Stock loan lotteries and individual investor performance

Jordan Moore [1]
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Abstract

Individual investors trade excessively, sell winners too soon, and overweight stocks with lottery features and low expected returns. This paper proposes and models a financial innovation, called stock loan lotteries, that improves individual investor performance. An individual investor signs a contract with an exchange promising to hold his shares of stock for multiple periods. The exchange operates a stock loan marketplace. Instead of paying each investor the lending fees on his individual shares, the exchange periodically holds a lottery for the entire pool of lending fees. I extend the Barberis and Xiong (2009) two-period model of realization utility to include stock loan lotteries. In frictionless markets, investors demand high fixed stock loan fees to hold shares for two periods. Because prospect theory investors overvalue low probability payoffs, they demand much lower fees denominated in stock loan lottery tickets. In many cases, introducing stock loan lotteries provides individual investors with greater expected utility and greater expected wealth. Stock loan lotteries provide the greatest benefits to the poorest investors, who typically exhibit the strongest lottery preferences. Introducing transactions costs, leverage constraints, and taxes to the model enhances the benefits of stock loan lotteries. I propose a mechanism for exchanges to structure stock loan lottery tickets as derivative securities.

[1] Simon Business School, University of Rochester. Email: jordan.moore@simon.rochester.edu. I am grateful to William Bazley, Anisha Nyatee, and participants at the 2017 Boulder Summer Conference on Consumer Financial Decision Making for helpful comments and suggestions. I thank the Montreal Institute of Structured Finance and Derivatives (IFSID) for financial support.

1. Introduction

Individual investors trade frequently and hold undiversified portfolios, so they earn lower returns after controlling for risk and transactions costs. What would it take to motivate individual investors to modify their costly behavior? This paper proposes and models a financial innovation, called stock loan lotteries, with the potential to improve individual investor performance. Stock loan lotteries are structured financial products that draw on insights from prospect theory to optimally compensate individual investors for refraining from excessive trading.

Barber and Odean (2000) find that individual investors turn over their portfolios frequently and there is a strong negative relation between returns and trading frequency.[2] French (2008) estimates that active trading costs US equity investors 67 basis points annually. Odean (1998) shows that the low average returns of individual investors are due to the disposition effect, the tendency to sell winning positions and realize a gain. Weber and Camerer (1998) identify a strong disposition effect in an experimental setting, suggesting that the behavior is not unique to the Odean (1998) sample. The tendency of individuals to trade frequently has substantial economic costs. Odean (1999) shows that the stocks that individuals sell outperform the stocks that individuals buy by more than 3% in the following year. Barber et al. (2009) analyze the complete record of Taiwanese equity trading activity over four years and find that stocks individual investors sell outperform stocks individual investors purchase by 3.8% over the next year. Several studies show that trading activity for investors with less experience and education exhibits a more pronounced disposition effect.[3]

[2] In data from a large discount broker between 1991 and 1996, the average household turns over more than 75% of its portfolio annually.

[3] These include Feng and Seasholes (2005), Dhar and Zhu (2006), and Frazzini (2006).

Individual investors tend to hold portfolios with a small number of individual stocks, hoping to earn extraordinary returns. Kumar (2009) finds that individual investors overweight stocks with "lottery features" including low share prices, high volatility and high skew. Green and Hwang (2012) show that when firms go public, those with greater expected skewness earn higher first-day returns and are purchased by more individuals. However, these lottery stocks earn poor returns relative to the market. Investors who are poor, inexperienced, and uneducated are especially likely to overweight lottery stocks. Dorn et al. (2014) provide evidence that when lottery jackpots in a particular state or country increase, trading activity by individual investors in the same state or country falls. Likewise, Barber et al. (2009) document a substantial reduction in the trading of Taiwanese equities following the introduction of legalized gambling. If investors treat lotteries and financial gambling as substitutes, then investors might accept lottery tickets as compensation for not gambling in the financial markets. I test this idea within the Barberis and Xiong (2009) discrete-time model of realization utility.[4]

Shefrin and Statman (1993) note that understanding the principles of prospect theory is necessary to designing successful financial products. For example, Breuer and Perst (2007) show that behavioral biases can predict the popularity of structured convertible bond products. Investors who participate in stock loan lotteries are paid, in the form of lottery tickets, to hold positions for multiple periods. This policy is similar in spirit to savings accounts and fixed income investments that incorporate lottery payoffs. Kearney et al. (2010) document a strong demand for these financial instruments across the world and over several centuries.

In the Barberis and Xiong (2009) model, investors only experience utility from realizing

[4]The model of realization utility is not the primary model in Barberis and Xiong (2009). However, it is fully specified, and serves as the benchmark model in this paper.

gains and losses. Investors have Tversky and Kahneman (1992) preferences, so they value gains and losses instead of wealth, react asymmetrically to gains and losses, and evaluate prospects using subjective "decision weights" rather than objective probabilities. Barberis and Xiong (2009) find that these investors aspire to realize gains over multiple episodes, resulting in a disposition effect. However, this tendency to reduce positions in assets with positive excess returns leads to lower expected wealth.

