Loan Insurance, Adverse Selection and Screening*

Toni Ahnert Martin Kuncl

June 5, 2018

Abstract

We propose a parsimonious model of loan default insurance with costly screening in primary markets and adverse selection in secondary markets. Lenders are heterogeneous in identifying high-quality borrowers and learn loan quality over time. With a competitive market for loan insurance, the usual equilibrium with an illiquid secondary market ceases to exist. The unique equilibrium exhibits liquid markets, in which high-quality loans of liquidity-shocked lenders and low-quality loans are sold to uninformed outside financiers, and higher welfare. Loan insurance exerts a positive externality on the price for uninsured loans via reduced adverse selection. A Pigouvian subsidy implements the constrained efficient allocation. It can be decentralized via subsidizing loan insurance, which Pareto-dominates outright purchases of uninsured loans in the secondary market. Our theory helps to evaluate policy options aimed at eliminating market freezes and contributes to the debate on regulatory reform.

JEL classifications: G01, G21, G28.

*Both authors are at the Bank of Canada. These are our views and not necessarily those of the Bank of Canada. Please send correspondence to Toni Ahnert, Financial Studies Division, Bank of Canada, 234 Wellington St, Ottawa, ON K1A 0G9, Canada. Email: toni.ahnert@gmail.com.

1 Introduction

Secondary markets for loans allow lenders access to market funding through loan sales. In the financial crisis of 2007-09, however, this funding source quickly dried up, perhaps due to low lending standards and adverse selection in secondary markets. One crisis response was an attempt to 'liquify' markets for securitized loans with a series of policies that included purchases of illiquid loans (TARP). In contrast, one category without market dry-up was loans insured by the government against default before the crisis. This category is large: in the US, 70% of all new residential mortgages are insured directly by the federal government or guaranteed by Government Sponsored Entreprises (GSE).[1] It is widely believed that this insurance was underpriced such that the government subsidized loan insurance (Congressional Budget Office, 2014).

This episode raises several important questions. What is the impact of loan insurance on the secondary market liquidity (allocation efficiency) and on lending standards (lending efficiency)? Is there a role for government intervention? If so, how do subsidies of loan insurance compare to policies of outright asset purchases?

In this paper, we propose a parsimonious model of lending with screening in primary markets and adverse selection in secondary markets. There are three dates and many risk-neutral and competitive lenders and outside financiers. Before extending a loan at an initial date, each lender chooses whether to screen at a cost in order to identify a high-quality borrower (who never default). Without screening, a lender faces a pool of high- and low-quality borrowers (who always default). The screening cost is heterogeneous across lenders, which helps to construct an interior equilibrium. The screening cost and screening choice are private information to the lender.

[1] About 50% of the total amount of mortgages are insured by Fannie Mae and Freddie Mac (GSEs with government backing) and the remaining 20% are insured mostly by the Federal Housing Administration (Congressional Budget Office, 2014).

At an interim date, all lenders privately learn the quality of their borrowers (Plantin, 2009). That is, non-screening lenders learn which of their borrowers will repay or default. Independent of their screening choice, a fraction of lenders is hit with a liquidity shock (a superior consumption or investment opportunity or a run in the case of a bank with fragile capital structure). Each lender privately observes whether the liquidity shock occurs. Because of the asymmetric information between lenders and outside financiers, the secondary market for loans is subject to adverse selection. A lender may sell high-quality loans because of the liquidity shock or low-quality loans. In the former case, there are gains from trade because of the lender's higher valuation of funds. In the latter case, the lender sells lemons. The secondary market price is set for deep-pocketed outside financiers to break even in expectation.

The benchmark model without loan insurance has multiple equilibria. An equilibrium with an illiquid secondary market always exists. We describe when the Pareto-superior equilibrium with liquid market exists, show that it is unique, and characterize its comparative statics. Our first contribution is to show that the market price can be non-monotonic in the share of high-quality borrowers. Although a higher average borrower quality directly supports the price, it also indirectly reduces the price through lower screening incentives. The equilibrium with liquid market displays a feedback between market liquidity and screening. The extent of adverse selection determines the liquidity in the secondary market, while screening incentives determine lending standards. More screening raises the price but a higher price reduces screening incentives due to the option to sell low-quality loans to uninformed financiers.

Our second contribution is to study competitive insurance against loan default. Such insurance allows non-screening lenders to pass default risk to insurers (outside financiers) before learning the quality of their loan. Thus, observable loan insurance effectively induces lenders to reveal their screening choice. Insured lenders and financiers are symmetrically informed and insured loans are free from both risk and adverse selection. With loan insurance, the illiquid equilibrium ceases to exist. If the price were zero, all non-screening lenders would insure their loans, so all uninsured loans offered for sale would be of high-quality, contradicting the supposed zero price.

Loan insurance is socially beneficial. It allows lenders to commit to not using their future private information about loan quality. Thus, loan insurance publicly reveals a lender's screening choice, since only non-screening lenders choose to insure loans. Loan insurance lowers adverse selection and increases the price in the secondary market for uninsured loans and, therefore, reduces screening incentives. For parameter values that induce loan insurance in the equilibrium with liquid markets, welfare increases relative to the benchmark model without loan insurance.

A planner attains the constrained efficient allocation by choosing the fraction of insured loans on behalf of all lenders to maximize welfare. In contrast to individual lenders, the planner internalizes the positive pecuniary externality of loan insurance on the secondary market price of uninsured loans. More insurance reduces the degree of adverse selection in the secondary market because fewer low-quality uninsured loans are sold. Compared to the decentralized equilibrium, the planner increases insurance at both the intensive margin (a larger fraction is insured) and the extensive margin (insurance occurs for a larger range of parameters). Higher usage of insurance increases secondary market price for uninsured loans and lowers screening incentives.

Our third contribution is to show how the constrained efficient allocation can be implemented. We consider a regulator subject to a balanced budget and without any informational advantage over outside financiers. A Pigouvian subsidy of loan insurance funded with lump-sum taxation of lenders achieves the constrained efficient allocation. This subsidy of the observable loan insurance choice induces price-taking lenders to internalize the positive externality on the secondary market price. Subsidizing outright purchases of uninsured loans in the secondary market does not achieve constrained efficiency. The intuition for the inferior welfare outcome is the higher cost due to purchases of lemons sold by informed lenders who choose not to screen.

Our normative implications contribute to the debate on the efficiency of policies designed to intervene in frozen markets plagued by adverse selection (Tirole, 2012; Philippon and Skreta, 2012; Fuchs and Skrzypacz, 2015; Chiu and Koeppl, 2016; Camargo et al., 2016). To the best of our knowledge, this paper is the first to show that insurance against loan default can help prevent market freezes and alleviate adverse selection in secondary markets for uninsured loans. Another contribution is to show that loan insurance subsidies can be more effective than asset purchases.

By uncovering a new beneficial welfare effect of loan insurance, we contribute to the debate on the reform of GSEs, contrasting with their negative effects studied after the financial crisis (e.g., Jeske et al. (2013); Elenev et al. (2016)). Our paper also relates to the literature on the trade-off between the quality of originated assets and their secondary market liquidity (Parlour and Plantin, 2008; Vanasco, 2017), building on the learning-by-holding literature (Plantin, 2009). In contrast to these papers, we examine the positive and normative implications of loan insurance on the quality of originated assets, secondary market liquidity, optimal second-best regulation.

The remainder of the paper is organized as follows. Section 2 presents the benchmark model with loan screening and adverse selection in secondary markets and its equilibrium is studied in section 3. Section 4 introduces competitive loan insurance against default. Section 5 characterizes the welfare benchmark of constrained efficiency to which we compare policies of subsidized insurance and subsidized uninsured loan purchases. Section 6 considers model extensions. All proofs are in the Appendix.

2 Model

There are three dates $t = 0, 1, 2$, a single good for consumption and investment, and universal risk-neutrality. A continuum of lenders $i \in [0, 1]$ has one unit of funds at $t = 0$ for loans. Loan quality is initially unknown: a fraction $\mu \in (0, 1)$ of loans pays $A > 1$ ('high-quality') at $t = 2$ and the fraction $1 - \mu$ and pays 0 ('low-quality').

By screening at $t = 0$, a lender identifies high-quality loans. Without screening, lenders can finance average-quality loans with expected payoff μA (see Figure 1). The non-pecuniary screening cost η_i is distributed across lenders according to a density function $f(\eta)$ with support $[0, \overline{\eta}]$ and cumulative distribution function $F(\eta)$. The screening choice $s_i \in \{0, 1\}$ is private information to lender i. Independent of screening, all lenders privately learn the quality of their loans at $t = 1$.

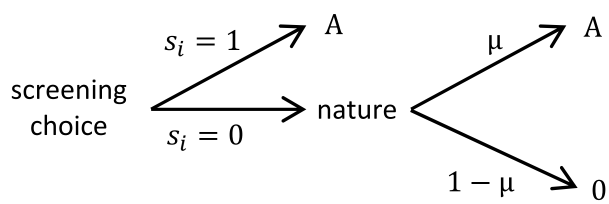

Figure 1: Loan payoffs and screening.

