower cost of funding and avoid the risk of bankruptcy. To prove the sufficient condition in claim (ii), notice that if $r_0 > P_N^B$, the bank will fail, because its liquidation value is insufficient to pay the risk premium demanded depositors.

We now prove the 'only if' (or necessary) condition. If the deposit rate is equal to r_0, the deposit must be risk-free. In particular, the relationship bank should not fail when it sells its asset in the bad state. Therefore, on the interim date in the bad state, the bank's liquidation value P_N^B must be greater than or equal to its liability r_0. On the other hand, if the deposit is risky and its rate equals $\widehat{r}_N$, by the definition of $\widehat{r}_N$, the bank must *only* fail in an asset sale in the bad state. This implies that the bank's liquidation value P_N^B must be smaller than its liability $\widehat{r}_N$. Furthermore, we can exclude the case that $r_0 \leq P_N^B \leq \widehat{r}_N$, because, if that is true, the relationship will reduce the deposit rate to r_0 and avoid the bankruptcy. Therefore, we must have $P_N^B < r_0$. Finally, recall that we have already shown that the relationship bank will not fail when state is good, or when the bank faces no run and does not need to sell its H-type loan in the bad state. This concludes the proof.

We now characterize deposit rates when the relationship bank participates in the information sharing. The deposit rates can now be conditional on the credit history of the borrower. If the borrower has a previous default (i.e., a D-history), depositors know the borrower as an L-type for sure, and expect to be paid only in State G.[17] This leads depositors to ask a deposit rate $r_S(D)$ that satisfies the break-even condition $\Pr(G)r_S(D) = r_0$. Accordingly we have

$$r_S(D) = \frac{r_0}{\pi} > r_0 \quad . \tag{7}$$

When the borrower has no previous default (i.e., a $\overline{D}$-history) the analysis is similar to the case without information sharing, and the equilibrium deposit rate depends on the asset price. Let $\widehat{r}_S(\overline{D})$ be the risky deposit rate by which depositors can break even, given that the relationship bank fails in an asset sale only when it owns a $\overline{D}$-loan. We have

$$[\Pr(G) + \Pr(B)\Pr(H \mid \overline{D})\Pr(\text{no run})]\widehat{r}_S(\overline{D}) = r_0$$

that implies

$$\widehat{r}_S(\overline{D}) = \frac{\alpha + (1-\alpha)\pi_0}{\alpha + (1-\alpha)\pi_0\pi - (1-\pi)\alpha\rho}r_0 > r_0 \quad .$$

Lemma 2 characterizes the equilibrium deposit rate $r_S(\overline{D})$ and how it depends on the secondary-market loan price.

Lemma 2 *When information sharing is in place and the borrower has a credit history of* $\overline{D}$, *the relationship bank pays an equilibrium deposit rate* $r_S(\overline{D})$ *such that, (i)* $r_S(\overline{D}) = r_0$ *if and only if* $P_S^B(\overline{D}) \geq r_0$, *and (ii)* $r_S(\overline{D}) = \widehat{r}_S(\overline{D})$ *if and only if* $P_S^B(\overline{D}) < r_0$.

The proof is provided in the Appendix. The intuition of the result is similar to that of Lemma 1. When the secondary-market loan price is sufficiently high, the relationship bank will survive the possible bank run, and the deposit will be risk-free. Otherwise, the deposit is risky due to the liquidity risk. Consequently, the relationship bank has to offer a premium over the risk-free rate for depositors to break even.

We now compute the deposit rates charged to the distant bank.[18] As the distant bank is assumed to face no liquidity risk, its deposit rates will depend only on the fundamental asset

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[17]This is because an L-type generates zero revenue in State B and the asset price $P_S^B(D) = 0$ under information sharing.

[18]As it is the relationship bank that finances the loan in equilibrium, these deposit rates for the distant bank would be off-equilibrium. Yet, they are necessary for the derivation of the loan market equilibrium.

risk. We denote these rates by r_i^E, for different information sharing regimes $i = \{N, S\}$. Without information sharing, the deposit rate r_N^E is determined by depositors' break-even condition as follows

$$\Pr(H)r_N^E + \Pr(L)\Pr(G)r_N^E = r_0,$$

which implies

$$r_N^E = \frac{r_0}{\alpha + (1-\alpha)\pi} > r_0. \tag{8}$$

With information sharing, the deposit rate $r_S^E(D)$ is charged when the borrower has a default history. The depositors' break-even condition $\Pr(G)r_S^E(D) = r_0$ implies

$$r_S^E(D) = r_0/\pi. \tag{9}$$

Finally, the deposit rate $r_S^E(\overline{D})$ is charged when the borrower has no previous default. The rate is determined by depositors' break-even condition

$$\Pr(H \mid \overline{D})r_S^E(\overline{D}) + \Pr(L \mid \overline{D})\Pr(G)r_S^E(\overline{D}) = r_0,$$

which implies

$$r_S^E(\overline{D}) = \frac{\alpha + (1-\alpha)\pi_0}{\alpha + (1-\alpha)\pi_0\pi}r_0 > r_0. \tag{10}$$

3.1.3 Loans Rates

When the credit market is contestable, the two banks will compete until the distant bank (the entrant) only breaks even. We denote by R_i^E the loan rate by which the distant bank breaks even under an information-sharing regime $i = \{N, S\}$.

Without information sharing, the distant bank holds the prior belief on the borrower's type. The break-even condition for the distant bank is

$$\Pr(H)(R_N^E - r_N^E) + \Pr(L)\Pr(G)(R_N^E - r_N^E) = c,$$

where c is the fixed entry cost and r_N^E is the distant bank's deposit rate as determined in equation (8). Combining the two expressions, we get

$$R_N^E = \frac{c + r_0}{Pr(H) + Pr(L)Pr(G)} = \frac{c + r_0}{\alpha + (1-\alpha)\pi}. \tag{11}$$

With information sharing in place, loan rates are contingent on the borrower's credit history. If the distant bank observes a previous default, it concludes that the borrower is surely an L-type, so that its break-even condition is

$$\Pr(G)[R_S^E(D) - r_S^E(D)] = c.$$

15

With $r_S^E(D) = r_0/\pi$ given in equation (9), we have

$$R_S^E(D) = \frac{c + r_0}{\pi}.$$

When the borrower has no previous default, the distant bank updates its belief according to Bayes' rule, and its break-even condition is

$$\Pr(H \mid \overline{D})[R_S^E(\overline{D}) - r_S^E(\overline{D})] + \Pr(L \mid \overline{D})\Pr(G)[R_S^E(\overline{D}) - r_S^E(\overline{D})] = c,$$

where $r_S^E(\overline{D})$ is given by (10). Combining the two expressions, we get

$$R_S^E(\overline{D}) = \frac{c + r_0}{Pr(H|\overline{D}) + Pr(L|\overline{D})Pr(G)} = \frac{\alpha + (1 - \alpha)\pi_0}{\alpha + (1 - \alpha)\pi_0\pi}(c + r_0). \qquad (12)$$

A simple comparison of the loan rates makes it possible to rank them as follows.

Lemma 3 *The ranking of the distant bank's break-even loan rates is* $R_S^E(\overline{D}) < R_N^E < R_S^E(D)$.

Intuitively, when information sharing is in place, and the borrower has a previous default, the distant bank charges the highest loan rate since the borrower is surely an L-type. On the contrary, when the borrower has no previous default, the distant bank offers the lowest loan rate since the borrower is more likely an H-type. Without information sharing, the distant bank offers an intermediate loan rate, reflecting its prior belief about the borrower's type.

The distant bank's break-even rates are not necessarily equal to the equilibrium loan rates. The latter also depends on the project return R which determines the contestability of the loan market. Suppose $R_i^E > R$, then R is too low and an entry into the loan market is never profitable for the distant bank. The relationship bank can charge a monopolistic loan rate and take the entire project return R from the borrower. Suppose, otherwise, $R_i^E \leq R$. In this case the payoff R is high enough to induce the distant bank to enter the market. The relationship bank, in this case, would have to undercut its loan rate to R_i^E, and the equilibrium loan rate equals the break-even loan rate charged by the distant bank.

Let us indicate the equilibrium loan rate as R_i^* under an information-sharing regime $i = \{N, S\}$. The following lemma characterizes the equilibrium loan rates.