Stock loan lotteries provide a potential solution. An individual investor and a centralized exchange enter into a contract. The investor agrees to hold stocks in his portfolio for multiple periods, and selling stocks early results in a severe penalty.[5] This contract is analogous to a Certificate of Deposit (CD) or a Guaranteed Investment Certificate (GIC). The exchange operates a competitive marketplace in securities lending. When the exchange lends shares, it allocates the fees, net of expenses and profits, to the lottery account of the individual investor. Periodically, the exchange holds a lottery and pays the winner the entire pool of stock lending fees. I find that investors with Tversky and Kahneman (1992) preferences are reluctant to forgo frequent trading for stock loan fees. However, these investors are willing to forgo something they overvalue, frequent trading, for something else they overvalue, lottery tickets. In the data, poor investors have especially strong lottery preferences. Since stock loan lotteries provide poorer investors a smaller probability of winning a larger payoff, they effectively target heterogeneous preferences among individual investors. Furthermore, the appeal of stock loan lotteries increases after introducing real market frictions such as transactions costs, leverage constraints, and taxes.

The remainder of this paper is organized as follows. Section 2 compares allocations and outcomes for three models of realization utility: the Barberis and Xiong (2009) model, a model with stock loan fees, and a model with stock loan lotteries. This section assumes that

[5] Throughout this paper, I assume that selling stocks early is impossible under the terms of the contract.

investors use objective probabilities to determine optimal allocations. Section 3 extends the models to incorporate subjective decision weights and defines conditions where individual investors have unconditionally greater welfare, meaning greater expected utility and greater expected wealth. This section also discusses the model implications for heterogeneous investors and market frictions. Section 4 discusses practical considerations in implementing stock loan lotteries. Section 5 concludes.

2. Models of Realization Utility with Objective Probabilities

This section evaluates investor allocations and outcomes in three models of realization utility, using objective probabilities to calculate expected utility. First, I describe the two-period model in Barberis and Xiong (2009) and replicate their results. Next, I construct a model where investors receive fixed stock loan fees if they commit to holding shares for two periods. Finally, I construct a model where investors receive stock loan lottery tickets if they commit to holding shares for two periods.

2.1. Baseline Two-Period Model of Realization Utility

There are three dates in the model: $t = 0$, $t = 1$, and $t = 2$. The duration of two periods is calibrated to span one year. Benartzi and Thaler (1995) suggest that the magnitude of the equity premium is consistent with investors who evaluate their performance at an annual frequency. There is a representative investor who can invest in two securities. The risk-free asset has a gross return of $R_f = 1$. The risky asset has an expected annual gross return of μ and annual standard deviation of σ. If one year consists of two periods, the gross one-period return of the risky asset is modeled as following a binomial distribution:

$$\begin{cases} R_t = R_u = \mu^{0.5} + ((\mu^2 + \sigma^2)^{0.5} - \mu)^{0.5} & \pi = 0.5 \\ R_t = R_d = \mu^{0.5} - ((\mu^2 + \sigma^2)^{0.5} - \mu)^{0.5} & 1 - \pi = 0.5 \end{cases}$$

R_u and R_d are gross risky-asset returns in the up and down states, π is the probability of realizing the up state in a particular period, and R_t is i.i.d. across periods.

The investor maximizes expected utility where the value function, defined over gains and losses, follows the Tversky and Kahneman (1992) cumulative prospect theory functional form:

$$\begin{cases} v(x) = x^{\alpha} & x \geq 0 \\ v(x) = -\lambda(-x)^{\beta} & x < 0 \end{cases}$$

The investor with prospect theory preferences is risk averse over gains and risk seeking over losses, so α and β are restricted to the interval $(0, 1)$. Also, the investor is more sensitive to losses, so $\lambda > 1$.

The investor is endowed with initial wealth W_0. At $t = 0$, the investor chooses to purchase x_0 shares of the risky asset at a price of P_0 per share. The investor is not allowed to have negative wealth at $t = 1$, so x_0 is restricted to the interval: $[0, \frac{W_0}{P_0 * (1 - R_d)}]$. The remainder of the investor's wealth is allocated to the risk-free asset. Therefore, at $t = 1$, the investor's wealth is distributed:

$$\begin{cases} W_u = W_0 + P_0 X_0 (R_u - 1) & \pi = 0.5 \\ W_d = W_0 + P_0 X_0 (R_d - 1) & 1 - \pi = 0.5 \end{cases}$$

Table 1 summarizes the parameters for the Barberis and Xiong (2009) two-period model of realization utility. At $t = 1$, the investor chooses x_u and x_d, state-contingent positions in the risky asset. If $x_u < x_0$ or $x_d < x_0$, the investor sells some or all of his initial position, realizing a gain or loss. The investor experiences a burst of prospect theory utility, its magnitude defined by the value function applied to the realized gain or loss. Since the investor cannot have negative wealth at $t = 2$, there are two state-specific nonnegativity constraints: x_u is restricted to the interval: $[0, \frac{W_u}{P_0 * R_u * (1-R_d)}]$ and x_d is restricted to the interval: $[0, \frac{W_d}{P_0 * R_d * (1-R_d)}]$. These maximum allocations depend on both the current stock price and, through the investor's wealth, his $t = 0$ allocation to the risky asset. At $t = 2$, the investor liquidates his position in the risky asset and experiences a second burst of prospect theory utility.