Lenders face an idiosyncratic liquidity shock at $t = 1$: their preference for interim-date consumption is $\lambda_i \in \{1, \lambda\}$ with $\lambda > 1$. The liquidity shock is identically and independently distributed across lenders and independent of loan quality. A shock arises with probability $\nu = Pr\{\lambda_i = \lambda\} \in (0, 1)$. The preferences of lenders are

$$u_i(c_{i1}, c_{i2}) = \lambda_i c_{i1} + c_{i2} - \eta_i s_i, \tag{1}$$

where c_{it} denotes consumption of lender i at date t. At $t = 1$, lenders can sell loans $q_i \in [0, 1]$ in a secondary market at a competitive price p to deep-pocketed outside financiers. These potential buyers are uninformed about the screening choices, liquidity shocks of lenders, and loan qualities. Figure 2 shows the timeline of events.

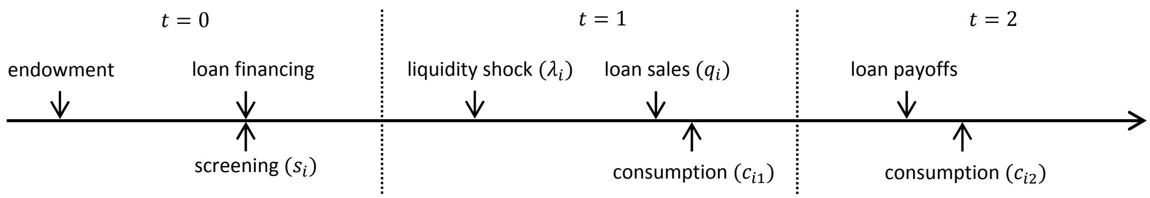

Figure 2: Timeline.

3 Equilibrium

Definition 1. *A competitive pure-strategy equilibrium comprises choices of screening* $\{s_i^*\}$*, sales in the secondary market* $\{q_i^*\}$*, and a market price* p^* *such that:*

1. *At* $t = 1$*, each lender* i *optimally chooses its sales in the secondary market for each realized liquidity shock* $\lambda_i \in \{1, \lambda\}$*, which is denoted by* $q_i^*(s_i^*, \lambda_i)$*, given the price* p^* *and the screening choice* s_i^**.*

2. *At* $t = 1$*, the price* p^* *is set for outside financiers to break even in expectation, given the screening choices of* $\{s_i^*\}$ *and sales schedules* $\{q_i^*(\cdot)\}$ *of all lenders.*

3. *At* $t = 0$*, each lender* i *chooses its screening* s_i *to maximize expected utility, given the price* p^* *and the sales schedule* $q_i^*(\cdot)$*:*

$$\max_{s_i, c_{i1}, c_{i2}} \quad E_\lambda \left[\lambda_i c_{i1} + c_{i2} - s_i \eta_i\right] \qquad \textit{subject to}$$
$$c_{i1} \leq q_i^*(s_i, \lambda_i)\, p^*,$$
$$c_{i2} \leq A\big[\mu + (1-\mu)\, s_i\big]\big[1 - q_i^*(s_i, \lambda_i)\big].$$

We focus on symmetric equilibria in which lenders use a threshold strategy. Each lender with a screening cost below a threshold $\eta^* \in [0, \overline{\eta}]$ chooses to screen:

$$s_i^* = \mathbf{1}\Big\{\eta_i \leq \eta^*\Big\}, \tag{2}$$

where $\mathbf{1}\{\cdot\}$ is the indicator function. Since the binary screening cost is the only source of heterogeneity across lenders and lender expected utility strictly decreases in this cost, our emphasis on threshold strategies is without loss of generality.

Sales in secondary market Due to the asymmetric information between lenders and outside financiers at $t = 1$, there is adverse selection. The price satisfies $p^* \in [0, A]$, so lenders always choose to sell low-quality loans and not to sell high-quality loans unless hit by a liquidity shock. A defining feature of an equilibrium is whether lenders sell high-quality loans when hit by a shock, which occurs whenever

$$p^* \lambda \geq A. \tag{3}$$

If condition (3) holds, we label it a good equilibrium. There always exists a bad equilibrium without sales of high-quality loans, $p^* = 0$, and there may also exist an unstable mixing equilibrium, in which some high-quality loans are sold. We focus on good equilibria because (i) it is the Pareto-dominant equilibrium; and (ii) it is the only equilibrium that survives the introduction of loan insurance in section 4.

In this equilibrium, sales in the secondary market at $t = 1$ are:

$$q_i^*(s_i^*, \lambda_i) = \mathbf{1}\Big\{\lambda_i = \lambda\Big\} + (1 - \mu)\mathbf{1}\Big\{\lambda_i = 1\Big\}\mathbf{1}\Big\{s_i^* = 0\Big\}. \tag{4}$$

Price in secondary market In equilibrium, the secondary market price at $t = 1$ ensures that outside financiers break even in expectation. Low-cost lenders $\eta_i \leq \eta^*$ (of mass $F(\eta^*)$) sell loans of high quality (worth A) when hit by a liquidity shock. High-cost lenders (of mass $1 - F(\eta^*)$) sell loans of average quality (worth μA) when hit by the liquidity shock and always sell loans of low quality (worth 0):

$$p^* = \nu A \frac{\mu + (1-\mu) F(\eta^*)}{\nu + (1-\nu)(1-\mu)(1-F(\eta^*))} \equiv p^*(\eta^*), \tag{5}$$

where we highlight the dependence of the equilibrium price on the screening threshold. More screening leads to fewer aggregate investment in low-quality loans at $t = 0$ and, therefore, reduces the degree of adverse selection at $t = 1$, $\frac{dp^*}{d\eta^*} > 0$. As a result, screening imposes a positive pecuniary externality as it raises the equilibrium price.

Screening An indifference condition of the marginal lender between screening and not screening determines the equilibrium threshold of the screening cost η^*. Screening allows to identify high-quality loans that are only sold on the secondary market after a liquidity shock, yielding $\nu\lambda p^* + (1-\nu)A - \eta^*$ to the marginal lender. Not screening results in sales of all loans with the exception of high-quality ones without a liquidity shock, yielding $\nu\lambda p^* + (1-\nu)(\mu A + (1-\mu)p^*)$. Equalizing these payoffs, the screening threshold for a given secondary market price is:

$$\eta^* = (1-\nu)(1-\mu)(A - p^*) \equiv \eta^*(p^*). \tag{6}$$

Intuitively, a low-cost lender benefits from the higher payoff $A - p^*$ only when not hit by a liquidity shock and for the fraction of low-quality loans. A higher price (e.g. due to lower degree of adverse selection) reduces the benefit of screening, $\frac{d\eta^*}{dp^*} < 0$.

Figure 3 shows the construction of the good equilibrium.

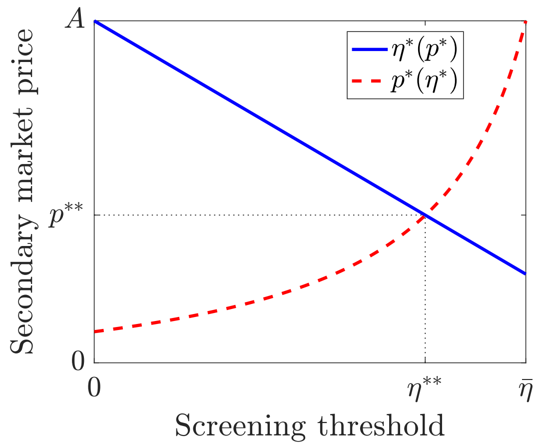

Figure 3: Existence of a unique good equilibrium.
Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\mu = 0.5$, $\lambda = 3$, $A = 3$.

Proposition 1. ***Existence of a unique good equilibrium.*** *If the liquidity shock is sufficiently large, $\lambda \geq \underline{\lambda} \equiv \frac{\nu+(1-\nu)\mu}{\nu\mu}$, then there exists a unique equilibrium in the class of good equilibria in which lenders sell all high-quality loans after a shock. This interior equilibrium is characterized by a secondary market price $p^{**} \in (0, A)$ and a cost threshold $\eta^{**} \in (0, \bar{\eta})$ below which all lenders screen.*

Proof. See Appendix A.1. ∎

We henceforth assume that this lower bound on the size of the liquidity shock holds. We turn to the impact of parameters on the good equilibrium.

Proposition 2. ***Comparative statics of good equilibrium.***

1. *The screening threshold increases in the payoff of high-quality loans, $\frac{d\eta^{**}}{dA} > 0$, and after a first-order stochastic dominance improvement of the cost distribution. It decreases in the fraction of high-quality loans, $\frac{d\eta^{**}}{d\mu} < 0$, and in the probability of a liquidity shock, $\frac{d\eta^{**}}{d\nu} < 0$.*

2. *The secondary-market price increases in the payoff of high-quality loans,* $\frac{dp^{**}}{dA} > 0$*, and decreases after a first-order stochastic dominance improvement of the cost distribution. Both the screening threshold and the price are independent of the liquidity preference.*

3. *If* $\eta_0^{**}\frac{f(\eta_0^{**})}{1-F(\eta_0^{**})} > 1$*, where* $\eta_0^{**} \equiv \lim_{\mu\to 0}\eta^{**}$*, then the market price is non-monotonic in the fraction of high-quality loans: it decreases and then increases.*

Proof. See Appendix A.2. ■

A higher payoff of high-quality loans A increases screening benefits and, as a result, raises the price directly and indirectly through a higher screening threshold. An improvement in the distribution of screening costs, in the first-order stochastic dominance sense, shifts down the pricing schedule in the secondary market, increasing the screening threshold and reducing the secondary market price. In the good equilibrium, all assets are sold when hit by the liquidity shock, so the liquidity preference parameter does not affect equilibrium values, $\frac{d\eta^{**}}{d\lambda} = 0 = \frac{dp^{**}}{d\lambda}$, though it affects the parameters conditions required to support this equilibrium.