Lemma 4 *In equilibrium, the relationship bank finances the loan. The equilibrium loan rates depend on the relationship between the distant bank's break-even loan rates and the project's return R. We have the following four cases:*

16

- *Case 0: If $R \in \mathbb{R}_0 \equiv \left(c + r_0, R_S^E(\overline{D})\right]$ then $R_S^*(\overline{D}) = R_N^* = R_S^*(D) = R$*

- *Case 1: If $R \in \mathbb{R}_1 \equiv \left(R_S^E(\overline{D}), R_N^E\right]$ then $R_S^*(\overline{D}) = R_S^E(\overline{D})$ and $R_N^* = R_S^*(D) = R$*

- *Case 2: If $R \in \mathbb{R}_2 \equiv \left(R_N^E, R_S^E(D)\right]$ then $R_S^*(\overline{D}) = R_S^E(\overline{D})$, $R_N^* = R_N^E$ and $R_S^*(D) = R$*

- *Case 3: If $R \in \mathbb{R}_3 \equiv \left(R_S^E(D), \infty\right)$ then $R_S^*(\overline{D}) = R_S^E(\overline{D})$, $R_N^* = R_N^E$ and $R_S^*(D) = R_S^E(D)$.*

Here $\mathbb{R}_j$, $j \in \{0, 1, 2, 3\}$, denotes the range of project's return R that defines Case j.[19] Each case shows a different degree of loan market contestability: the higher R, the more contestable the loan market. In Case 0, for example, the project payoff R is so low that the distant bank does not find it profitable to enter the market even if the borrower has no previous default. The loan market, therefore, is least contestable. In Case 3, for example, the loan market is the most contestable, since R is high enough that the distant bank competes for a loan even if the borrower has a previous default. The four cases are mutually exclusive and jointly cover the whole range of possible R, as depicted in Figure 2.

[Put Figure 2 here]

3.1.4 The Benefit of Information Sharing

We now show that in all of the four cases characterized in Lemma 4, information sharing can be beneficial to the relationship bank. To be more specific, there always exists a set of parameters in which the relationship bank owning a $\overline{D}$-loan will survive a run in State B by sharing the borrower's credit history. In other words, information sharing can save the relationship bank from illiquidity.

Information sharing has such a positive impact because it boosts the secondary-market loan price in State B. Recall that, in the bad state, an L-type loan will generate a zero payoff, so that when asset buyers do not observe the quality of a loan, there will be adverse selection and an H-type loan will be underpriced. Consequently, the relationship bank

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[19]The case index j indicates the number of interior solutions, i.e., the number of equilibrium loan rates that are strictly smaller than R.

17

can fail in a run even if it holds a safe H-type loan. As we pointed out in Section 3.1.1, the shared credit history provides an informative signal for loan qualities. In particularly, the perceived loan quality is higher for a loan with a $\overline{D}$ history than for a loan with unknown credit history. This mitigates the adverse selection and boosts the asset price in the secondary loan market.

The reduced adverse selection, however, is not the only impact information sharing has on secondary-market asset prices. Recall expressions (2) and (6) that the secondary-market loan prices depend both on the perceived loan quality and the equilibrium loan rates R_N^* and $R_s^*(\overline{D})$. As the distant bank competes more aggressively in the primary loan market for a borrower with no default history, the loan rate will decline, i.e., $R_S^*(\overline{D}) \leq R_N^*$. It appears that information sharing *may* result in $P_S^B(\overline{D}) < P_N^B$ as it reduces loan rate from R_N^* to $R_S^*(\overline{D})$. Lemma 5 in below establishes that information sharing *always* leads to a higher asset price in the bad state under information sharing. The positive effect of mitigating adverse selection is of the first order importance and dominates the negative effect of lower equilibrium loan rates.

Lemma 5 *The equilibrium asset prices are such that $P_S^B(\overline{D}) > P_N^B$. That is, in the bad state, the secondary-market price for a loan with $\overline{D}$ history is always higher than that for a loan with unknown credit history.*

The complete proof is in the Appendix. To provide the intuition, we discuss here Case 2—a core case upon which the general proof builds. The equilibrium asset prices P_N^B and $P_S^B(\overline{D})$ are determined in expressions (2) and (6), respectively. In Case 2, the equilibrium loan rates are $R_N^* = R_N^E$, given in equation (11), and $R_s^*(\overline{D}) = R_s^E(\overline{D})$, given in (12). Plugging the equilibrium loan rates into the expressions that characterize the equilibrium asset prices, we can compute the ratio between P_N^B and $P_S^B(\overline{D})$.[20]

$$
\begin{aligned}
\frac{P_N^B}{P_S^B(\overline{D})} &= \left(\frac{\Pr(L|\overline{D}) + \Pr(H|\overline{D})\Pr(\text{run})}{\Pr(L) + \Pr(H)\Pr(\text{run})} \right) \left(\frac{\Pr(H|\overline{D}) + \Pr(L|\overline{D})\Pr(G)}{\Pr(H) + \Pr(L)\Pr(G)} \right) \\
&= \underbrace{\left(\frac{(1-\alpha)\pi_0 + \alpha\rho}{(1-\alpha) + \alpha\rho} \right)}_{(A)} \underbrace{\left(\frac{\alpha + (1-\alpha)\pi_0\pi}{(\alpha + (1-\alpha)\pi)(\alpha + (1-\alpha)\pi_0)} \right)}_{(B)}
\end{aligned}
$$

---

[20]In Case 0, the equilibrium loan rates are $R_S^*(\overline{D}) = R_N^* = R$. Plugging these values into (2) and (6), it is stragithforward to verify that $P_S^B(\overline{D}) > P_N^B$.

This ratio between P_N^B and $P_S^B(\overline{D})$ can be decomposed into a product of two elements. Expression (A) reflects how information sharing affects the adverse selection in the secondary loan market, and expression (B) captures the impact of information sharing on the adverse selection in the primary loan market. Specifically, expression (A) is the ratio between the expected quality of a loan with a $\overline{D}$-history under information sharing and that of a loan with an unknown credit history under no information sharing.[21] This ratio is smaller than 1, implying an increase in the expected asset quality conditional on no default history. Expression (B) is the ratio between the probability of no fundamental credit risk (either that the borrower is an H-type or an L-type in the G state) under no information sharing and that under information sharing. This ratio is greater than 1, implying a decline in the perceived credit risk and the corresponding drop in the primary-market loan rates.

The adverse selection in both primary and secondary loan markets is rooted in the uncertainty of the borrower's type. But these two markets differ in two aspects. First, the strategic asset sale by the relationship bank aggravates the adverse selection in the secondary market. The strategic asset sale disappears when the relationship bank is selling the loan because it is facing a run for sure, i.e. when $\rho = 1$. Second, the uncertainty about State s has resolved when the secondary market opens. For this reason, the secondary-market prices P_N^B and P_S^B are conditional on State $s = B$, whereas the loan rates from the primary market is not conditional on the state. This difference in the uncertainty of State s disappears when $\pi = 0$. Therefore, the primary and secondary loan markets have the same level of adverse selection when both $\rho = 1$ and $\pi = 0$. The impact of information sharing in the two markets is symmetric and the price ratio $P_N^B/P_S^B(\overline{D})$ equals 1.

Otherwise, the price ratio $P_N^B/P_S^B(\overline{D})$ is smaller than 1 for either $\rho < 1$ or $\pi > 0$. To see so, first notice that expression (A) increases in ρ. Intuitively, as the probability of a run decreases from 1, it becomes more likely that the asset is on sale for strategic reasons. As a result, the adverse selection in the secondary market aggravates, and the gap in the expected qualities widens across the two information sharing regimes, leading to a lower value for expression (A). Second, notice that expression (B) decreases in π. Intuitively, as π increases, the difference between H- and L-type borrower diminishes.[22] The credit history becomes less relevant as an informative signal of the borrower's type, and the gap between

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[21]The expected quality is defined as the probability that the loan is granted to an H-type borrower.

[22]When $\pi \to 1$, the H- and L-type borrower no longer differ in terms of $t = 2$ loan performance.

the two loan rates narrows, leading to a lower value of expression (B). Therefore, we have that $P_N^B < P_S^B(\overline{D})$ whenever either $\rho < 1$ or $\pi > 0$. In terms of supporting the asset price for a $\overline{D}$-loan, information sharing's positive imapct of increasing secondary-market asset quality always dominates its negative impact of decreasing primary-market loan rates.