The investor chooses x_0, x_u, and x_d to maximize expected prospect theory utility:

$$\max_{x_0, x_u, x_d} E_0[v((x_0 - x_1)(P_1 - P_0)) * 1_{x_1 < x_0} + v(x_1 * (P_2 - P_b)) * 1_{x_1 > 0}]$$

The first term measures the investor's realization utility at $t = 1$, while the second term measures the investor's realization utility at $t = 2$. P_b is the investor's cost basis, the reference price for evaluating gains and losses at $t = 2$. The cost basis depends on whether the investor purchases shares at $t = 1$ and whether the purchase follows an up state or down state:

$$\begin{cases} P_{bu} = P_{bd} = P_0 & x_1 \leq x_0 \\ P_{bu} = \frac{x_0 * P_0 + (x_1 - x_0) * P_0 * R_u}{x_1} & x_1 > x_0 \\ P_{bd} = \frac{x_0 * P_0 + (x_1 - x_0) * P_0 * R_d}{x_1} & x_1 > x_0 \end{cases}$$

The value function at each of the four possible $t = 2$ outcomes (uu, ud, du, dd) can be written in terms of the choice variables:

$$v_{uu}(x_0, x_u, x_d) = v((x_0 - x_u)(P_u - P_0)) * 1_{x_u < x_0} + v(x_u * (P_{uu} - P_{bu})) * 1_{x_u > 0}$$

$$v_{ud}(x_0, x_u, x_d) = v((x_0 - x_u)(P_u - P_0)) * 1_{x_u < x_0} + v(x_u * (P_{ud} - P_{bu})) * 1_{x_u > 0}$$

$$v_{du}(x_0, x_u, x_d) = v((x_0 - x_d)(P_d - P_0)) * 1_{x_d < x_0} + v(x_d * (P_{du} - P_{bd})) * 1_{x_d > 0}$$

$$v_{dd}(x_0, x_u, x_d) = v((x_0 - x_d)(P_d - P_0)) * 1_{x_d < x_0} + v(x_d * (P_{dd} - P_{bd})) * 1_{x_d > 0}$$

Because $\pi = 0.5$, each node of the binomial tree is equally likely, and the investor maximizes the average value associated with each outcome:

$$\max_{x_0, x_u, x_d} 0.25 * (v_{uu} + v_{ud} + v_{du} + v_{dd})$$

I solve the model numerically by calculating $E_0(v)$ for all feasible values of (x_0, x_u, x_d) and choosing arguments that maximize the value function. Table 2 summarizes the model solutions for different values of μ, holding all other parameter values constant. When the expected gross annual return of the risky asset is below 1.08, the investor's optimal decision is to invest all his wealth in the risk-free asset. Because the gross return of the risk-free asset is calibrated to $R_f = 1$, this conservative investment strategy guarantees $E_0(v) = 0$.[6] Because prospect theory investors are more sensitive to losses than gains, they require a substantial risk premium to invest in the risky asset.

When the expected return of the risky asset is between 1.09 and 1.11, the investor exhibits a disposition effect. He chooses to invest some of his wealth in the risky asset at $t = 0$ and takes some profits at $t = 1$ when the up state occurs. Because the investor has concave realization utility over gains, he prefers to experience gains over multiple bursts. Once the risk premium exceeds 12%, the investor prefers to increase his initial investment following the realization of the up state. Because the investor's portfolio appreciates in the first period, he is able to take more risk before exhausting the nonnegative wealth constraint. For assets with sufficiently high expected returns, the marginal expected prospect theory utility of increasing expected gains in the second period exceeds the marginal utility of

[6]Setting $R_f = 1$ assumes investors evaluate the performance of the risky investment relative to the risk-free rate instead of relative to 0.

realizing gains at $t = 1$.

2.2. Two-Period Model of Realization Utility with Fixed Stock Loan Fees

In this model, the investor can only choose to purchase shares of the risky asset at $t = 0$ and promise to hold the position until $t = 2$. This commitment allows the centralized exchange to lend shares to institutions who want to short sell the stock. The exchange retains some proportion of the securities lending proceeds as a commission and pays the remainder to the investor at $t = 2$.

Since the investor cannot execute closing trades at $t = 1$, there are two new constraints: $x_u \geq x_0$ and $x_d \geq x_0$.[1] The one new parameter in this model is f, the lending fee. I consider two values for f: five basis points (0.0005), a realistic fee for US large-cap equities, and 50 basis points (0.005), a realistic fee for US small-cap equities or foreign equities. These parameter estimates are within the range of stock lending fees in the D'Avolio (2002) and Cohen et al. (2003) data.

Because the investor is guaranteed to receive fees at $t = 2$, x_0 is restricted to the interval: $[0, \frac{W_0}{P_0 * (1 - R_d^2 - f)}]$. This constraint ensures that if the down state occurs in both periods, the investor's initial wealth and the stock loan fees he receives at $t = 2$ will exactly cover his losses. Since the investors do not receive fees for any $t = 1$ purchases, the restrictions on x_u and x_d are unchanged from the baseline model.

In this model, the investor maximizes:

$$\max_{x_0} E_0(v) = 0.25 * (v_{uu} + v_{ud} + v_{du} + v_{dd})$$

The value at each possible $t = 2$ outcome is:

[1] Technically these constraints are in absolute value terms, but it is never optimal for the investor to take a short position in the risky asset.