The price can be non-monotonic in the proportion of high-quality loans μ. A higher proportion reduces the benefit of screening, which indirectly lowers the price, but a lower proportion of low-quality loans ('lemons') also directly raises the price. Figure 4 shows how the price and screening threshold vary with the proportion of high-quality loans. For $\mu \to 1$, only high-quality loans are financed, so the secondary market price is $p \to A$. The price is lowest for an intermediate proportion that satisfies $(1-\mu)\left(-\frac{d\eta^{**}}{d\mu}\right) = \frac{1-F(\eta^{**})}{f(\eta^{**})}$. For $\mu \to 0$, the low average quality of loans provides lenders with incentives to screen and, therefore, increases the price. Similarly, the price can be non-monotonic in the probability of a liquidity shock (Figure 5). A higher ν reduces the benefit of screening, which indirectly lowers the price, but more liquidity sellers also lower the adverse selection and directly raise the price.

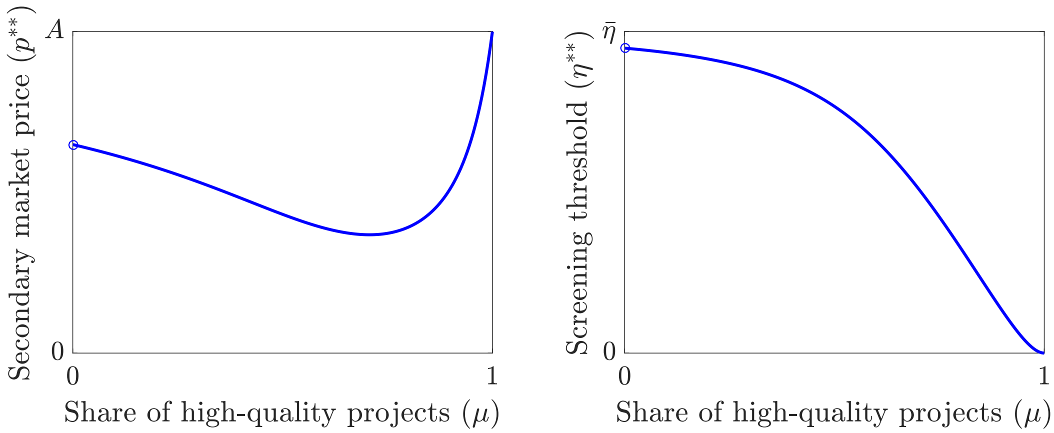

Figure 4: Non-monotonic secondary market price in μ in good equilibrium. Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\lambda = 3$, $A = 3$.

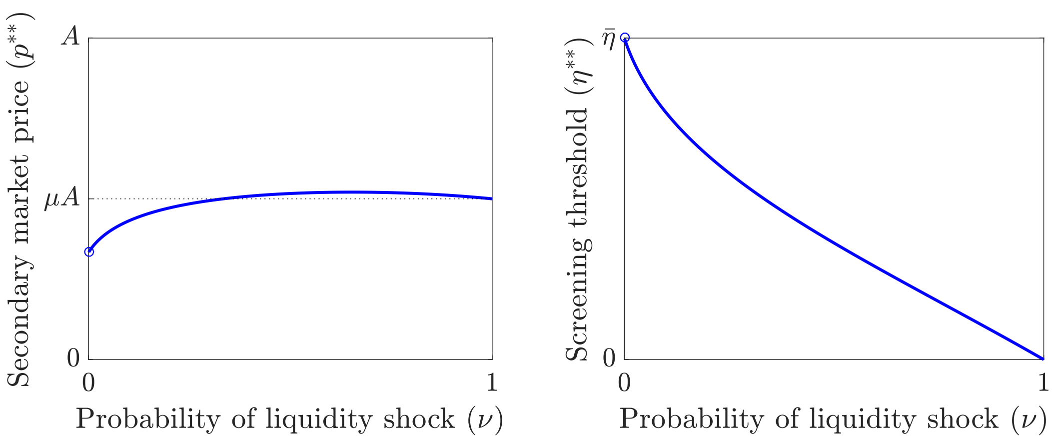

Figure 5: Non-monotonic secondary market price in ν in good equilibrium. Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\mu = 0.5$, $\lambda = 3$, $A = 3$.

4 Loan insurance

This section introduces insurance against loan default. At $t = 0$, each lender can fully insure an amount $\ell_i^I \in [0, 1]$ of loans against default. We focus on full insurance, that is the transferral of all risk to the insurer, without loss of generality.[2] If a loan

[2] In section 6, we allow for partial insurance and show that full insurance is privately optimal.

is insured, its idiosyncratic default risk passes to a competitive and deep-pocketed insurer (outside financiers), resulting in a risk-free payoff at $t = 2$. That is, competitive insurance replaces the risky loan payoff $(0, A)$ with an average payoff μA, since insurers cannot observe a lender's screening effort such that no lender screens prior to insurance. At $t = 1$, a lender sells $q_i^I \in [0, \ell_i^I]$ of insured and $q_i \in [0, 1 - \ell_i^I]$ of uninsured loans for prices p_I and p, respectively. Since insured loans are risk-free, these are not subject to adverse selection. Figure 6 shows the adjusted timeline.

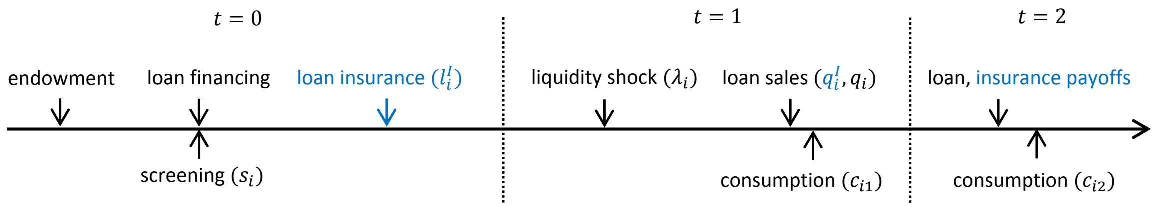

Figure 6: Timeline with loan insurance.

Definition 2. *A competitive equilibrium with loan insurance comprises choices of screening* $\{s_i\}$, *insurance* $\{\ell_i^I\}$, *sales of insured and uninsured loans in the secondary market* $\{q_i^I, q_i\}$, *and market prices* p^I *and* p *such that:*

1. *At* $t = 1$, *each lender* i *optimally chooses its sales of insured and uninsured loans in secondary markets for each realized liquidity shock* $\lambda_i \in \{1, \lambda\}$, *denoted by* $q_i^I(s_i, \lambda_i, \ell_i^I)$ *and* $q_i(s_i, \lambda_i, \ell_i^I)$, *given the prices* p, p^I *and the screening* s_i *and insurance choice* ℓ_i^I .

2. *At* $t = 1$, *prices* p^I *and* p *are set for outside financiers to break even in expectation, given the screening* $\{s_i\}$ *and insurance choices* $\{\ell_i^I\}$ *and sales schedules* $\{q_i^I(\cdot), q_i(\cdot)\}$ *of all lenders.*

3. *At* $t = 0$, *each lender* i *chooses its screening* s_i *and loan insurance* ℓ_i^I *to maximize*

expected utility, given the prices p^I and p and sales schedules $q_i^I(\cdot)$ and $q_i(\cdot)$:

$$\begin{aligned}\max_{s_i,\ell_i^I,c_{i1},c_{i2}} \quad & E_\lambda\left[\lambda_i c_{i1} + c_{i2} - s_i\eta_i\right] \qquad \textit{subject to} \\ c_{i1} \;\leq\; & q_i(s_i,\lambda_i,\ell_i^I)\, p + q_i^I(s_i,\lambda_i,\ell_i^I)\, p^I, \\ c_{i2} \;\leq\; & A\big[\mu + (1-\mu)\, s_i\big]\big[1 - \ell_i^I - q_i(s_i,\lambda_i,\ell_i^I)\big] + \mu A\big[\ell_i^I - q_i^I(s_i,\lambda_i,\ell_i^I)\big].\end{aligned}$$

Let $m \equiv \int_0^1 l_i^I di$ denote the fraction of loans insured by high-cost lenders. Low-cost lenders choose to screen in equilibrium and have no incentive to insure. High-cost lenders, in contrast, do not screen and may insure loans.