Once we proved that $P_N^B < P_S^B(\overline{D})$, by continuity, there must exist a set of parameters where the risk-free rate r_0 lies between the two prices. We show this is equivalent to say that in such parametric configurations, information sharing saves a relationship bank with a $\overline{D}$-loan from the bank run in State B.

Proposition 1 *There always exist a range of parameters such that $P_N^B < r_0 < P_S^B(\overline{D})$. For such parameters, the relationship bank's equilibrium deposit rates are $r_S(\overline{D}) = r_0$ and $r_N = \widehat{r}_N$ with and without information sharing respectively. The relationship bank with a $\overline{D}$-loan will be saved by information sharing from a bank run in the bad state.*

Given $P_N^B < P_S^B(\overline{D})$, the result of existence is based on a continuity argument. The corresponding deposit rates follow directly from the sufficient conditions established in Lemma 1 and Lemma 2. Those deposit rates indicate that a relationship bank with a $\overline{D}$-loan will be risky without information sharing, but risk-free with information sharing. The difference in the risk is due to the fact that the relationship bank will fail in a run in State B under no information sharing.

This relationship between bank risk and information sharing can be best understood from the perspective of depositors. Consider first the case where the relationship bank does not share information. When the depositors expect the bank to fail in an asset sale in State B, they will accept the risky deposit rate $\widehat{r}_N > r_0$. Given this belief and the deposit rate, the bank will indeed fail in an asset sale in State B, provided that the parametric condition is satisfied. This is because the bank's asset liquidation value P_N^B is smaller than r_0 which is in turn smaller than its liability $\widehat{r}_N$. On the other hand, a belief that the bank will be risk-free cannot be rationalized. To see this, suppose that depositors accept the risk-free rate r_0. Given $P_N^B < r_0$, the bank will still be unable to meet its interim-date liability in an asset sale happens in State B. Therefore, the deposit rate cannot be risk-free in the first place. In other words, when the relationship bank does not share information, the deposit rate can only be equal to $\widehat{r}_N$ in a rational expectation equilibrium. On the equilibrium path, the bank will fail in an asset sale in the bad state. The same argument also applies to the case where the relationship bank shares information and is risk-free. When the

depositors expect the bank to survive a run even in the bad state, they will perceive their claims to be risk-free and accept the risk-free rate r_0. Such a belief is rational, since given the deposit rate $r_s(\overline{D}) = r_0$, the bank will indeed be able to repay its liability on the interim date with the liquidation value $P_S^B(\overline{D})$.

We now characterize the set of parameters in Proposition 1. For each Case $j = \{0, 1, 2, 3\}$ as defined in Lemma 4, we characterize a set of parameters $\mathbb{F}_j$ where the condition $P_N^B < r_0 < P_S^B(\overline{D})$ holds. We define the intersection $\Psi_j \equiv \mathbb{R}_j \bigcap \mathbb{F}_j$, with $j = \{0, 1, 2, 3\}$. Set Ψ_j is non-empty for all cases.

- $\Psi_0 \equiv \mathbb{R}_0 \bigcap \mathbb{F}_0$ with $\mathbb{F}_0 \equiv \{(c + r_0, R) | \frac{(1-\alpha)\pi + \alpha\rho}{\alpha\rho} r_0 < R < \frac{(1-\alpha)+\alpha\rho}{\alpha\rho} r_0\}$.

- $\Psi_1 \equiv \mathbb{R}_1 \bigcap \mathbb{F}_1$ with $\mathbb{F}_1 \equiv \{(c+r_0, R) | R < \frac{\alpha\rho+(1-\alpha)}{\alpha} r_0 \text{ and } c+r_0 > \frac{\alpha\rho+(1-\alpha)\pi}{\alpha\rho} \frac{\alpha+(1-\alpha)\pi_0\pi}{\alpha+(1-\alpha)\pi_0} r_0\}$.

- $\Psi_2 \equiv \mathbb{R}_2 \bigcap \mathbb{F}_2$ with $\mathbb{F}_2 \equiv \{(c + r_0, R) | \frac{(1-\alpha)\pi + \alpha\rho}{\alpha\rho} \frac{\alpha+(1-\alpha)\pi_0\pi}{\alpha+(1-\alpha)\pi_0} r_0 < c + r_0 < \frac{(1-\alpha)+\alpha\rho}{\alpha\rho}[\alpha + (1-\alpha)\pi] r_0\}$.

- $\Psi_3 \equiv \mathbb{R}_3 \bigcap \mathbb{F}_3$ with $\mathbb{F}_3 \equiv \mathbb{F}_2$.[23]

Figure 3 gives the graphic representation of the sets Ψ_j. The shaded area corresponds the set of parameters that satisfy the inequality in Proposition 1. Only in those regions, the relationship bank with a $\overline{D}$-loan will survive a bank run in State B when sharing information, but will fail without information sharing.

[Put Figure 3 here]

Information sharing can endogenously emerge only inside those shaded areas, because for all other possible parametric combinations information sharing does not reduce the relationship bank's liquidity risk, but only leads to the loss of information rent. To see so, suppose $r_N = r_S(\overline{D}) = r_0$, then the relationship bank is riskless with and without information sharing. Given that information sharing does not reduce liquidity risk but only intensifies primary loan market competition, the relationship bank will not share the borrower's credit history. Similarly, suppose $r_N = \widehat{r}_N$ and $r_S(\overline{D}) = \widehat{r}_S(\overline{D})$. Then the

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[23]Notice that the prices P_N^B and $P_S^B(\overline{D})$ are the same in Case 2 and Case 3. This is because the prime loan market is rather contestable under these two cases. The distant bank competes with the relationship bank for a loan with unknown credit history as well as for a loan with no default credit history. Therefore, we have $\mathbb{F}_3 = \mathbb{F}_2$.

relationship bank will fail in a run in State B, with or without information sharing. Again, the bank does not gain anything to disclose the borrower's credit history. In fact, these two cases correspond to the non-shaded areas in Figure 3. Finally, the combination $r_N = r_0$ and $r_S(\overline{D}) = \widehat{r}_S(\overline{D})$ simply cannot exist. Otherwise, we will have $P_N^B \geq r_0$ and $P_S^B(\overline{D}) < r_0$, which contradicts Lemma 5. In fact, $P_N^B < P_S^B(\overline{D})$ is a necessary condition for information sharing to endogenously emerge. We will focus on this parametric configuration in the rest of the paper and characterize the sufficient condition in the next section.

3.1.5 Ex-ante Decision on Information Sharing

We are now in a position to determine when information sharing emerges as an equilibrium of our game. We focus on the regions Ψ_j with $j = \{0, 1, 2, 3\}$. At $t = 0$, the relationship bank decides whether to choose the information sharing regime or the no information sharing regime by comparing the expected profits in those two regimes. Let us call the relationship bank's expected profits at $t = 0$ with V_i, where like before $i = \{N, S\}$.

The relationship bank's expected profits under no information sharing regime is

$$V_N = [\Pr(G) + \Pr(B)\Pr(H)\Pr(\text{ no run })](R_N^* - r_N) .$$

In the good state, the relationship bank will always survive irrespective of the type of its loan. However, in the bad state the relationship bank holding an H-type loan will survive only if there is no bank run.[24] Without an information sharing scheme, the relationship bank cannot charge discriminative prices conditional on the borrower's type. Otherwise, it will reveal the borrower's type to the distant bank. Recall that the equilibrium deposit rate r_N under no information sharing regime is risky. That is, $r_N = \widehat{r}_N$ which is determined by $[\Pr(G) + \Pr(B)\Pr(H)\Pr(\text{no run})]\widehat{r}_N = r_0$. Therefore, we obtain

$$V_N = [\alpha + (1 - \alpha)\pi_0 \pi]R_N^* + (1 - \alpha)(1 - \pi_0)\pi R_N^* - \alpha(1 - \pi)\rho R_N^* - r_0 .$$

When the relationship bank participates in the information sharing regime, its expected profits V_S are

$$V_S = \Pr(\overline{D})[\Pr(H|\overline{D})V_S^H(\overline{D}) + \Pr(L|\overline{D})V_S^L(\overline{D})] + \Pr(D)V_S^L(D) , \qquad (13)$$

---

[24]Recall that we focus on the case where the relationship bank with an H-type loan will survive from bank run when sharing information but will fail otherwise.

where $V_S^H(\overline{D})$ and $V_S^L(\overline{D})$ are the expected profits of financing an H-type and an L-type borrower, respectively, when they generate a credit history $\overline{D}$. While $V_S^L(D)$ is the expected profit of financing an L-type borrower with default credit history D. Notice that when a loan has a credit history $\overline{D}$, with posterior probability $\Pr(H|\overline{D})$ it is an H-type loan. Moreover, $\Pr(D) = \Pr(L)\Pr(B) = (1-\alpha)(1-\pi_0)$ and $\Pr(\overline{D}) = 1 - \Pr(D) = \alpha + (1-\alpha)\pi$.