$$v_{uu}(x_0) = v((P_0 * x_0 * f + x_0 * (P_{uu} - P_0))$$

$$v_{ud}(x_0) = v((P_0 * x_0 * f + x_0 * (P_{ud} - P_0))$$

$$v_{du}(x_0) = v((P_0 * x_0 * f + x_0 * (P_{du} - P_0))$$

$$v_{dd}(x_0) = v((P_0 * x_0 * f + x_0 * (P_{dd} - P_0))$$

Table 3 summarizes the investor's optimal allocations and outcomes in the two-period model with fixed stock loan fees. The investor's $t = 2$ allocation is always a corner solution. If the prospective two-period gamble has positive expected utility for some positive allocation, then the gamble has strictly greater expected utility for a larger positive allocation.[2] Therefore, the investor either invests fully in the risky asset, exhausting the nonnegative wealth constraint, or invests fully in the risk-free asset. If the investor has a full position in the risky asset at $t = 0$, he holds the position at $t = 1$ following the realization of a down state. He is unable to sell any shares, and he is unable to purchase any more because the nonnegative wealth constraint still binds. When the up state is realized at $t = 1$, the nonnegative wealth constraint no longer binds. The investor always chooses to purchase more shares, but never exhausts the new constraint. Beyond a certain point, the marginal utility over losses, which are weighted more heavily, starts exceeding the marginal utility over gains.

For any particular fee, there is a small range of moderate μ in which the investor's best outcome with fixed stock loan fees of five basis points is worse than the investor's best outcome in the baseline model. Once the fees are sufficient to encourage the investor to add to his position following positive returns in the first period, the investor's best outcome is better in the fees model. When the fees are higher, the break-even value of μ is higher. For any given expected risky-asset return, increasing fees increases the investor's maximum

[2] This follows from the functional form of the value function. See Appendix A for details.

expected utility.

2.3. Two-Period Model of Realization Utility with Stock Loan Lotteries

In this model, the investor still promises to hold any shares he purchases at $t = 0$ until $t = 2$. However, instead of receiving a stock loan fee $(f * P_0 * x_0)$ at $t = 2$, the investor receives stock loan lottery tickets. Periodically, the exchange holds a lottery and pays the winner the entire pool of stock loan fees. The stock loan fees are net of the expenses and profits of the exchange and the lottery itself is actuarially fair. In Kahneman and Tversky (1979) notation, a single stock loan lottery at $t = 2$ is a gamble of $(\frac{f*P_0*x_0}{p}, p; 0, 1-p)$, where p is the probability of winning the lottery.[7] In this model, the parameters provide three sources of variation. First, as in section 2.2, the stock loan fee (f) is either five basis points or 50 basis points. Second, p is either 0.01 or 0.1. Third, there is either a single lottery at $t = 2$ for all of the loan fees, or two lotteries at $t = 1$ and $t = 2$, each for half of the loan fees. Each of these lotteries is equivalent to the Kahneman and Tversky (1979) gamble: $(\frac{0.5*f*P_0*x_0}{p}, p; 0, 1-p)$.

When there is a single lottery at $t = 2$, the investor maximizes:

$$\max_{x_0} E_0(v) = 0.25*[p*(v_{uuw}+v_{udw}+v_{duw}+v_{ddw})+(1-p)*(v_{uul}+v_{udl}+v_{dul}+v_{ddl})]$$

In this notation, w is the state where the investor wins the stock loan lottery and l is the state where the investor loses the stock loan lottery. The values in the two outcomes following the realization of two up states are:

$$v_{uuw}(x_0) = v((\frac{f*p_0*x_0}{p} + x_0 * (P_{uu} - P_0))$$

$$v_{uul}(x_0) = v((x_0 * (P_{uu} - P_0))$$

In the contingent value formulas for the six other outcomes $(udw, udl, duw, dul, ddw, ddl)$,

[7] I assume the investor references the gamble to the baseline model instead of the fees model.

the only difference is the share price after two periods, which is either P_{ud}, P_{du}, or P_{dd}. In the version of the model with lotteries at $t = 1$ and $t = 2$, there are 16 possible outcomes, depending on whether the up or down state is realized in each period, and whether the investor wins or loses the two lotteries. The investor maximizes:

$$\max_{x_0} 0.25 * [p^2 * (v_{uuww} + v_{udww} + v_{duww} + v_{ddww}) + p(1-p) * (v_{uuwl} + v_{udwl} + v_{duwl} + v_{ddwl} + v_{uulw} + v_{udlw} + v_{dulw} + v_{ddlw}) + (1-p)^2 * (v_{uull} + v_{udll} + v_{dull} + v_{ddll})]$$

Because the investor savors each burst of prospect theory utility distinctly, a lone lottery win at $t = 1$ by itself has a different value than a lone lottery win at $t = 2$ accompanied by realized gains or losses. The four representative value formulas are:

$$v_{uuww}(x_0) = v(\frac{0.5f*p_0*x_0}{p}) + v(\frac{0.5f*p_0*x_0}{p} + x_0 * (P_{uu} - P_0))$$

$$v_{uuwl}(x_0) = v(\frac{0.5f*p_0*x_0}{p}) + v(x_0 * (P_{uu} - P_0))$$

$$v_{uulw}(x_0) = v(\frac{0.5f*p_0*x_0}{p} + x_0 * (P_{uu} - P_0))$$

$$v_{uull}(x_0) = v(x_0 * (P_{uu} - P_0))$$

Whether there are one or two lotteries, the non-negative wealth constraint requires that x_0 is restricted to the interval: $[0, \frac{W_0}{P_0*(1-R_d^2)}]$. The worst case scenario is that both periods are down states and the investor doesn't win any lotteries. The optimal strategy for the investor is the same in the fees model and the lottery model. If the expected return is below some threshold, the investor keeps all his wealth in the risk-free asset. If the expected return is above the threshold, the investor exhausts the nonnegative wealth constraint at $t = 0$. If the risky asset earns a positive return in the first period, the investor buys more shares at $t = 1$, but not enough to exhaust the new constraint. If the risky asset earns a negative return in the second period, the $t = 0$ constraint still binds.