Proposition 3. ***Loan insurance.*** *The equilibrium with the option to insure loans is unique. It is a good equilibrium in which lenders sell high-quality uninsured loans after a liquidity shock; the Pareto-dominated bad or mixing equilibria do not exist.*

1. *If $A \geq \bar{A} \equiv \frac{\nu\lambda+(1-\nu)(1-\mu)}{(1-\nu)(1-\mu)^2\kappa} F^{-1}\left(\frac{\mu(\lambda-1)(1-\nu)}{\kappa+\mu(\lambda-1)(1-\nu)}\right)$, then no loans are insured, $l_i^{I**} = 0$, and the good equilibrium described in Proposition 1 occurs.*

2. *If $A < \bar{A}$, then loan insurance occurs, $m^{**} \in (0,1)$, and the prices of insured and uninsured loans are $p^{I**} = \mu A$ and $p^{**} \equiv \frac{\nu\lambda\mu A}{\nu\lambda+(1-\nu)(1-\mu)} \in (0, \mu A)$, respectively. Lenders with costs below the threshold $\eta^{**} \equiv \frac{(1-\nu)(1-\mu)^2\kappa A}{\nu\lambda+(1-\nu)(1-\mu)}$ choose to screen, where $\kappa \equiv \nu\lambda + 1 - \nu$ is the expected marginal utility of consumption at $t = 1$. Compared to the good equilibrium without loan insurance, the price for uninsured loans is higher, the screening threshold is lower and welfare is higher.*

Proof. See Appendix A.3. ■

With competitive loan insurance, welfare increases on the extensive margin since the bad and mixing equilibria fail to exist. The bad equilibrium without loan insurance is characterized by $p = 0$. But at this price high-cost lenders strictly prefer to insure their loans at $t = 0$ and trade them for $p_I^{**} = \mu A$ with outside financiers at $t = 1$. Thus, all low-quality loans are insured, $m = 1$, so only high-quality loans are traded in the market for uninsured loans at $t = 1$, which is inconsistent with $p = 0$.

Welfare also increases on the intensive margin and exceeds welfare without loan insurance. The social benefit of insurance is that there is no adverse selection between lenders and insurers at $t = 0$, as lenders have not yet learned the quality of their loans. Effectively, a lender commits to not exploiting its private information in the future, reducing the severity of adverse selection in the market for uninsured loans at $t = 1$.

Figure 7 shows the effect of the availability of competitive loan insurance on the secondary market price for uninsured loans and on screening. On the interval with insurance ($A < \bar{A}$), the average quality of uninsured loans is higher, increasing the price. In turn, a higher price reduces the incentives to screen (moral hazard).

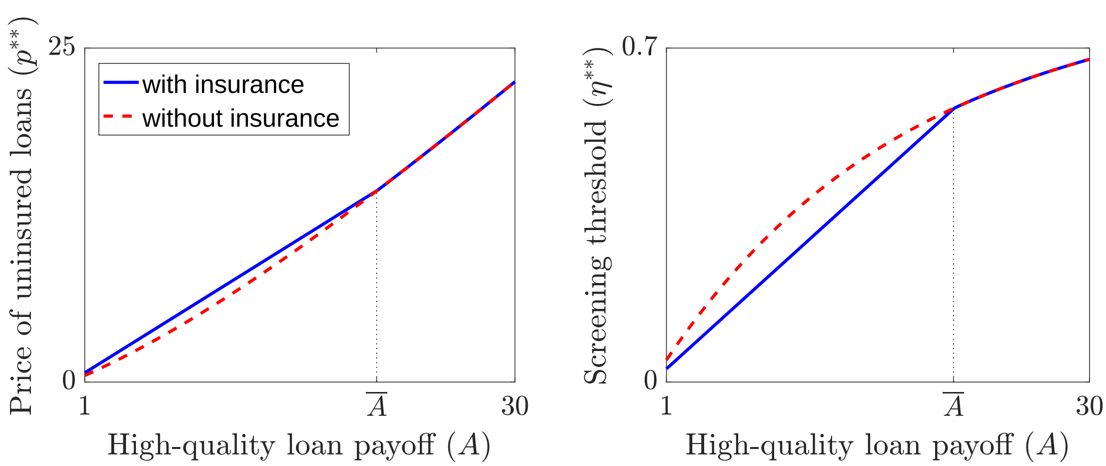

Figure 7: Loan insurance, the screening threshold, and the price for uninsured loans. Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\lambda = 3$, $\mu = 0.9$

5 Constrained Efficiency and Regulation

As constrained-efficient benchmark for the unique equilibrium with loan insurance, we consider a social planner who chooses loan insurance of all lenders. In contrast to the decentralized equilibrium, the planner internalizes the impact of insurance on the secondary market price of uninsured loans. This positive pecuniary externality arises because lenders insure their loans at $t = 0$ before information about loan quality arises at $t = 1$, so they effectively commit to not acting on future private information. The planner maximizes welfare subject to the marginal lender's indifference condition for screening and the break-even condition of outside financiers:

$$\max_{m,p,\eta} \quad \overbrace{\nu\lambda p\left(F+(1-F)(1-m)\right)+(1-\nu)\left(FA+(1-F)(1-m)(\mu A+(1-\mu)p)\right)}^{\text{Value to uninsured lenders}}$$
$$+\underbrace{\kappa\mu Am(1-F)}_{\text{Value to insured lenders}} - \underbrace{\int_0^\eta \tilde{\eta}dF(\tilde{\eta})}_{\text{Screening costs}} \tag{7}$$

$$\text{s.t.} \quad (6) \quad \text{and } p=\frac{\nu\left(F(\eta)+(1-F(\eta))(1-m)\mu\right)A}{\nu\left(F(\eta)+(1-F(\eta))(1-m)\mu\right)+(1-\mu)(1-F(\eta))(1-m)}. \tag{8}$$

Proposition 4. ***Constrained Efficiency.*** *There exists an interior constrained efficient allocation $(m^{CE}, p^{CE}, \eta^{CE})$. The constrained efficient level of loan insurance exceeds the unregulated level at both the intensive and the extensive margin:*

1. *$m^{CE} > m^{**} > 0$ when insurance arises in the unregulated equilibrium $(A < \bar{A})$;*
2. *a positive amount of insurance occurs for a larger set of parameters ($m^{CE} > 0 = m^{**}$ for $A^{CE} > A > \bar{A}$).*

Higher fractions of loan insurance, $m^{CE} > m^{**}$, implies a higher price in secondary market for uninsured loans, $p^{CE} > p^{**}$, and a lower screening effort, $\eta^{CE} < \eta^{**}$.

Proof. See Appendix A.4. ∎

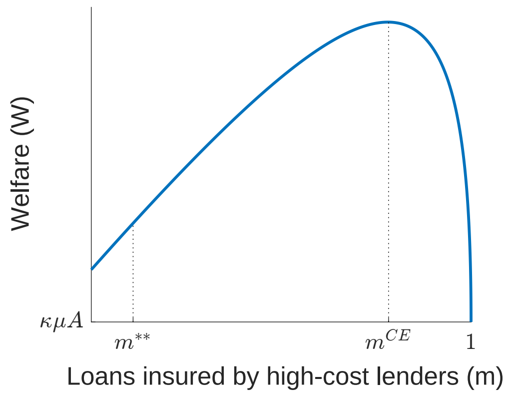

Figure 8: Welfare as a function of the fraction of insured loans by high-cost lenders. Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\lambda = 3$, $\mu = 0.9$

Figure 8 shows how the constrained efficient level of loan insurance exceeds the unregulated level at the intensive margin (case $A < \bar{A}$). Figure 9 shows the effect of the constrained efficient level of loan insurance on the secondary market price for uninsured loans and screening. Higher insurance level on the interval $A < A^{CE}$ increases the average quality of uninsured loans, resulting in higher price, which in turn reduces the incentives to screen.

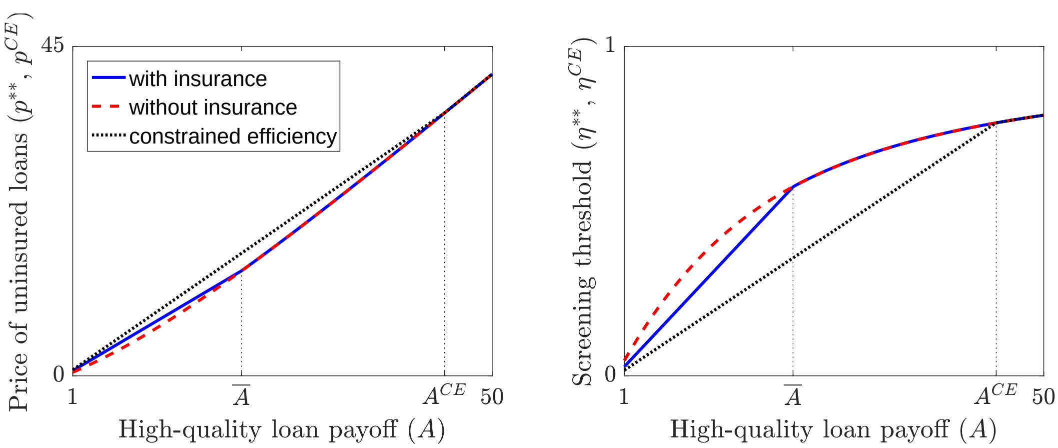

Figure 9: Constrained efficiency, the screening threshold, and uninsured loans price. Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\lambda = 3$, $\mu = 0.9$

We proceed by examining how the constrained-efficient allocation may be implemented. We consider a regulator with a balanced budget and no information advantage over uninformed outside financiers. As a result, direct implementation by picking $\{l_i^I\}$ for lenders is infeasible, since only high-cost lenders should insure themselves but the screening cost of lenders are private information. In particular, we study subsidizing either loan insurance or purchases of uninsured loans in its secondary market. These Pigouvian subsidies are accompanied with lump-sum taxation, so the cost of these policies are charged to lenders at the same date, preventing the planner from transferring funds across dates with different average utility of consumption.