The expected profit of financing an H-type borrower with credit history $\overline{D}$ is

$$V_S^H(\overline{D}) = [\Pr(G) + \Pr(B)\Pr(\text{ no run })]R_S^*(\overline{D}) + \Pr(B)\Pr(\text{ run })P_S^B(\overline{D}) - r_0 \ .$$

Notice that, given that we focus on the case in which information sharing saves the relationship bank from illiquidity, we have $r_S(\overline{D}) = r_0$. Moreover, the relationship bank will hold an H-type loan to maturity if no bank run occurs because $P_S^B(\overline{D}) = \frac{\alpha\rho}{(1-\alpha)\pi+\alpha\rho}R_S^*(\overline{D}) < R_S^*(\overline{D})$. Similarly, the expected profit of financing an L-type borrower with credit history $\overline{D}$ is given by

$$V_S^L(\overline{D}) = \Pr(G)R_S^*(\overline{D}) + \Pr(B)P_S^B(\overline{D}) - r_0 \ .$$

When the relationship bank holds an L-type loan, the bank will sell it on the secondary market in State B, even without facing a run. Finally, a borrower that generates a default credit history D must be an L-type borrower. The equilibrium deposit rate is risky, that is $r_S(D) = r_0/\pi$. The expected profit of financing such a loan is

$$V_S^L(D) = \Pr(G)[R_S^*(D) - r_0/\pi] = \Pr(G)R_S^*(D) - r_0 \ .$$

Insert the expressions of $V_S^H(\overline{D})$, $V_S^L(\overline{D})$ and $V_S^L(D)$ into equation (13), and we get after rearranging

$$V_S = [\alpha + (1-\alpha)\pi_0\pi]R_S^*(\overline{D}) + (1-\alpha)(1-\pi_0)\pi R_S^*(D) - r_0 \ .$$

Information sharing is preferred by the relationship bank if and only if $V_S - V_N > 0$. The difference between the expected profits in the two regimes can be rewritten as

$$V_S - V_N = \underbrace{[\alpha + (1-\alpha)\pi_0\pi](R_S^*(\overline{D}) - R_N^*)}_{(1)} + \underbrace{(1-\alpha)(1-\pi_0)\pi(R_S^*(D) - R_N^*)}_{(2)} + \underbrace{\alpha(1-\pi)\rho R_N^*}_{(3)} \ .$$

The interpretation of the three terms is quite intuitive. Term (1) represents the *competition* effect, and it has a negative consequence on the adoption of the information sharing regime since $R_S^*(\overline{D}) \leq R_N^*$. Sharing information about the credit history encourages the

23

distant bank to compete for the borrower with good credit history, i.e. $\overline{D}$-history. The expected profits of the relationship bank is reduced due to this effect because the entrant bank undercuts the loan rate when $\overline{D}$-history is observed. Term (2) is understood as the *capturing* effect, and it has positive impact on sharing information since $R_S^*(D) \geq R_N^*$. Sharing information about the borrower with bad credit history, i.e. D-history, deters the entry of distant bank. Thus the relationship bank can discriminate the borrower with D-history by charging higher loan rate. The expected profits of the relationship bank increases due to this effect. Finally, Term (3) denotes the *liquidity* effect, which is always positive. Sharing credit information of a borrower with good credit history reduces the adverse selection in the secondary credit market. In State B, the relationship bank will be saved from potential bank run. This effect increases the expected profits of the relationship bank by avoiding costly asset liquidation.

The overall effect crucially depends if the capturing effect together with the liquidity effect dominate the competition effect. In that case the relationship bank chooses information sharing regime to maximize its expected profits. Denote with φ_j where $j = \{0, 1, 2, 3\}$ the set of parameters in which $V_S > V_N$ holds, then we have

Proposition 2 *The relationship bank chooses voluntarily to share information on $\varphi_j = \Psi_j$ with $j = \{0, 3\}$ and on $\varphi_j \subseteq \Psi_j$ with $j = \{1, 2\}$.*

Corollary 1 *When $\rho > (1 - \alpha)(1 - \pi_0)$, the relationship bank will choose to share credit information on $\varphi_j = \Psi_j$, $\forall j \in \{0, 1, 2, 3\}$.*

The proof is in the Appendix. The intuition is the following. In Cases 0 and 3 the set of parameters φ_j in which the relationship bank decides to share information coincides with the set Ψ_j in which information sharing saves the relationship bank from illiquidity. The reason is that there is no cost for the relationship bank to share information in both cases. In Case 0 because the distant bank never competes for the borrower, and in Case 3 because the distant bank always competes for the borrower. This is not true in Cases 1 and 2. In those two cases, the competition effect could overcome the sum of the capturing and the liquidity effect, and the relationship bank would find it profitable to not share information. This reduces the set of parameters φ_j in which sharing information is actually chosen versus the set of parameters Ψ_j in which is actually beneficial. However, when the probability of

a bank run is sufficiently high, the benefit from sharing information becomes sufficiently high that the relationship bank finds it convenient to share information.

[Put Figure 4 here]

Figure 4 shows Cases 0, 1, 2 and 3 corresponding to different degrees of loan market contestability. In each graph, the double-shaded area corresponds to the set of parameters φ_j. Clearly the double-shaded areas in Cases 0 and 3 correspond to the shaded areas in Figure 3. When ρ is low, the double-shaded areas in the graphs of Cases 1 and 2 are smaller than the corresponding areas in Figure 3 (the red line is the boundary of the double-shaded area in which the relationship bank voluntarily chooses to share information). When ρ is sufficiently high Figure 3 and 4 coincide.

3.2 Unverifiable Credit History

In this section we relax the assumption of verifiable credit history. If the reported borrower's credit history is not verifiable, the relationship bank that chooses to share such information may have an incentive to misreport the borrower's credit history after observing it. In particular, the relationship bank may have an incentive to overstate the borrower's credit history, that is to report a default credit history D as a non default credit history $\overline{D}$.[25] We have the following

Proposition 3 *The relationship bank truthfully discloses the borrower's credit history only if it leads to an increase in the loan rate for borrowers who have a default history D.*

Corollary 2 *Truth telling cannot be sustained for parameters in φ_j with $j = \{0,1\}$. It can be sustained for parameters in φ_2 when ρ is sufficiently low, and can be sustained for all parameters in φ_3.*

The proof in the Appendix. In order to sustain truthful reporting as an equilibrium, a necessary condition is that the relationship bank must suffer a loss when deviating from the

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[25]We assume misreporting $\overline{D}$ as D to be impossible, that is the relationship bank cannot claim non-defaulted borrower as defaulted. This is because borrowers have means and incentive to correct it or act against it (e.g., FCA in US). Moreover, according to the documentations in www.doingbusiness.com, borrowers can access their own credit record. A false report about defaulting can result in a legal dispute.

equilibrium strategy. Consider the case in which the relationship bank lends to an L-type of borrower, which generated a default credit history D. If the relationship bank truthfully reveals the credit history, it is able to charge the loan rate $R_S^*(D)$. Yet, the relationship bank will not survive if the state is bad (i.e., with probability $1 - \pi$), because the asset buyers know that a loan with a credit history D is L-type and will generate zero payoff in state B. If the relationship bank lies about the credit history, the asset buyers as well as the distant bank will perceive the borrower to be more likely an H-type. Accordingly, the loan rate charged by the relationship bank is $R_S^*(\overline{D})$, which could be lower than $R_S^*(D)$ due to the intensified competition. However, cheating gives the relationship bank more resilience against the future liquidity shock since it can sell the loan in the secondary market at the price $P_S^B(\overline{D}) > r_0$ when the state is bad. The relationship bank always survives when cheating. Thus, the relationship bank trades off the benefit of market liquidity (surviving in state B) versus the loss in profitability (potential decrease in loan rate from $R_S^*(D)$ to $R_S^*(\overline{D})$) when deciding to tell the truth about the reported credit history. Notice that a pre-requisite for the relationship bank to manipulate the reported credit history is that it must choose the information sharing regime in the first place. Thus, we focus our discussion on the intuition in each case, on the parameter sets φ_j, with $j = \{0, 1, 2, 3\}$, defined in Section 3.1.5.