Table 4 summarizes the investor's best outcomes in each version of the model with stock loan lotteries. The investor always prefers earning a fixed stock loan fee to participating in a single risky lottery at $t = 2$ with the same expected value. Furthermore, holding the expected value of the lottery constant, the investor always prefers the lottery with $p = 0.1$ to the lottery with $p = 0.01$. The investor always prefers participating in two smaller lotteries at $t = 1$ and $t = 2$ to one larger lottery at $t = 2$. These results follow from the concavity of the prospect theory value function over gains. It is possible for the investor to have a slight preference for the model with two small lotteries at $t = 1$ and $t = 2$ to the model with the equivalent fixed fees because of the possibility of realizing utility in two time periods. Of course, the investor always prefers receiving a stock loan fee of 50 basis points to receiving a fee of five basis points.

3. Model Extensions

This section develops the two-period models of realization utility. Following Tversky and Kahneman (1992), the prospect theory investor calculates expected utility using decision weights, not probabilities. Investors are willing to pay more than the actuarially fair price to participate in lotteries, suggesting that introducing lotteries increases the maximum expected utility. Investors who enter into stock loan lottery contracts do not realize gains prematurely and therefore can also earn higher returns. In an economy with many agents, investors with different levels of wealth will have different Kahneman and Tversky (1979) gambles in the lottery. Poor investors have greater exposure to lottery features. This feature is appealing because, in the data, poor investors hold larger proportions of lottery stocks. Finally, market frictions increase the appeal of lotteries by increasing the motivation for investors to refrain from excessive trading.

3.1. Decision Weights

Tversky and Kahneman (1992) use experimental data to estimate a functional form for

$w(p)$, the "weighting function" that converts objective probabilities into decision weights. Other studies confirm the shape of the decision weighting function in non-experimental settings. Wakker et al. (1997) show that these decision weights are necessary to explain the additional premium individuals require to purchase insurance with a nonzero default probability. Polkovnichenko and Zhao (2013) show that the probability weights implied by US index options prices are consistent with the prospect theory decision weights. A lottery is a gamble of the form (1,p;0,1-p).[8] For nonnegative gambles, Tversky and Kahneman (1992) estimate the weighting function as a two-part power function:

$w^{+}(p) = \frac{p^{\gamma}}{(p^{\gamma}+(1-p)^{\gamma})^{1/\gamma}}$

Likewise, for nonpositive gambles, the weighting function is:

$w^{-}(p) = \frac{p^{\delta}}{(p^{\delta}+(1-p)^{\delta})^{1/\delta}}$

Using experimental data, Tversky and Kahneman (1992) estimate $\gamma = 0.61$ and $\delta = 0.69$. These values imply that investors overvalue gambles, both positive and negative, with low probabilities of success. Although the experimental evidence in Tversky and Kahneman (1992) uses small wagers, [9]

Figure 1 shows the potential for stock loan lotteries to increase the investor's maximum expected utility. If an investor with Tversky and Kahneman (1992) preferences receives a certain gain of one dollar, his utility is $v(1) = 1^{0.88} = 1$. On the other hand, suppose the investor receives a free lottery ticket with an expected payoff of one dollar. This lottery ticket can be written as a Kahneman and Tversky (1979) gamble $(1/p, p; 0, 1-p)$ with expected utility $E_0(v|p) = w^{+}(p) * v(1/p)$. The dotted line shows the expected utility from

[8] The investor's reference point is receiving a free lottery ticket.

[9] Kachelmeier and Shehata (1992) show that for large wagers, risk seeking over small probabilities holds for both positive and negative rewards. They offer lotteries in China, where the amount of the wager comprises a substantial portion of the subject's income. Likewise, Cameron (1999) shows that Indonesian subjects fail to play the optimal strategy in the ultimatum game for high stakes.

the lottery payoff as a function of p. For values of p such that $E_0(v|p) > 1$, the investor prefers the lottery payout to a sure gain. Individuals overvalue all lotteries with $p \leq 0.24$. For these values of p, the expected utility from the lottery payoff is strictly decreasing in p. This relation holds because the curvature of $w^+(p)$ function is stronger than the concavity of the value function over gains in this region. Consistent with this functional form, Cook and Clotfelter (1993) show that lottery ticket sales are positively related to jackpot size. In the context of this model, the optimal lottery is not necessarily the one with the smallest odds of winning the biggest prize. The prospect theory investor places an emphasis on reducing losses. Lotteries that pay off more frequently have more potential to reduce the negative utility from negative risky-asset returns.

Table 5 summarizes the best investor outcomes in two-period models of realization utility, calculating expected value using both objective probabilities and decision weights. I solve all 11 specifications of the model from Section 2. In specification 1, the Barberis and Xiong (2009) baseline model (B), the investor chooses $t = 0$ and $t = 1$ risky asset positions subject to wealth constraints. In the remaining models, the investor cannot liquidate positions at $t = 1$. Specifications 2 and 3 are models with fixed loan fees (F). The investor receives a fee (f) for allowing the exchange to lend shares between $t = 0$ and $t = 2$. The remaining models include stock loan lotteries. In specifications 4 through 7, there is a single lottery at $t = 2$ and the investor has probability p of winning $f * P_0 * x_0/p$. In specifications 8 through 11, there are lotteries at $t = 1$ and $t = 2$ and the investor has probability p of winning $0.5 * f * P_0 * x_0/p$ in each lottery. For each specification, the top panel presents the investor's maximum expected prospect theory utility using objective probabilities. The bottom panel presents the investor's maximum expected prospect theory utility calculated using decision weights.