5.1 Subsidized loan insurance

Consider a Pigouvian subsidy $b \geq 0$ to the owner of insured loans at $t = 1$ before loan trading. Figure 10 shows the timeline with a loan insurance subsidy. Since the lump-sum tax is charged in proportion to a lender's observable insurance choice, $T_i = b\, l_i^I$, it has always enough resources to pay the tax. Since taxes are lump sum, lenders ignore the effect of insurance choice on taxes but incorporate the effect on the Pigouvian subsidy.

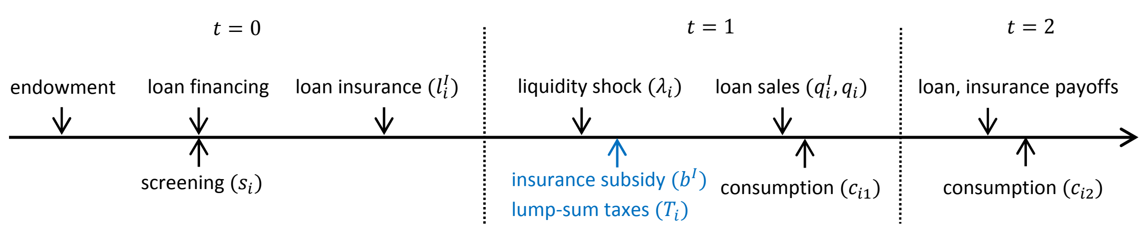

Figure 10: Timeline with loan insurance subsidy.

Definition 3. *A competitive equilibrium with loan insurance subsidy comprises lenders' choices of screening $\{s_i\}$, insurance $\{\ell_i^I\}$ and sales of insured and uninsured loans in the secondary market $\{q_i^I, q_i\}$, and government choices of insurance subsidy b and lump-sum taxes $\{T_i\}$, and market prices p^I and p such that:*

1. *At $t = 1$, each lender i optimally chooses its sales of insured and uninsured loans in secondary markets for each realized liquidity shock $\lambda_i \in \{1, \lambda\}$, denoted by $q_i^I(s_i, \lambda_i, \ell_i^I)$ and $q_i(s_i, \lambda_i, \ell_i^I)$, given the prices p, p^I and the screening s_i and insurance choice ℓ_i^I .*

2. *At $t = 1$, prices p^I and p are set for outside financiers to break even in expectation, given the screening $\{s_i\}$ and insurance choices $\{\ell_i^I\}$ and sales schedules $\{q_i^I(\cdot), q_i(\cdot)\}$ of all lenders.*

3. *At $t = 0$, each lender i chooses its screening s_i and loan insurance ℓ_i^I to maximize expected utility, given the prices p^I and p, sales schedules $q_i^I(\cdot)$ and $q_i(\cdot)$, insurance subsidy b and taxes T_i:*

$$\begin{aligned}
\max_{s_i, \ell_i^I, c_{i1}, c_{i2}} \quad & E_\lambda \left[\lambda_i c_{i1} + c_{i2} - s_i \eta_i\right] \qquad \textit{subject to} \\
c_{i1} \ \leq \ & q_i(s_i, \lambda_i, \ell_i^I)\, p + q_i^I(s_i, \lambda_i, \ell_i^I)\, p^I + b\ell_i^I - T_i, \\
c_{i2} \ \leq \ & A\big[\mu + (1-\mu)\, s_i\big]\big[1 - \ell_i^I - q_i(s_i, \lambda_i, \ell_i^I)\big] + \mu A\big[\ell_i^I - q_i^I(s_i, \lambda_i, \ell_i^I)\big].
\end{aligned}$$

4. *At $t = 0$, planner chooses insurance subsidy b to maximize welfare of all lenders subject to the balance budget constraint, specifically $T_i = b\ell_i^I$ for each lender i.*

This policy makes insurance more attractive and increases the fraction of insured loans, m^{**}, which in turn indirectly increases the secondary market price for uninsured loans, p^* (see equation (34) in Appendix A.3). In equilibrium, the price for uninsured loans p^* is determined by an indifference condition of high-cost lenders between insurance and no insurance, $(\mu A + b)\kappa = \nu\lambda p^* + (1-\nu)\,(\mu A + (1-\mu)\, p^*)$, which is rewritten as:

$$p^*(b) = \frac{\nu\lambda\mu A + \kappa b}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{9}$$

A higher price p^*, in turn, decreases the screening threshold η^*, given in equation (6).

The planner chooses an optimal insurance subsidy b to maximize welfare:[3]

$$\max_b W = \max_b \overbrace{\nu\lambda p^* \left(F + (1-F)\left(1-m^*\right)\right) + (1-\nu)\left(FA + (1-F)(1-m^*)(\mu A + (1-\mu)p^*)\right)}^{\text{Value to uninsured lenders}}$$
$$+ \underbrace{\kappa(\mu A + b)m^* \left(1-F\right)}_{\text{Value to insured lenders}} - \underbrace{\int_0^{\eta^*} \eta dF}_{\text{Screening costs}} - \underbrace{\kappa b m^* \left(1-F\right)}_{\text{Cost of loan subsidy}}, \tag{10}$$

subject to the indifference condition that determines the screening threshold in equation (6), the break-even condition of outside financiers in equation (8), and the indifference condition of high-cost lenders regarding insurance in equation (9).

Proposition 5. ***Loan insurance subsidy.*** *For all $A < A^{CE}$, an optimal Pigouvian subsidy for loan insurance implements the constrained-efficient allocation:*

$$b^* = \frac{[\kappa - (1-\nu)\mu]p^{CE} - \nu\lambda\mu A}{\kappa} \tag{11}$$

Proof. See Appendix A.5. ■

5.2 Subsidized purchases of uninsured loans

Consider the alternative policy of subsidized purchases of uninsured loans. This policy is equivalent to a minimum price guarantee for uninsured loans p_{min} to lenders. The cost of the policy is charged lump-sum to all lenders at $t = 1$. For the minimum price to be effective, it must exceed the price without intervention, $p_{min} > p^{**}$. An increase in the minimum price on the interval (p^{**}, A) implies a lower screening effort,

[3] We focus on the interval $b \leq (1-\mu)A$ without loss of generality. Higher subsidies have no effect on the welfare, as the payoff of insured loans $\mu A + b$ would exceed the payoff from high-quality loans, so all lenders insure and do not screen, resulting in lower welfare $W = \kappa\mu A$ (see also Figure 8).

$\eta_{min} \equiv \eta^*(p_{min}) = \max\{(1-\nu)(1-\mu)(A-p_{min}), 0\} < \eta^{**}$, and an increase in the quantity of sold uninsured loans, $q_{min} = \nu + (1-\nu)(1-\mu)(1-F(\eta_{min}))$.

When loan insurance occurs in the unregulated equilibrium, $A < \bar{A}$, an effective minimum price guarantee eliminates insurance, since it breaks the insurance indifference condition (30) by making insurance less attractive. The benefit of a minimum price guarantee for uninsured loans is higher value to liquidity sellers and the cost are lower screening incentives, which reduce the average quality of investment.

The planner chooses the optimal price floor p_{min} to maximize welfare:[4]

$$\begin{aligned}\max_{p_{min}} W &= \max_{p_{min}} \overbrace{\nu\lambda \max\{p^{**}, p_{min}\} + (1-\nu)[(\mu + (1-\mu)F)A + (1-F)(1-\mu)\max\{p^{**}, p_{min}\}]}^{\text{Value to lenders}} \\ &\quad - \underbrace{\int_0^{\eta_{min}} \eta dF}_{\text{Screening costs}} - \underbrace{\kappa\left[\max\{p^{**}, p_{min}\} q_{min} - \nu[\mu + (1-\mu)F]A\right]}_{\text{Cost of purchases}}, \end{aligned} \tag{12}$$

where the value to lenders in (12) is independent of the fraction of insured loans because high-cost lenders have the same payoff from insurance and non-insurance in the equilibrium with positive insurance.

Proposition 6. *Subsidizing loan insurance dominates subsidizing outright purchases of uninsured loans. The latter policy does not achieve constrained efficient allocation.*

Proof. See Appendix A.6. ∎

While the planner can use both policies (p_{min}, b) simultaneously, only one of them is effective. For $p_{min} > p^*(b)$, the indifference condition (9) is not satisfied, no loans are insured and the planner effectively subsidizes only purchases of uninsured

[4] When the price after policy $\max\{p^{**}, p_{min}\}$ is below A/λ, low-cost lenders stop selling uninsured loans and welfare is given by $W = A(F + (1-F)\mu)$. If $p_{min} \geq A$ no lender screens and welfare collapses to $W = \kappa\mu A$

loans. For $p_{min} < p^*(b)$, in contrast, the equilibrium price given by the insurance subsidy exceeds the minimum price guarantee for uninsured loans p_{min}, so the latter policy is ineffective. For direct comparison of the two policies, we compare the welfare if the government decides to target a price of uninsured loans p_T – either directly with subsidized purchases of uninsured loans ($p_{min} = p_T$) or indirectly with subsidized loan insurance ($p^*(b) = p_T$). The value of the screening threshold is $\eta^*(p_T)$ in both cases. Comparing the level of welfare under both policies, subsidized loan insurance is more efficient as long as the policy target for the secondary market price is lower than the payoff of high-quality assets, $p_T < A$. This result arises because fewer assets are bought because the planner does not buy lemons sold by informed high-cost lenders.[5]

Figure 11 compares welfare under both policies as the target price p_T varies. Welfare is the same when intervention does not take place ($p_T = p^{**}$) and when $p_T = A$. Subsidizing insurance is optimal in this case. If only the policy of minimum price in market for uninsured loans is available, it would be optimal not to intervene. The discontinuity between no intervention and an effective minimum price for uninsured loans is due to the fact that this policy eliminates insurance and its positive effects on price for uninsured loans has to be compensated by costly asset purchases.