Consider Case 0. We have $R_S^*(D) = R_S^*(\overline{D}) = R$ therefore the relationship bank always has incentive to misreport the true D-history as $\overline{D}$-history in the parameters space φ_0. The loan market is least contestable and we have $R_S^*(D) = R_S^*(\overline{D}) = R$. Assuming truthful reporting, ex-ante information sharing is more profitable for the relationship bank in the parameter set φ_0. However, when the relationship bank observes a credit history D ex-post, it will incur no loss in profit to misreport the credit history as $\overline{D}$ because $R_S^*(D) = R_S^*(\overline{D})$. Consequently, the relationship bank will always misreport the true D-history as $\overline{D}$-history in the parameters set φ_0. Truthfully reporting the credit history can never be an equilibrium in Case 0.

Since in the other cases we have $R_S^*(D) > R_S^*(\overline{D})$, there is hope for the relationship bank to report the true credit history. However, as noticed, this is only a necessary condition. Even if ex-post the relationship bank has an incentive to tell the truth, it is possible that ex-ante it is not willing to share information. The parameters that guarantee the ex-post truth telling have to be consistent with those that induce ex-ante information sharing.

Consider Case 1. On the one hand, assuming truthful reporting, the relationship bank ex-ante prefers an information sharing scheme when R is low. This is because its expected profit without sharing information is increasing in R ($R_N^* = R$), while the expected profit with information sharing is increasing in R only if the relationship bank lends to an L-type borrower. On the other hand, in order to make the relationship bank report the true credit history ex-post, R must be high. This is because the deviating penalty increases with R, that is $R_S^*(D) = R$ while $R_S^*(\overline{D})$ is an internal solution thus it does not depend on R. It turns out that the ex-ante and ex-post conditions on R determine an empty set, and therefore, truthful reporting cannot be sustained as an equilibrium in the parameter space φ_1.

Consider Case 2. On the one hand, assuming truthful reporting, the relationship bank ex-ante prefers an information sharing scheme when R is high. This is because the loan market becomes more contestable, the expected profit without information sharing does not depend on R any more (R_N^* becomes an internal solution), while the expected profit with information sharing is increasing in R (with L-type borrower, the loan rate is $R_S^*(D) = R$). On the other hand, in order to make the relationship bank report the true credit history ex-post, the return R must be high since, as in Case 1, $R_S^*(D) = R$ and $R_S^*(\overline{D})$ is an internal solution. It turns out that the ex-ante condition on R is more restrictive than the ex-post condition only if ρ is lower than a critical value $\widehat{\rho}$ (i.e., the value in which the relationship bank is indifferent between reporting the true credit history D and the false credit history $\overline{D}$). Under this condition, whenever the relationship bank finds ex-ante optimal to share information it also will report ex-post the true credit history, and truthful reporting can be sustained as an equilibrium in the parameter space φ_2.

Finally, consider Case 3. Assuming truthful reporting, the relationship bank ex-ante always prefers information sharing (irrespective of R). Moreover, the prime loan market is most contestable, $R_S^*(D) = \frac{c+r_0}{\pi} > R_S^*(\overline{D})$. It turns out that the relationship bank earns a strictly negative profit by ex-post misreporting D history with $\overline{D}$. This is because, $R_S^*(D)$ is substantially higher than $R_S^*(\overline{D})$, so the relationship bank's expected loss in profit overcomes its expected gain from market liquidity by misreporting the credit history. As a result, truthful reporting is sustained as an equilibrium in the parameter space φ_3.

To sum up, by truthfully reporting the credit history, the relationship bank can discriminate an L-type borrower by charging a higher loan rate. When misreporting the credit

history, the relationship bank has to charge a lower loan rate but benefits from the higher market liquidity to survive potential runs. If the market is less contestable, the profit from the discriminative loan pricing is bounded above by the loan's return R. Thus, in Case 0 and 1, the benefit from the higher market liquidity to save the bank from run in state B, dominates the loss in profit. The relationship bank will lie in those Cases. However, in Case 2 and 3, the return R is sufficiently large and the profit from discriminative loan pricing tends to dominate the benefit from market liquidity. Truthfully reporting the credit history can be sustained as equilibrium in those two Cases.

Figure 5 shows Cases 0, 1, 2 and 3 each with its respective dark-blue area corresponding to the set of parameters in which truth-telling is an equilibrium. In Case 0 and 1 such area is empty since truth-telling is not possible under these Cases. In Case 2 we show a situation where truth-telling can be sustained in a subset of φ_2, which occurs when $\rho < \min[\widehat{\rho}, (1-\alpha)(1-\pi_0)]$. In Case 3, since truth-telling is always sustained in the entire region φ_3, the green area coincides with the area in Figure 4.

[Put Figure 5 here]

4   Robustness and Discussion

To be written (computation available upon requests)

4.1   Timing

We explain why information sharing has to be an ex-ante decision. If the relationship bank only shares information when the liquidity risk strikes, no truthful information sharing can be sustained.

4.2   Risky H-type

In this section, we allow also the good type to have a positive bankruptcy probability. We show that our main result: information sharing increases market liquidity in the presence of adverse selection in both primary and secondary markets still holds. In contrast to our main model, We assume that H type loan defaults with probability $0 < \delta < 1$ in state B, while all other assumptions are maintained.

28

We again start the analysis by computing the loan rates without and with information sharing.

Without information sharing, the loan rate in the good state is still $P_N^G = R_N$. While in the bad state, the break-even condition of asset buyers is

$$Pr(L)(0 - P_N^B) + Pr(H)Pr(run)(\delta R_N - P_N^B) = 0$$

As a counterparty of equation 2, the asset price in state B changes to

$$P_N^B = \frac{\alpha\rho}{(1-\alpha)+\alpha\rho}\delta R_N. \tag{14}$$

Then, we compute the loan rates with information sharing. In the good state, we have $P_S^G(\overline{D}) = R_S(\overline{D})$ for a loan with non default history, and $P_S^G(D) = R_S(D)$ for a loan with default history. In the bad state, the break even condition of asset buyers for $\overline{D}$ loan is

$$\Pr(L \mid \overline{D})[0 - P_S^B(\overline{D})] + \Pr(H \mid \overline{D})\Pr(run)[\delta R_S(\overline{D}) - P_S^B(\overline{D})] = 0 \ ,$$

which implies

$$P_S^B(\overline{D}) = \frac{\alpha\rho}{(1-\alpha)\pi + \alpha\rho}\delta R_S(\overline{D}). \tag{15}$$

Second, we turn to the discussion of deposit rates. The discussion exactly follows the one in the main model, thus we focus on the break-even deposit rates: $\hat{r}_N$, $r_S(\overline{D})$ and $r_S(D)$. As before, without information sharing we have

$$[\Pr(G) + \Pr(B)\Pr(H)\Pr(\text{no run})\delta]\hat{r}_N = r_0 \ ,$$

which implies

$$\widehat{r}_N = \frac{r_0}{\pi + \alpha(1-\pi)(1-\rho)\delta} > r_0 \ . \tag{16}$$

With information sharing, because H loan is also allowed to default with probability δ, the less risky deposit rate charged will be

$$[Pr(G)+Pr(B)Pr(\text{no run})Pr(H|\overline{D})\delta+Pr(B)Pr(\text{no run})Pr(L|\overline{D})+Pr(B)Pr(run)]r_S(\overline{D}) = r_0$$

which implies

$$r_S(\overline{D}) = \frac{\alpha + (1-\alpha)\pi_0}{\alpha + (1-\alpha)\pi_0 - \alpha(1-\pi)(1-\rho)(1-\delta)}r_0 \tag{17}$$

Note that, when $\delta = 1$ $r_S(\overline{D})$ equals to r_0 again. For the loan with default history, the deposit rate is still $r_S(D) = \frac{1}{\pi}$.