In the baseline model, using decision weights has little effect on optimal allocations because

$w^{-}(0.25)$ is about 1% larger than $w^{+}(0.25)$. The dd outcome always corresponds to a loss and the uu outcome always corresponds to a gain. For the range of μ values, the ud and du outcomes correspond to gains because $R_u * R_d > 1$. In the baseline model, the investor chooses similar optimal allocations using decision weights instead of objective probabilities. For all specifications with stock loan lotteries, the investor's maximum expected utility is greater when prospect theory utility is calculated with decision weights rather than with objective probabilities. The investor's best outcome with stock loan lotteries is better than the investor's best outcome with fixed stock loan fees if utility is calculated with decision weights, rather than similar or worse if utility is calculated with objective probabilities. The fixed stock loan fees increase gains or decrease losses by a marginal amount with certainty. On the other hand, stock loan lotteries increase gains or decrease losses dramatically for a small proportion of outcomes that investors substantially overvalue.

As a result, relative to the Barberis and Xiong (2009) two-period model, it is much easier to increase expected utility through stock loan lotteries than through stock loan fees. For instance, in the baseline model, investors only hold a risky-asset position when the risk premium is at least 10%. A fixed stock loan fee of 50 basis points motivates the investor to hold a risky-asset position when the risk premium is 9%. When this 50 basis point fee is restructured as two small lotteries with low win probabilities, the minimum risk premium falls to 6%. Benartzi and Thaler (1995) argue that investors require a high risk premium to own stocks because they reevaluate their portfolios too frequently. Implementing the stock loan lotteries forces investors to hold risky assets for longer horizons. Stock loan lotteries are optimal compensation for these investors and effectively lower the required rate of return.

3.2. Conditions for Welfare Gains

Barberis (2013) argues that prospect theory complements traditional economic theory. Individuals care about expected wealth and variability of wealth as well as gains or losses relative to a reference point. In this section, I argue that when two conditions are satisfied, introducing stock loan lotteries yields unconditional welfare gains. First, the investor must experience greater expected prospect theory utility in a model with stock loan lotteries than he experiences in the baseline model. The expected prospect theory utility calculation should use decision weights instead of objective probabilities because utility is a subjective measure of the investor's happiness:

$$E_0^*(v_L) > E_0^*(v_B)$$

Second, if the investor maximizes $E[U(W_2)]$ and has risk-neutral preferences, he must have higher expected wealth at $t = 2$ in the model with stock loan lotteries than he has in the baseline model:[10]

$$E_0^*[(W_{2,L})] > E_0^*[(W_{2,B})]$$

The expected wealth calculation should use objective probabilities to measure the investor's actual financial position at $t = 2$. In the baseline model, the only source of wealth is the terminal portfolio value:

$$E_0[(W_{2,B})] = 0.25 * [W_{uu} + W_{ud} + W_{du} + W_{dd}]$$

In the model with stock loan lotteries, the investor also earns the expected value of the lottery payoff:

$$E_0[(W_{2,L})] = [P_0 * x_0 * f] + 0.25 * [W_{uu} + W_{ud} + W_{du} + W_{dd}]$$

The terminal wealth at each of the possible $t = 2$ nodes depends on the $t = 0$ and condi-

[10] Prospect theory investors are risk averse over gains and risk seeking over losses. The assumption of risk neutrality splits the difference.

tional $t = 1$ allocations:

$$W_{uu}(x_0, x_u, x_d) = W_0 + P_0 * x_0 * (R_u^2 - 1) + P_u * (x_u - x_0) * (R_u - 1)$$

$$W_{ud}(x_0, x_u, x_d) = W_0 + P_0 * x_0 * (R_u * R_d - 1) + P_u * (x_u - x_0) * (R_d - 1)$$

$$W_{du}(x_0, x_u, x_d) = W_0 + P_0 * x_0 * (R_d * R_u - 1) + P_d * (x_d - x_0) * (R_u - 1)$$

$$W_{dd}(x_0, x_u, x_d) = W_0 + P_0 * x_0 * (R_d^2 - 1) + P_d * (x_d - x_0) * (R_d - 1)$$

Because $R_u + R_d = 2\sqrt{\mu}$, the expected wealth calculations simplify to:

$$E_0[(W_{2,B})] = W_0 + P_0 x_0(\mu - 1) + 0.5[P_u(x_u - x_0) + P_d(x_d - x_0)](\sqrt{\mu} - 1)$$

$$E_0[(W_{2,L})] = [P_0 x_0 f] + W_0 + P_0 x_0(\mu - 1) + 0.5[P_u(x_u - x_0) + P_d(x_d - x_0)](\sqrt{\mu} - 1)$$

Figure 2 presents conditions in which introducing stock loan lotteries delivers unconditional welfare gains. For all points above the solid curve, investors in the model with stock loan lotteries have strictly higher welfare than investors in the baseline model. The lottery model uses a single lottery with $p = 0.1$ at $t = 2$. In other words, for these points in (μ, f) space, the utility-maximizing allocation in the model with lotteries produces greater expected utility and greater expected wealth than the utility-maximizing allocation in the baseline model. The solid curve is kinked, first decreasing in μ to some threshold value of μ. For risky-asset returns above this threshold, the investor already wants to maximize his holdings and any positive f is sufficient to produce better welfare.