6 Extension: Partial insurance

Consider insurance contracts that allow lenders to choose the fraction ω of default costs that are covered by the insurance. Such insurance contracts are equivalent to guaranteeing non-default payment A with a deductible $(1 - \omega)A$ and with insurance

[5] This advantage disappears for $p_T \geq A$ because $\eta^* = 0$ and subsidized insurance would result in every lender insuring $m^* = 1$. Thus, the cost of subsidy and the welfare are equalized under both options: $W|_{p_T \geq A} = \kappa\mu A$. However, the corner $m^* = 1$ ($p_T = A$) is not optimal (see Proposition 4).

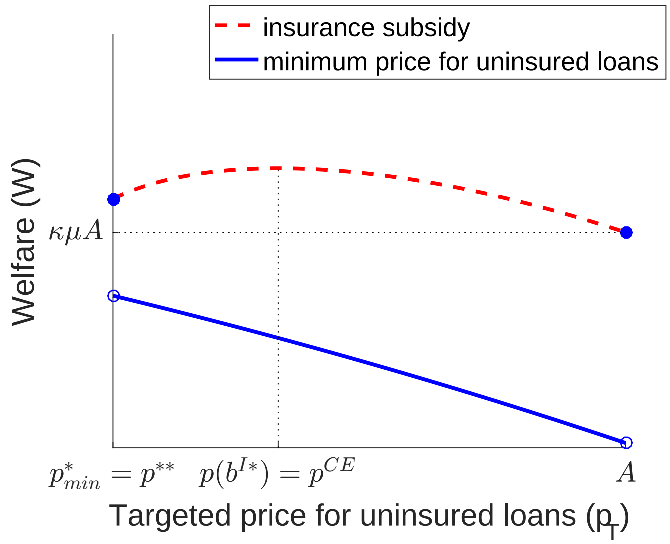

Figure 11: Comparison of welfare under the two policies.
Parameters: uniform distribution $\eta_i \sim \mathcal{U}$, $\bar{\eta} = 1$, $\nu = 0.1$, $\lambda = 3$, $\mu = 0.78$

fee paid by the loan owner at time of loan and insurance payoffs ($t = 2$). The fee covers the average insurance costs, $k = \omega(1 - \mu)A$.

Proposition 7. *Lenders always choose full insurance,* $\omega^{**} = 1$*.*

Proof. See Appendix A.7. ■

The intuition for this result is the following. Unlike in the full insurance case, when the insurance is partial ($\omega < 1$), low-quality assets have a fundamental value $\omega A - k = \omega\mu A$ that is lower than the fundamental value of high-quality assets $A - k = A\left(1 - (1 - \mu)\omega\right)$. Therefore, there is adverse selection in the market for partially insured loans, since lenders without liquidity shock decide to sell only low-quality loans. Adverse selection redistributes wealth from lenders with liquidity shock, who sell all loans, to lenders without liquidity shock, who sell only low-quality loans. But since lenders have higher utility of consumption in states with liquidity shock, they choose full coverage $\omega^* = 1$ to avoid the negative effect of adverse selection.

References

Camargo, B., Kim, K., Lester, B., 2016. Information Spillovers, Gains from Trade, and Interventions in Frozen Markets. Review of Financial Studies 29 (5), 1291–1329.

Chiu, J., Koeppl, T. V., 2016. Trading Dynamics with Adverse Selection and Search: Market Freeze, Intervention and Recovery. Review of Economic Studies 83 (3), 969–1000.

Congressional Budget Office, 2014. Transition to alternative structures for housing finance. Tech. rep.

Elenev, V., Landvoigt, T., Nieuwerburgh, S. V., 2016. Phasing out the gses. Journal of Monetary Economics 81, 111 – 132.

Fuchs, W., Skrzypacz, A., 2015. Government interventions in a dynamic market with adverse selection. Journal of Economic Theory 158, 371 – 406.

Jeske, K., Krueger, D., Mitman, K., 2013. Housing, mortgage bailout guarantees and the macro economy. Journal of Monetary Economics 60 (8), 917–935.

Parlour, C. A., Plantin, G., 2008. Loan Sales and Relationship Banking. Journal of Finance 63 (3), 1291–1314.

Philippon, T., Skreta, V., 2012. Optimal interventions in markets with adverse selection. American Economic Review 102 (1), 1–28.

Plantin, G., 2009. Learning by Holding and Liquidity. Review of Economic Studies 76 (1), 395–412.

Tirole, J., 2012. Overcoming adverse selection: How public intervention can restore market functioning. American Economic Review 102 (1), 29–59.

Vanasco, V., 2017. The downside of asset screening for market liquidity. The Journal of Finance 72 (5), 1937–1982.

A Appendix

A.1 Proof of Proposition 1

The equilibrium value of the screening threshold, η^{**}, is obtained by substituting equation (5) in equation (6). It is implicitly given by:

$$\eta^{**} = \frac{(1-\nu)(1-\mu)^2(1-F(\eta^{**}))}{\nu + (1-\nu)(1-\mu)(1-F(\eta^{**}))}A. \tag{13}$$

Does a unique equilibrium exist within the class of good equilibria? Regarding uniqueness, the left-hand side (LHS) of equation (13) increases in η and the right-hand side (RHS) decreases in it, so at most one intersection exists. Regarding existence, we evaluate both sides of (13) at the bounds of the screening cost distribution, using $F(0) < 1 = F(\bar{\eta})$. Since $LHS(0) < RHS(0)$ and $LHS(1) > RHS(1)$, there exists a unique interior screening threshold $\eta^{**} \in (0, \bar{\eta})$. The resulting equilibrium price is:

$$p^{**} \equiv p^*(\eta^{**}) = \nu A \frac{\mu + (1-\mu)F(\eta^{**})}{\nu + (1-\nu)(1-\mu)(1-F(\eta^{**}))}. \tag{14}$$

To verify the supposition of a good equilibrium in which high-quality loans are sold in the secondary market, we combine conditions (5) and (3). A necessary condition for the good equilibrium is the parameter constraint

$$\lambda \geq \frac{\nu + (1-\nu)(1-\mu)(1-F(\eta^{**}))}{\nu(\mu + (1-\mu)F(\eta^{**}))} > 1. \tag{15}$$

To simplify the lower bound on the liquidity shock, we note it decreases in $F(\eta^{**})$. Since $F \geq 0$, a simpler but more restrictive sufficient condition is $\lambda \geq \underline{\lambda} = \frac{\nu+(1-\nu)\mu}{\nu\mu}$.

A.2 Proof of Proposition 2

Effect on screening threshold Using equation (13), we define the function:

$$H \equiv \eta - \frac{(1-\nu)(1-\mu)^2(1-F(\eta))A}{\nu + (1-\nu)(1-\mu)(1-F(\eta))} \equiv \eta - \frac{N}{D}, \tag{16}$$

with $H(\eta^{**}) = 0$ and N and D are the numerator and denominator, respectively. To use the implicit function theorem, we obtain the following partial derivatives of H:

$$\frac{\partial H}{\partial \eta} = 1 + \frac{1}{D^2}\left[(1-\nu)(1-\mu)^2 \nu A f(\eta)\right] > 0, \tag{17}$$

$$\frac{\partial H}{\partial \nu} = \frac{(1-\mu)^2(1-F(\eta))A}{D^2} > 0, \tag{18}$$

$$\frac{\partial H}{\partial \mu} = \frac{(1-\nu)(1-\mu)[1-F(\eta)]\,A\,[2\nu + (1-\nu)(1-\mu)(1-F(\eta))]}{D^2} > 0, \tag{19}$$

$$\frac{\partial H}{\partial \lambda} = 0, \tag{20}$$

$$\frac{\partial H}{\partial A} = -\frac{(1-\nu)(1-\mu)^2(1-F(\eta))}{\nu + (1-\nu)(1-\mu)(1-F(\eta))} < 0. \tag{21}$$

These partial derivatives implies the following comparative statics, where we use the notation g_x to denote the partial derivative of some generic function $g(x;\cdot)$:

$$\frac{d\eta^{**}}{d\nu} = -\frac{H_\nu}{H_\eta} < 0, \qquad \frac{d\eta^{**}}{d\mu} = -\frac{H_\mu}{H_\eta} < 0, \tag{22}$$

$$\frac{d\eta^{**}}{d\lambda} = -\frac{H_\lambda}{H_\eta} = 0, \qquad \frac{d\eta^{**}}{dA} = -\frac{H_A}{H_\eta} > 0. \tag{23}$$

Effect on price Using equation (5), we note that there are direct effects of parameters on the price as well as indirect effects via the screening threshold:

$$\frac{\partial p^{**}}{\partial \lambda} = 0, \qquad \frac{\partial p^{**}}{\partial A} = \frac{p^{**}}{A} > 0. \tag{24}$$

$$\frac{dp^{**}}{d\eta^{**}} = \frac{(1-\mu)\ A\,\nu\, f(\eta^{**})}{D^2} > 0, \tag{25}$$

$$\frac{\partial p^{**}}{\partial \nu} = \frac{[\mu + (1-\mu)\,F(\eta^{**})](1-\mu)(1-F(\eta^{**}))A}{D^2} > 0, \tag{26}$$

$$\frac{\partial p^{**}}{\partial \mu} = \frac{\nu\,(1-F(\eta^{**}))A}{D^2} > 0. \tag{27}$$