Lastly, we calculate the loan rates. They are still derived from the distant bank's break even conditions in primary loan market competition. Without information sharing, the break-even condition is

$$\Pr(H)[Pr(G) + Pr(B)\delta](R_N^E - r_N^E) + \Pr(L)\Pr(G)(R_N^E - r_N^E) = c \ ,$$

We get

$$R_N^E = \frac{c + r_0}{\pi + \alpha(1 - \alpha)\delta} \ .$$

Without information sharing, the break-even condition for $\overline{D}$ loan is

$$\Pr(H|\overline{D})[Pr(G) + Pr(B)\delta][R_S^E(\overline{D}) - r_S^E(\overline{D})] + \Pr(L|\overline{D})\Pr(G)[R_S^E(\overline{D}) - r_S^E(\overline{D})] = c \ ,$$

We get

$$R_S^E(\overline{D}) = \frac{\alpha + (1 - \alpha)\pi_0}{\alpha[\pi + (1 - \pi)\delta] + (1 - \alpha)\pi_0\pi}(c + r_0) \ .$$

For a D loan, the break-even loan rate will be the same $R_S^E(D) = \frac{c+r_0}{\pi}$.

With the loan rates, asset prices in place, we can again follow the same procedure to analyze the benefit of information sharing. The result is presented in the following proposition.

Proposition 4 *When H type loan is allowed to default in state B with probability δ, our main results from Lemma 5 to Proposition 1 still hold. In each of the four cases, there always exist a range of parameters such that a relationship bank with a loan of $\overline{D}$ history will fail without information sharing, but will survive with information sharing.*

One can also check that there is no qualitative change in the endogenous information choice and the truth telling results.

4.3 Micro-foundation for deposit rates

We consider depositors strategically bid instead of being price takers. The equilibrium deposit rates will not change.

4.4 Mutual information sharing agreement

We model information sharing as a mutual agreement between the two banks instead of a unilateral decision.

5   Welfare and Policy Implication

We first notice what is the socially efficient level of information sharing. Suppose a benevolent social planner knows borrower's type, then the planner would always invest (all positive NPV projects). Moreover, there are two sources of frictions: i) information power of the relationship bank over the borrower ; ii) adverse selection in the secondary market for loan sale. Since both frictions are reduced by information sharing, from a social perspective maximum information sharing is preferred. Indeed, the planner does not care about friction i), but reducing friction ii) is better for everybody.

From a private perspective, relationship bank values information sharing since it reduces the adverse selection problem in the secondary asset market, enhancing asset market liquidity. But it also reduces market power vis a vis the borrower. This can generate a private level of information sharing that is less than the efficient one.

This is seen comparing the shaded areas in Figure 3 and the double-shaded areas in Figure 4. In Cases 0 and 3 the two areas coincide so there is no inefficient choice. However, in Cases 1 and 2 the relationship bank chooses a level of information sharing that is less than what would be (socially) optimal. In these Cases sharing information is costly, and the private cost of the relationship bank is higher than the social cost.

The endogenous rise of private registries is rational from the bank's point of view, but can be inefficiently low in some circumstances. A public registry can increase welfare in Cases 1 and 2, without harming welfare in Cases 0 and 3.

6   Conclusion

This paper formally analyzes the conjecture according to which banks' decision to share information about the credit history of their borrowers is driven by the needs for market liquidity. To meet urgent liquidity needs, banks have to make loan sale in the secondary market. However, the information friction in loan markets makes this sale costly, and good loans can be priced below their fundamental value. This concern became very evident during the financial crisis started in the summer of 2007. Several potentially solvent banks risk failures because they could not raise enough liquidity.

This basic observation implies that banks could find it convenient to share information on their loans in order to reduce the information asymmetry about their quality in case they

have to sell them in the secondary market. Information sharing can be a solution to reduce the cost of urgent liquidity needs so to make banks more resilient to funding risk. Clearly, sharing information makes banks lose the rent they extract if credit information was not communicated. Banks may be no longer able to lock in their loan applicants because competing banks also know about the quality of those loans. Eventually, the benefit of a greater secondary market liquidity has to be traded off with the loss in information rent. We show under which conditions information sharing is feasible, and when it is actually chosen by the banks in equilibrium.

We also show that our rationale for information sharing is robust to truth telling. A common assumption in the literature is that when banks communicate the credit information, they share it truthfully. We allow banks to manipulate the information they release by reporting bad loans as good ones. The reason for mis-reporting is for the banks to increase the liquidation value in the secondary market. We show that when banks lose too much in information rent from good borrowers with bad credit history, then information sharing is a truth telling device.

Consistent with previous theoretical models of information sharing, the existing empirical literature has mostly focused on the impact of information sharing on bank risks and firms' access to bank financing. Our theoretical contribution generates new empirical implications. In particular, information sharing should facilitate banks liquidity management and loan securitization. The model also suggests that information sharing can be more easily established, and work more effectively, in countries with a competitive banking sector, and in credit market segments where competition is strong.

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Appendix: Proofs

Proof of Lemma 2. TBC. Similarly to the proof of Lemma 1, for $r_0 < P_S^B(\overline{D}) < \widehat{r}_S(\overline{D})$, the depositor will also break even when the offered deposit rate is $\widehat{r}_S(\overline{D})$. This cannot be an equilibrium, because the relationship bank will optimally choose deposit rate equal to r_0 to benefit from a lower cost of funding and avoid the risk of bankruptcy. ∎

Proof of Lemma 5. Recall expressions (2) and (6) that determine equilibrium asset prices in the secondary market. They are

$$P_N^B = \frac{\alpha\rho}{(1-\alpha) + \alpha\rho} R_N^*$$

and

$$P_S^B(\overline{D}) = \frac{\alpha\rho}{(1-\alpha)\pi_0 + \alpha\rho} R_S^*(\overline{D}) \ ,$$

where R_N^* and $R_S^*(\overline{D})$ are the equilibrium loan rates under no information sharing and information sharing regime, respectively. Notice that the average loan quality in the secondary market without information sharing ($\frac{\alpha\rho}{(1-\alpha)+\alpha\rho}$) is lower than the average loan quality with information sharing ($\frac{\alpha\rho}{(1-\alpha)\pi_0+\alpha\rho}$).

Consider Case 0. The distant bank does not compete for any loan even if the relationship bank shared the credit history of the borrower. The relationship bank extracts the entire payoff of the loan irrespective of the information sharing regime, that is $R_S^*(\overline{D}) = R_N^* = R$. Information sharing solely brings in the benefit from boosting asset liquidity for loan with $\overline{D}$ history. Consequently, $P_S^B(\overline{D}) > P_N^B$.

Consider Case 2 (for the ease of exposition, it is convenient to analyze this case first). Distant bank competes both under information sharing (and the borrower has no default history $\overline{D}$) and when there is no information sharing. The equilibrium loan rates are therefore

$$R_N^* = \frac{c + r_0}{\alpha + (1-\alpha)\pi} > \frac{\alpha + (1-\alpha)\pi_0}{\alpha + (1-\alpha)\pi_0\pi}(c + r_0) = R_S^*(\overline{D}) \ .$$

We want to show that

$$P_N^B = \frac{\alpha\rho}{(1-\alpha) + \alpha\rho}\frac{c + r_0}{\alpha + (1-\alpha)\pi} < \frac{\alpha\rho}{(1-\alpha)\pi_0 + \alpha\rho}\frac{\alpha + (1-\alpha)\pi_0}{\alpha + (1-\alpha)\pi_0\pi}(c + r_0) = P_S^B(\overline{D}) \ ,$$

which can be rewritten as

$$\frac{(1-\alpha)\pi_0 + \alpha\rho}{(1-\alpha) + \alpha\rho}\frac{\alpha + (1-\alpha)\pi_0\pi}{[\alpha + (1-\alpha)\pi_0][\alpha + (1-\alpha)\pi]} < 1 \ .$$

34

To show that the last inequality holds, we notice that the ratio $\frac{(1-\alpha)\pi_0 + \alpha\rho}{(1-\alpha) + \alpha\rho}$ is increasing in ρ, so its maximum value is reached when $\rho = 1$ and it equal to $(1 - \alpha)\pi_0 + \alpha$ $(= Pr(\overline{D}))$. Therefore, the maximum value of the LHS of the last inequality can written as

$$[(1 - \alpha)\pi_0 + \alpha]\frac{\alpha + (1 - \alpha)\pi_0\pi}{[\alpha + (1 - \alpha)\pi_0][\alpha + (1 - \alpha)\pi]} = \frac{\alpha + (1 - \alpha)\pi_0\pi}{\alpha + (1 - \alpha)\pi} \ ,$$

which is smaller than 1 since $\pi \in (0, 1)$. Thus, $P_S^B(\overline{D}) > P_N^B$.