The dotted curve is also kinked, but for a wide range of expected asset returns and lending fees, introducing stock loan lotteries unconditionally improves investor welfare, while introducing stock loan fees does not. This region is represented by the area between the curves. For moderate values of μ, this range could represent a significant proportion of all stocks. For example, investors require a flat fee of 143 basis points to hold stocks with $\mu = 1.08$, but an expected fee of only 30 basis points denominated in lottery tickets. This range widens for lower values of μ, however the wider range is likely to include fewer stocks since it is rare for US stocks to command high lending fees. Table 5 suggests that lotteries that pay off more frequently with lower probabilities have the potential to further widen the region of potential welfare gains.

3.3. Implications for Equilibrium with Heterogeneous Agents

In equilibrium, there are many individual investors. All investors benefit from the lottery because the probability of any individual investor winning the lottery is in the region where the decision weight exceeds the objective probability. Individual investors have different risky asset holdings because of income inequality, and a number of studies suggest that poor individual investors exhibit especially strong biases. For instance, Dhar and Zhu (2006) show that the trading activity of poor investors shows a larger disposition effect, while Kumar (2009) shows that poor investors hold a larger proportion of "lottery stocks" in their portfolios.

Consider a stock loan marketplace with many individual investors, and each investor, i, owns $X_{0,i}$ shares of the risky asset at $t = 0$. Each investor's probability of winning the lottery depends on the ratio of his allocation to the aggregate allocation of all investors, $X_{0,A}$. Since investors in the lottery model who buy the risky asset always exhaust their non-negative wealth constraints, each investor's probability of winning the lottery is also equivalent to his wealth share:

$p = \frac{X_{0,i}}{X_{0,A}} = \frac{W_{0,i}}{W_{0,A}}$

One way to examine the relative appeal of stock loan lotteries to different investors is to see how a standardized measure of utility varies according to wealth share. Define Π as this standardized measure of utility per unit of wealth:

$\Pi_i = \frac{E_0[U(W_{2,i})|p,v(),w^+(p)]}{W_{0,i}}$

Figure 3 shows how standardized utility varies according to wealth share in four different models of realization utility. The solid curves show standardized utility for models with a single stock loan lottery at $t = 2$ and stock loan fees of five and 50 basis points. The dotted curves show standardized utility for models with stock loan fees of five and 50 basis points. For all models, the annualized expected gross return of the risky asset (μ), is 1.12. In all four models, standardized utility is strictly downward sloping with respect to wealth share. In other words, these models are regressive in the sense that poor investors benefit disproportionately in utility terms.

Models with stock loan fees are regressive because the prospect theory value function is concave over gains, so the marginal utility of wealth is strictly decreasing with increasing wealth. The concavity of the value function implies that increasing loan fees also benefits the poorest investors disproportionately. The economy with stock loan lotteries is even more regressive. This follows from the functional form of the decision weighting function. The ratio $w^+(p)/p$ is strictly decreasing with increasing p. The poorest investors have the lowest probabilities of winning the lotteries and place the highest value on the lotteries relative to the value implied by objective probabilities. For this reason, the regressive effect of increasing f is greater in the model with stock loan lotteries than in the model with stock loan fees.

It is also straightforward to show that the idiosyncratic volatility and idiosyncratic skew of the lotteries are both strictly decreasing in wealth. Since the investment environment consists of only a single risky asset, the lottery payoffs are the only source of idiosyncratic returns. In a model with a single lottery, the proceeds from winning are $f * W_{0,A}$. So for any given investor i, the gross returns from winning the lottery are:

$R_L = \frac{f * W_{0,A}}{W_{0,i}} = \frac{f}{p}$.

The gross returns from losing the lottery are 0. The idiosyncratic volatility and skew are:

$E(R_L^2) = p * (\frac{f}{p})^2 = \frac{f^2}{p}$

$E(R_L^3) = p * (\frac{f}{p})^3 = \frac{f^3}{p^2}$

Kumar (2009) shows that poor investors have especially strong preferences for stocks with positive idiosyncratic volatility and idiosyncratic skew. Since idiosyncratic volatility and idiosyncratic skew of the lotteries are both strictly decreasing in wealth share, introducing the lotteries would effectively target heterogeneous preferences.

3.4. Market Frictions: Transactions Costs, Leverage, and Taxes

The three models of realization utility assume that markets are frictionless. However, excessive trading by individual investors is a problem, in part, because trading is costly. French (2008) estimates that the annual total cost of active trading consistently ranges from 61 to 74 basis points from 1990 to 2006. These constant trading costs are the result of two countervailing trends. As studies such as Novy-Marx and Velikov (2016) document, the cost of trading a share of stock decreases significantly over time. On the other hand, French (2008) and others document a significant upward time trend in share turnover. I model trading costs by introducing a new parameter, ρ, representing round-trip transactions costs. Since the two periods in the model correspond to a year, and all positions are closed at $t = 2$, I follow the French (2008) estimates and set $\rho = 0.013$.