The total derivatives for A and λ yield unambiguous results, while the total derivatives for ν and μ may yield ambiguous results:

$$\frac{dp^{**}}{dA} = \frac{\partial p^{**}}{\partial A} + \frac{dp^{*}}{d\eta^{*}}\frac{d\eta^{*}}{dA} > 0, \qquad \frac{dp^{**}}{d\lambda} = \frac{\partial p^{**}}{\partial \lambda} + \frac{dp^{*}}{d\eta^{*}}\frac{d\eta^{*}}{d\lambda} = 0, \tag{28}$$

$$\frac{dp^{**}}{d\nu} = \underbrace{\frac{\partial p^{**}}{\partial \nu}}_{>0} + \underbrace{\frac{dp^{**}}{d\eta^{**}}\frac{d\eta^{**}}{d\nu}}_{<0} \lesseqgtr 0, \qquad \frac{dp^{**}}{d\mu} = \underbrace{\frac{\partial p^{**}}{\partial \mu}}_{>0} + \underbrace{\frac{dp^{**}}{d\eta^{*}}\frac{d\mu}{d\mu}}_{<0} \lesseqgtr 0.. \tag{29}$$

Increases in these parameters increase the price directly but decrease it indirectly via a negative effect on the screening threshold. A set of sufficient condition for the non-monotonicity of p^{**} in μ is $\frac{dp}{d\mu}\mid_{\mu\to 1} > 0$ and $\frac{dp}{d\mu}\mid_{\mu\to 1} < 0$. Substituting into (29) from conditions (27), (25), (17) and (19), we evaluate derivatives for the two limits:

$$\frac{dp}{d\mu}\mid_{\mu\to 1} = \frac{A}{\nu} > 0,$$

$$\frac{dp}{d\mu}\mid_{\mu\to 0} = \frac{\nu A(1-F(\eta^{**}))}{D^2}\left\{1 - \frac{A(1-\nu)f(\eta^{**})\,[2\nu + (1-\nu)(1-f(\eta^{**}))]}{D^2 + (1-\nu)\nu A f(\eta^{**})}\right\}.$$

The first derivative is always positive and the latter is negative if $\eta_0^{**}\frac{f(\eta_0^{**})}{1-F(\eta^{**})} > 1$, where $\eta_0^{**} = \eta^{**}\mid_{\mu\to 0}$.

Finally, the result on the first-order stochastic dominance improvement, $\tilde{F} \leq F$, arises as the pricing schedule shifts down: $\frac{dp}{dx} = \frac{(1-\mu)\nu A}{(\nu+(1-\nu)(1-\mu)(1-x))^2} > 0$ for $x \equiv F(\eta)$.

A.3 Proof of Proposition 3

We start by constructing the equilibrium secondary market price of insured loans. Since insurers cannot observe screening by lenders, low-cost lenders would only screen and insure a loan if $p_I = A$. At this price, however, high-cost lenders strictly prefer not to screen and insure, so the average quality of loans falls short of A and no insurer can break even. Thus, $p_I^{**} < A$ and only high-cost lenders may insure loans. Hence, the expected value of insured loans is μA at $t = 2$. Because of risk-neutrality, the competitive loan arrangement agreed upon at $t = 0$ converts a risky unscreened loans into a safe payment of the average quality μA at $t = 2$. Since the market for insured loans is not subject to adverse selection at $t = 0$ (when high-cost lenders do not yet know the quality of their loan), competitive insurers are willing to pay $p_I^{**} = \mu A$.

For $m^{**} \in (0, 1)$, high-cost lenders must be indifferent between insuring and not insuring,

$$p_I^{**}\kappa = \nu\lambda p^{**} + (1-\nu)\left(\mu A + (1-\mu)\, p^{**}\right), \tag{30}$$

which simplifies to $\nu\lambda\left(\mu A - p^{**}\right) = (1-\nu)\,(1-\mu)\, p^{**}$, where the benefits of insurance on the left-hand side are a higher price if the liquidity shock hits. The costs of insurance on the right-hand side are the cost of giving up the option to sell only low-quality loans at $t = 1$ while keeping high-quality loans.

Next, we show that not all of the unscreened loans are insured, $m^{**} < 1$. Proof by contradiction: suppose all unscreened loans were insured, $m = 1$. Then, there would be no adverse selection in the market for uninsured loans, $p = A$. Since $m = 1$ requires high-cost lenders to prefer insurance over no insurance, or $p_I^{**}\kappa > \nu\lambda p^{**} + (1-\nu)\left(\mu A + (1-\mu)\, p^{**}\right)$ instead of equation (30). This condition simplifies to $\mu > 1$, a contradiction. In sum, only some high-cost lenders insure their loans.

We are now equipped to derive the equilibrium price, sales volume and screening threshold. Condition (30) pins down the price for uninsured loans at $t = 1$:

$$p^{**} = \frac{\nu\lambda\mu A}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{31}$$

Substituting equation (31) into equation (6), we obtain the threshold screening effort:

$$\eta^{**} = \frac{(1-\nu)(1-\mu)^2 \kappa A}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{32}$$

The equilibrium price in equation (31) has to satisfy condition (3) in order to ensure the existence of a unique good equilibrium. That is, high-quality loans have to be sold at $t = 1$. It can be shown that the maintained sufficient condition, $\lambda \geq \underline{\lambda}$, suffices for the existence of a good equilibrium.

To pin down the fraction of loans high-cost lenders insure, m^*, the price of uninsured loans is also given by the outside financiers break-even condition (8). Combined with (31), we obtain

$$m^{**} = 1 - \frac{\kappa F(\eta^{**})}{\mu(\lambda-1)(1-\nu)(1-F(\eta^{**}))}. \tag{33}$$

Therefore, $m^{**} > 0$ whenever $A \leq \bar{A}$ as defined in the main text, where the parameter constraint arises from substituting in η^{**} from equation (32) into equation (33).

We turn to the welfare result on the intensive margin. In particular, we study the direct impact of changes in loan insurance on the price of uninsured loans, screening threshold, and welfare. For the effect on the price, the total derivative of (8) is:

$$\frac{dp^*}{dm^*} = \frac{\partial p^*}{\partial m^*} + \frac{dp^*}{d\eta^*}\frac{d\eta^*}{dp^*}\frac{dp^*}{dm^*} = \frac{\frac{\partial p^*}{\partial m^*}}{1 - \frac{dp^*}{d\eta^*}\frac{d\eta^*}{dp^*}} > \frac{\partial p^*}{\partial m^*} > 0, \tag{34}$$

since $\frac{dp^*}{d\eta^*} > 0$, $\frac{d\eta^*}{dp^*} = -(1-\nu)(1-\mu) < 0$, and

$$\frac{\partial p^*}{\partial m^*} = \frac{\nu A F(\eta^{**})(1-F)(1-\mu)}{\left[\nu\left(F + (1-F)\left(1-m^*\right)\mu\right) + (1-\mu)\left(1-F\right)\left(1-m^*\right)\right]^2}. \tag{35}$$

Since the price increases in loans insurance, the screening threshold falls, $\frac{d\eta^*}{dm^*} = \frac{d\eta^*}{dp^*}\frac{dp^*}{dm^*} < 0$. Finally, we use that outside financiers and insurers break even in expectation, so welfare adds up the expected value to lenders. Without loan insurance (for $A \geq \bar{A}$) and with loan insurance, welfare can be expressed as:

$$\begin{aligned} W(m=0) \quad = \quad & \underbrace{\nu\lambda p^{**}}_{\text{Liquidity shock}} + \underbrace{(1-\nu)\Big\{F(\eta^{**})A + \left[1-F(\eta^{**})\right]\left(\mu A + (1-\mu)p^{**}\right)\Big\}}_{\text{No shock}} \\ & - \underbrace{\int_0^{\eta^{**}} \eta dF(\eta)}_{\text{Screening costs}}. \end{aligned} \tag{36}$$

$$\begin{aligned} W(m>0) \quad = \quad & \overbrace{\nu\lambda\Big\{p^{**}(F(\eta^{**}) + (1-F(\eta^{**}))(1-m^{**})) + m^{**}(1-F(\eta^{**}))p_I^{**}\Big\}}^{\text{Liquidity shock}} \\ & + \overbrace{(1-\nu)[F(\eta^{**})A + (1-m^{**})(1-F(\eta^{**}))(\mu A + (1-\mu)p^{**})]}^{\text{No shock uninsured}} \\ & + \underbrace{(1-\nu)m^{**}(1-F(\eta^{**}))p_I^{**}}_{\text{No shock insured}} - \underbrace{\int_0^{\eta^{**}} \eta dF(\eta)}_{\text{Screening costs}}. \end{aligned} \tag{37}$$

For the subspace $A \geq \bar{A}$, there is no loan insurance and and equation (37) collapses to equation (36). For $A < A$, however, some loans are insured at $t = 0$ and sold at $t = 1$, and we can substitute p_I^{**} from condition (30) to get the same functional form as (36) but with different arguments (p^{**} is higher and $\eta^{**}(p^{**})$ is lower).