Consider Case 1. The distant bank only competes for the loan with a credit history $\overline{D}$. The equilibrium loan rate $R_S^*(\overline{D})$ is determined by the distant bank. Without information sharing, the relationship bank can discriminate the borrower by charging $R_N^* = R > R_S^*(\overline{D})$. The competition effect is clearly smaller than under Case 2. Since $P_S^B(\overline{D}) > P_N^B$ always holds in Case 2, then it necessarily holds also in Case 1.

Consider Case 3. The distant bank competes no matter the past history of the borrower. The relevant equilibrium loan rates R_N^* and $R_S^*(\overline{D})$ do not change with respect Case 2. The relationship between the prices $P_S^B(\overline{D})$ and P_N^B is the same as the one analyzed in Case 2. Thus, $P_S^B(\overline{D}) > P_N^B$.

Since we have that is all cases $P_N^B < P_S^B(\overline{D})$, by continuity when r_0 is located in between these two prices the relationship bank survives from illiquidity under information sharing regime and fails under no information sharing regime. ∎

Proof of Proposition 2. For each Case $j = \{0, 1, 2, 3\}$ we consider the parameter set Ψ_j defined in Proposition 1.

Note that,

$$V_S = [\alpha + (1 - \alpha)\pi_0\pi]R_S^*(\overline{D}) + (1 - \alpha)(1 - \pi_0)\pi R_S^*(D) - r_0,$$

while

$$V_N = [\pi + (1 - \pi)\alpha(1 - \rho)]R_N^* - r_0$$
$$= [\alpha + (1 - \alpha)\pi_0\pi]R_N^* + (1 - \alpha)(1 - \pi_0)\pi R_N^* - \alpha(1 - \pi)\rho R_N^* - r_0$$

So the difference in profit with and without information sharing:

$$V_S - V_N = [\pi + (1 - \pi)\alpha(1 - \rho)][R_S^*(\overline{D}) - R_N^*]$$
$$+ (1 - \alpha)(1 - \pi_0)\pi[R_S^*(\overline{D}) - R_N^*]$$
$$+ \alpha(1 - \pi)\rho R_N^*$$

35

In Consider Case 0. We have: $V_S = [\alpha + (1-\alpha)\pi]R - r_0$ and $V_N = [\alpha - \alpha(1-\pi)\rho + (1-\alpha)\pi]R - r_0$. Then $V_S > V_N$ for the entire region Ψ_0. Thus $\varphi_0 = \Psi_0$.

In Consider Case 1. We have:

$$V_S - V_N = [\alpha + (1-\alpha)\pi_0](c + r_0) - [(1-\alpha)\pi_0\pi + \alpha - \alpha(1-\pi)\rho]R \ .$$

Notice that $(1-\alpha)\pi_0\pi + \alpha - \alpha(1-\pi)\rho > 0$. We have that $V_S - V_N > 0$ if and only if

$$R < \frac{\alpha + (1-\alpha)\pi_0}{\alpha - \alpha(1-\pi)\rho + (1-\alpha)\pi_0\pi}(c + r_0) \equiv R_1 \ .$$

We define the region φ_1 as follows

$$\varphi_1 = \Psi_1 \bigcap \{R|R < R_1\} \subseteq \Psi_1 \ .$$

If R_1 is greater than the upper bound R_N^E of R defining Case 1 then information sharing is preferred for the entire region Ψ_1. That is, if

$$R_1 = \frac{\alpha + (1-\alpha)\pi_0}{\alpha - \alpha(1-\pi)\rho + (1-\alpha)\pi_0\pi}(c + r_0) > \frac{1}{\alpha + (1-\alpha)\pi}(c + r_0) = R_N^E$$

the set φ_1 coincides with Ψ_1. We can simplify the last inequality as

$$\rho > (1-\alpha)(1-\pi_0) \ .$$

Otherwise, when $\rho < (1-\alpha)(1-\pi_0)$, we have $\varphi_1 \subset \Psi_1$. Recall the definition of region Ψ_1, we always have such $\varphi_1 = \Psi_1 \bigcap \{R|R < R_1\}$ non-empty for any value of $\rho \in (0,1)$ and $\varphi_1 \subset \Psi_1$ when $\rho < (1-\alpha)(1-\pi_0)$.

Consider Case 2. We have:

$$V_S - V_N = \left[\alpha + (1-\alpha)\pi_0 - 1 + \frac{\alpha(1-\pi)\rho}{\alpha + (1-\alpha)\pi}\right](c + r_0) + (1-\alpha)(1-\pi_0)\pi R; \ .$$

We have $V_S - V_N > 0$ if and only if

$$R > \left[1 - \frac{1-\pi}{1-\pi_0}\frac{\alpha\rho}{(1-\alpha)[\alpha + (1-\alpha)\pi]}\right]\frac{c + r_0}{\pi} \equiv R_2 \ .$$

We define the set φ_2 as follows

$$\varphi_2 = \Psi_2 \bigcap \{R|R > R_2\} \subseteq \Psi_2 \ .$$

If R_2 is lower than the lower bound of R defining Case 2 then information sharing is preferred for the entire region Ψ_2. That is, if

$$\left[1 - \frac{1-\pi}{1-\pi_0}\frac{\alpha\rho}{(1-\alpha)[\alpha + (1-\alpha)\pi]}\right]\frac{c + r_0}{\pi} < \frac{c + r_0}{\alpha + (1-\alpha)\pi}$$

the set $\varphi_2 = \Psi_2$. We can simplify the last inequality again as

$$\rho > (1 - \alpha)(1 - \pi_0) \ .$$

Otherwise, when $\rho < (1 - \alpha)(1 - \pi_0)$ we have $\varphi_2 \subset \Psi_2$. Indeed, also R_2 is decreasing in ρ. When $\rho \to 0$, we have $R_2 \to \frac{c + r_0}{\pi} = R_S^E(D)$. Recall the definition of region Ψ_2, we always have such φ_2 non-empty for all $\rho \in (0, 1)$ and $\varphi_2 \subset \Psi_2$ when $\rho < (1 - \alpha)(1 - \pi_0)$.

Consider Case 3. We have $V_S = c$ and $V_N = c - \alpha(1 - \pi)\rho\frac{c + r_0}{\alpha + (1 - \alpha)\pi}$, therefore

$$V_S - V_N = \alpha(1 - \pi)\rho\frac{c + r_0}{\alpha + (1 - \alpha)\pi} > 0 \ .$$

In this case we have $\varphi_3 = \Psi_3$ and information sharing is preferred by the relationship bank. ∎

Proof of Proposition 3. Suppose the distant bank, depositors and asset buyers all hold the belief that the relationship bank will tell the truth about the credit history of the borrower. We analyze the profitable deviation of the relationship bank to announce truthfully a defaulted D-history under such belief. We focus our discussion on the parameter set φ_j with $j = \{0, 1, 2, 3\}$ defined in Proposition 2.

Consider Case 0. We first compute the relationship bank's expected profit at $t = 1$ of truthfully reporting a loan with default credit history D. Recalling that $R_S^*(D) = R$ in this case, we have

$$V_S(D) = \pi R_S^*(D) - r_0 = \pi R - r_0 \ . \tag{18}$$

The expected profit of misreporting the borrower's true credit history (i.e., reporting the false $\overline{D}$-history) is

$$V_S(D, \overline{D}) = \Pr(G)R_S^*(\overline{D}) + \Pr(B)P_S^B(\overline{D}) - r_0 = \pi R + (1 - \pi)\frac{\alpha\rho}{\alpha\rho + (1 - \alpha)\pi_0}R - r_0 \ .$$

Notice the relationship bank does not fail by misreporting the credit history. Clearly we have $V_S(D) - V_S(D, \overline{D}) < 0$. The relationship bank finds it profitable to misreport the borrower's credit history. The benefit from the deviation $(1 - \pi)\frac{\alpha\rho}{\alpha\rho + (1 - \alpha)\pi_0}R$ is the expected liquidation loss in case of bank run. Under this case, the belief of outsiders can not be rationalized, and truthful information sharing can not be sustained as a Perfect Bayesian Equilibrium in the set of parameter φ_0.

Consider Case 1. Like in Case 0, the relevant equilibrium loan rate is $R_S^*(D) = R$. Then reporting the true default history gives the same expected profit as in (18). The expected

profit of misreporting the true credit history with the false $\overline{D}$-history can be expressed as

$$V_S(D, \overline{D}) = \Pr(G)R_S^*(\overline{D}) + \Pr(B)P_S^B(\overline{D}) - r_0$$
$$= \pi \frac{\alpha + (1-\alpha)\pi_0}{\alpha + (1-\alpha)\pi_0\pi}(c + r_0) + (1 - \pi)\frac{\alpha\rho}{\alpha\rho + (1-\alpha)\pi_0}\frac{\alpha + (1-\alpha)\pi_0}{\alpha + (1-\alpha)\pi_0\pi}(c + r_0) - r_0 \ ,$$

since $R_S^*(\overline{D}) = \frac{\alpha + (1-\alpha)\pi_0}{\alpha + (1-\alpha)\pi_0\pi}(c + r_0)$ in this Case. Then we have

$$V_S(D, \overline{D}) = \frac{\alpha\rho + (1-\alpha)\pi_0\pi}{\alpha\rho + (1-\alpha)\pi_0}\frac{\alpha + (1-\alpha)\pi_0}{\alpha + (1-\alpha)\pi_0\pi}(c + r_0) - r_0 \ . \tag{19}$$

Then the ex-post incentive compatibility constraint to tell the truth is

$$V_S(D) - V_S(D, \overline{D}) = \pi R - \frac{\alpha\rho + (1-\alpha)\pi_0\pi}{\alpha\rho + (1-\alpha)\pi_0}\frac{\alpha + (1-\alpha)\pi_0}{\alpha + (1-\alpha)\pi_0\pi}(c + r_0) > 0 \ ,$$

which can be simplified as

$$R > \left[\frac{\alpha\rho + (1-\alpha)\pi_0\pi}{\alpha\rho + (1-\alpha)\pi_0}\frac{\alpha + (1-\alpha)\pi_0}{\alpha + (1-\alpha)\pi_0\pi}\right]\frac{c + r_0}{\pi} \equiv \underline{R} \ . \tag{20}$$

Information sharing is ex-ante chosen in Case 1 when (recall the definition of R_1 in the proof of Proposition 2)

$$R < \frac{\alpha + (1-\alpha)\pi_0}{\alpha - \alpha(1-\alpha)\rho + (1-\alpha)\pi_0\pi}(c + r_0) \equiv R_1 \ .$$

It can be calculated that

$$\frac{1}{R_1} - \frac{1}{\underline{R}} = \frac{1}{\alpha + (1-\alpha)\pi_0}\frac{\alpha^2(1-\rho)\rho(1-\pi)}{\alpha\rho + (1-\alpha)\pi_0\pi}\frac{1}{c + r_0} > 0$$

Consequently, $R_1 < \underline{R}$, there exists no R such that the relationship bank will ex-ante participate in an information sharing scheme and ex-post report the true default credit history of a borrower. The belief of outsiders can not be rationalized, and truthful information sharing can not be sustained as a Perfect Bayesian Equilibrium in the set of parameter φ_1.

Consider Case 2. We again have $R_S^*(D) = R$. Reporting the true default history gives the same expected profit as in (18). The expected profit of misreporting the true credit history is the same as in expression (19), since $R_S^*(\overline{D}) = \frac{\alpha + (1-\alpha)\pi_0}{\alpha + (1-\alpha)\pi_0\pi}(c + r_0)$ also in this Case. Therefore the condition on R to ensure ex-post the relationship bank tells the truth is the same as in (20). Information sharing is ex-ante chosen in Case 2 when (recall the definition of R_2 in the proof of Proposition 2)

$$R > \left[1 - \frac{1 - \pi}{1 - \pi_0}\frac{\alpha\rho}{(1-\alpha)[\alpha + (1-\alpha)\pi]}\right]\frac{c + r_0}{\pi} \equiv R_2 \ .$$

Information sharing can be sustained as a Perfect Bayesian Equilibrium only if both the inequality $R > R_2$ and the condition (20) are satisfied. In particular, we find a region of parameters in which whenever it is ex-ante optimal for the relationship bank to share information is also ex-post profitable for it to tell the true credit history. This implies to impose the following restriction

$$\left[1 - \frac{1-\pi}{1-\pi_0}\frac{\alpha\rho}{(1-\alpha)[\alpha+(1-\alpha)\pi]}\right] > \frac{\alpha\rho+(1-\alpha)\pi_0\pi}{\alpha\rho+(1-\alpha)\pi_0}\frac{\alpha+(1-\alpha)\pi_0}{\alpha+(1-\alpha)\pi_0\pi} \quad . \tag{21}$$

We define a function

$$F(\rho) = \left[1 - \frac{1-\pi}{1-\pi_0}\frac{\alpha\rho}{(1-\alpha)[\alpha+(1-\alpha)\pi]}\right] - \frac{\alpha\rho+(1-\alpha)\pi_0\pi}{\alpha\rho+(1-\alpha)\pi_0}\frac{\alpha+(1-\alpha)\pi_0}{\alpha+(1-\alpha)\pi_0\pi}.$$

It can be checked that

$$F'(\rho) = -\frac{1-\pi}{1-\pi_0}\frac{\alpha}{(1-\alpha)[\alpha+(1-\alpha)\pi]} - \frac{\alpha(1-\alpha)\pi_0(1-\pi))}{[\alpha\rho+(1-\alpha)\pi_0]^2}\frac{\alpha+(1-\alpha)\pi_0}{\alpha+(1-\alpha)\pi_0\pi} < 0 \quad .$$

Moreover, we can take the limits

$$\lim_{\rho\to 0} F(\rho) = 1 - \frac{\alpha\pi+(1-\alpha)\pi_0\pi}{\alpha+(1-\alpha)\pi_0\pi} = \frac{\alpha(1-\pi)}{\alpha+(1-\alpha)\pi_0\pi} > 0$$

$$\lim_{\rho\to 1} F(\rho) = 1 - \frac{1-\pi}{1-\pi_0}\frac{\alpha}{(1-\alpha)[\alpha+(1-\alpha)\pi]} - 1 = -\frac{1-\pi}{1-\pi_0}\frac{\alpha}{(1-\alpha)[\alpha+(1-\alpha)\pi]} < 0 \quad .$$

Thus, there exists a unique $\hat{\rho}$ such that $F(\hat{\rho}) = 0$. Whenever $0 < \rho < \hat{\rho}$, we have $F(\rho) > 0$, and expression (21) holds. Then truth telling can be sustained as a Perfect Bayesian Equilibrium in the set of parameter φ_2. Recall that we established in Proposition 2 that φ_2 is non-empty for all $\rho \in (0,1)$.

Consider Case 3. In this Case we have $R_S^*(D) = (c + r_0)/\pi$ since the distant bank competes also for the defaulted borrower. Reporting the true default history gives an expected profit equal to

$$V_S(D) = \pi R_S^*(D) - r_0 = c \quad .$$

The expected profit of misreporting the credit history is the same as in (19), and since

$$\frac{\alpha\rho+(1-\alpha)\pi_0\pi}{\alpha\rho+(1-\alpha)\pi_0}\frac{\alpha+(1-\alpha)\pi_0}{\alpha+(1-\alpha)\pi_0\pi} < 1 \quad ,$$

we have $V_S(D,\overline{D}) - V_S(D) < 0$. The belief of outsiders can be rationalized, and truthful information sharing can be sustained as a Perfect Bayesian Equilibrium in the set of parameters φ_3. ∎