To prove that introducing the option of loan insurance increases welfare, we show that the function $W(p^{**})$ increases in p^{**}, using equation (6):

$$\frac{dW}{dp^{**}} = \overbrace{\frac{\partial W}{\partial p^{**}}}^{>0} + \overbrace{\frac{\partial W}{\partial \eta^{**}}}^{=0} \overbrace{\frac{d\eta^{**}}{dp}}^{<0} > 0, \tag{38}$$

where $\frac{\partial W}{\partial p^{**}} = \nu\lambda + (1-\nu)(1-\mu)(1-F(\eta^{**})) > 0$ and $\frac{\partial W}{\partial \eta^{**}} = [(1-\nu)(1-\mu)(A-p) - \eta^{**}]f(\eta^{**}) = 0$ by an envelope-theorem-type argument.

A.4 Proof of Proposition 4

We prove existence and the result on the intensive margin by showing that (i) welfare increases in insurance probability m on interval $m \in [0, m^{**}]$; and (ii) welfare decreases in m at the limit $m \to 1$. Since the welfare function in equation (7) is continuous and defined everywhere in the interval $m \in (0,1)$, the optimal insurance probability lies within the interval $m^{CE} \in (m^{**}, 1)$, thus exceeding the decentralized choice m^{**} .

Total derivative of welfare can be expressed as follows:

$$\begin{aligned}
\frac{dW}{dm} &= \frac{\partial W}{\partial m} + \frac{\partial W}{\partial p^*}\frac{dp^*}{dm} + \frac{\partial W}{\partial \eta^*}\frac{d\eta^*}{dm} \\
\frac{\partial W}{\partial m} &= (1-F)(\kappa\mu A - \nu\lambda p^* - (1-\nu)(\mu A + (1-\mu)p)) = 0 \\
\frac{\partial W}{\partial p^*} &= \nu\lambda(F + (1-F)(1-m)) + (1-\nu)(1-F)(1-m)(1-\mu) > 0 \\
\frac{\partial W}{\partial \eta^*} &= f\left[(1-\nu)(1-\mu)(A-p^*) - \eta^* + m(\nu\lambda p^* + (1-\nu)(\mu A + (1-\mu)p^*)\right] = 0.
\end{aligned}$$

Since $\frac{dp^*}{dm} > 0$ and $\frac{d\eta^*}{dm} < 0$, the total derivative is positive at the decentralized equilibrium due to the positive pecuniary externality, $\frac{dW}{dm} |_{m=m^{**}} = \frac{\partial W}{\partial p^*}\frac{dp^*}{dm} > 0$. The total derivative is also positive for any $\tilde{m} < m^{**}$ since $\frac{\partial W}{\partial m} |_{\tilde{m}} > 0$, $\frac{\partial W}{\partial p} |_{\tilde{m}} > 0$, and $\frac{\partial W}{\partial \eta^*} |_{\tilde{m}} < 0$.

Next we consider the limit $m \to 1$. In this case price of uninsured loans equals payoff of high-quality loans and there is no screening: $\lim_{m\to 1} p = A$, $\lim_{m\to 1} \eta = 0$, therefore we can evaluate partial derivatives: $\lim_{m\to 1} \frac{\partial W}{\partial m} = -\kappa(1-\mu)A$, $\lim_{m\to 1} \frac{\partial W}{\partial p} = 0$ and $\lim_{m\to 1} \frac{\partial W}{\partial \eta^*} = \infty$. This implies that total derivative is negative $\lim_{m\to 1} \frac{dW}{dm} < 0$.

Since welfare is continuous and defined everywhere in $m \in (0,1)$, the above two results imply that the optimal insurance probability exceeds decentralized choice m^{**} and lies at the interior of the interval $m^{CE} \in (m^{**}, 1)$. From the proof of Proposition 3 (see equation 34), a higher insurance probability increases price in secondary markets for uninsured loans and decreases the screening effort, $p^{CE} > p^{**}$ and $\eta^{CE} < \eta^{**}$.

To prove the result on the extensive margin, we compare the threshold of A at which insurance stops being used in decentralized equilibrium, $\bar{A}$, and in the constrained efficient case, A^{CE}. $\bar{A}$ satisfies $m^{**} = 0$ and $\frac{\partial W}{\partial m} = (1-F)(\kappa\mu A - \nu\lambda p^* - (1-\nu)(\mu A + (1-\mu)p)) = 0$. Substituting p^{**} from the break-even condition in $\frac{\partial W}{\partial m} = 0$, we get:

$$\frac{\nu(F + (1-F)\mu)}{\nu(F + (1-F)\mu) + (1-\mu)(1-F)} = \frac{\nu\lambda\mu}{\nu\lambda + (1-\nu)(1-\mu)}. \tag{39}$$

Payoff A^{CE} has to satisfy $m^{CE} = 0$ and $\frac{dW}{dm} = 0$. The latter condition can be rearranged and after substituting p^* from the break-even condition we get

$$\frac{\nu(F + (1-F)\mu)}{\nu(F + (1-F)\mu) + (1-\mu)(1-F)} = \frac{\nu\lambda\mu}{\nu\lambda + (1-\nu)(1-\mu)} + \frac{\frac{\partial W}{\partial p}\frac{dp}{dm}}{(1-F)(\nu\lambda + (1-\nu)(1-\mu))A^{CE}}. \tag{40}$$

Combining this condition with (39), we obtain

$$\frac{\nu(F + (1-F)\mu)}{\nu(F + (1-F)\mu) + (1-\mu)(1-F)} \Big|_{A=\bar{A}} < \frac{\nu(F + (1-F)\mu)}{\nu(F + (1-F)\mu) + (1-\mu)(1-F)} \Big|_{A=A^{CE}}.$$

Since $\frac{d\eta^*}{dA} > 0$, this condition implies $A^{CE} > \bar{A}$.

A.5 Proof of Proposition 5

The objective functions of the planner in (7) and of the regulator in (10) are identical, and so are the indifference condition for screening and the break-even condition of outside financiers. It follows that the optimal Pigouvian subsidy is set to achieve the constrained efficient price in the secondary market for uninsured loans setting, thus achieving the constrained efficient allocation. Solving equation (9) and evaluating at $p^*(b^*) = p^{CE}$, we obtain the expression for b^* stated in Proposition 5.

A.6 Proof of Proposition 6

When the subsidized uninsured loan-purchase policy is effective, the welfare is given by (12). When the planner subsidizes insurance, the welfare is given by (10). Partially substituting in the latter equation $p^*(b)$ from the indifference condition (9), we obtain a functional form for the welfare that differs from (12) only in the policy cost term:

$$\begin{aligned} W(b) &= \nu\lambda p^* - \int_0^{\eta^*} \eta dF + (1-\nu)(F + \mu(1-F))A + (1-\nu)(1-F)(1-\mu)p^* \\ &\quad -\kappa b m^* (1-F), \end{aligned} \tag{41}$$

where $p^* = p^*(b)$, $\eta^* = \eta^*(p^*) = \eta^*(p^*(b))$, and $m^* = m^*(\eta^*(p^*(b)))$.

Comparing the costs of the two policies, we find that welfare is higher when planner chooses subsidized loan insurance as opposed to subsidized purchases of uninsured loans for a given target price $p_T \equiv p_{min} = p^*(b) > p^{**}$ whenever

$$p_T(\nu + (1-\nu)(1-\mu)(1-F)) - \nu(F + \mu(1-F))A > bm^*(1-F). \tag{42}$$

Substituting for b from (9) and for $(1-F)m^* = \frac{p_T(\nu+(1-\nu)(1-\mu)(1-F)) - \nu(F+\mu(1-F))A}{p_T(1-\mu+\mu\nu) - \mu\nu A}$ from

(8), we can rewrite (42) as $1 > \frac{1}{\kappa}\frac{[\nu\lambda+(1-\nu)(1-\mu)]p_T-\nu\mu A\lambda}{p_T(1-\mu+\mu\nu)-\nu\mu A}$, which collapses to $p_T < A$.

A price target $p_T = A$ would require $m = 1$ in the case of insurance subsidies. But we already know from the Proposition 4 that constrained efficient insurance level is never full insurance ($m = 1$). Therefore, we can conclude that on the interval where planner wants to increase the price of uninsured loans $p_T > p^{**}$, the policy of loan insurance subsidies strictly dominates subsidized purchases of uninsured loans.

A.7 Proof of Proposition 7

Due to adverse selection, the price in secondary markets for insured loans is given by:

$$p_I^{**} = \frac{\nu + (1-\nu)(1-\mu)\omega}{\nu + (1-\nu)(1-\mu)}\mu A, \tag{43}$$

which implies that p_I^{**} monotonically increases in insurance coverage ω: $\frac{dp_I^{**}}{d\omega} > 0$. Lenders who insure do not screen, so they maximize the following payoff:

$$\begin{aligned} &\max_{\omega} \nu\lambda p_I + (1-\nu)\left(\mu(A-k) + (1-\mu)p_I\right) \\ &= \max_{\omega} \frac{\nu\kappa + (1-\nu)(1-\mu)\left(\kappa + \nu(\omega-1)(\lambda-1)\right)}{\nu + (1-\nu)(1-\mu)}\mu A. \end{aligned} \tag{44}$$

Since equation (44) increases in ω, the optimum is the corner solution $\omega^* = 1$.