Another important market friction is the cost and availability of leverage. The Barberis and Xiong (2009) model assumes that investors can borrow or lend at the risk-free rate, and the non-negative wealth constraint ultimately determines the maximum leverage. In fact, Frazzini and Pedersen (2014) show that leverage constraints lead to high demand and low expected returns for high-beta assets. Barberis and Xiong (2009) acknowledge that while optimal allocations imply the use of substantial leverage, only a small proportion of individual investors use leverage. Since the US Federal Reserve Board has maintained a 50% initial margin requirement since 1974, I model leverage constraints by limiting the investor's risky asset investment to twice his wealth.[3] In all three models, this restricts the $t = 0$ allocation to $x_0 \leq \frac{2*W_0}{P_0}$, and restricts the state-contingent investments at $t = 1$: $x_u \leq \frac{2*W_u}{P_u}$ and $x_d \leq \frac{2*W_d}{P_d}$.

Figure 4 shows how market frictions change the potential welfare benefits of stock loan lotteries. For different values of μ, the annualized expected gross return of the risky asset, I solve all three models in a perfect market as well as a market with trading costs and leverage constraints. For each value of μ, in each environment, I calculate f_F and f_L, the minimum fee that provides investors in the fee and lottery models unconditionally greater welfare than in the baseline model. For each μ, a proxy for the potential welfare benefits of introducing stock loan lotteries is $max[f_F(\mu) - f_L(\mu), 0]$. The solid line shows the potential for welfare improvement with market frictions, while the dotted line shows the potential for welfare improvement with perfect markets. Market frictions increase the potential for lotteries to improve welfare for two reasons. First, trading costs reduce the effective return of all risky investments. As Figure 2 shows, it is far easier to persuade investors to buy risky assets with low expected returns by using stock loan lotteries than by using stock loan fees. Second, even when allocations using maximum leverage are lower than those chosen in a model without leverage constraints, investors still typically choose allocations with $x_u < x_0$ to spread out the realization of gains over multiple episodes. In a model with capital gains taxes, the value of stock loan lotteries increases even further.

[3]Regulation T allows the Federal Reserve to change this margin requirement, but the Federal Reserve has never changed it.

4. Implementing Stock Loan Lotteries in Practice

This section addresses practical questions about how to implement stock loan lotteries. What type of laboratory experiment could test whether there is a demand among individual investors for stock loan lotteries? What are the relevant lottery laws and regulations in countries with developed equity markets? Is it possible for exchanges to offer financial securities that replicate lottery payouts?

4.1. Survey Evidence

The Tversky and Kahneman (1992) decision weighting function specifies that prospect theory investors prefer fair lotteries with low win probabilities to certain gains. In the Barberis and Xiong (2009) model of realization utility, providing lottery payoffs to investors as compensation to hold positions for longer can lead to improved outcomes. In practice, would individuals have demand for stock loan lottery tickets? This section proposes a randomized controlled trial (RCT) experiment to test this question.[4] In this research design, there is a preliminary stage in which the participant allocates a portfolio among a risky investment and a risk-free investment at $t = 0$, observes the risky asset return at $t = 1$, and rebalances his portfolio. In the second stage, the participant allocates his portfolio between a risky asset and a risk-free asset at $t = 0$ and must hold the portfolio until $t = 2$. As compensation for forgoing the opportunity to rebalance at $t = 1$, the control group is offered a fixed fee, while the treatment group is offered a lottery payoff.

Survey participants will be chosen using Amazon's Mechanical Turk (MTurk) platform. Casler et al. (2013) find that the MTurk community is more demographically diverse than samples recruited on college campuses or via social media. Survey participants will receive $2 for a task which takes an average of 15 minutes to complete. Camerer and Hogarth

[4] At this time, the survey has not been administered pending institutional review board approval.

(1999) review the literature on field experiments with varying levels of financial incentives. The authors find that in similar types of experiments, increasing incentives does not have a significant impact on average behavior. Cryder et al. (2012) show that MTurk participants exhibit standard behavioral biases and their average responses are similar to other survey populations. However, some MTurk participants do not pay attention to or understand the instructions, introducing noise into the responses.

The dependent variable is the percentage of the portfolio allocated to the risky investment in the second stage. The coefficient of interest is the treatment dummy. Control variables include demographic information and proxies for financial sophistication. The demographic controls are age, income, education, marital status, gender, and state of residence. Barber and Odean (2001) show that men turn over their portfolios more frequently than women. Kumar (2009) provides evidence that socioeconomic factors explain investment in stocks with lottery features. Dorn et al. (2014) provide evidence that the state of residence could impact financial gambling through differences in major lottery jackpots. Variables that proxy for financial sophistication include home ownership status, employment status, and employment experience in the financial industry. Feng and Seasholes (2005), Dhar and Zhu (2006), and Frazzini (2006) document a more pronounced disposition effect in the trading activity of less experienced investors. Alevy et al. (2007) show that professional traders outperform students in experimental tasks.

Other control variables include proxies for risk aversion. Risk aversion can be ascertained by the respondent's allocations in the preliminary task. Furthermore, participants will complete questions developed by Weber et al. (2002) to measure risk aversion in the areas of investments and gambling.[5] The risky asset allocation should relate positively

[5]Weber et al. (2002) ask respondents: "Please indicate your likelihood of engaging in each activity or behavior. Provide a rating from 1 (Very unlikely) to 5 (Very likely)." One activity in the investment risk taking questionnaire is: "Investing 5% of your annual income in a very speculative stock." One activity in

to both the Sharpe ratio of the risky asset and the amount of the fee. In the research design, individuals participating in the survey will be required to complete the preliminary task and the control or treatment task five times each. This will allow me to estimate a specification of the model that includes individual fixed effects. If the coefficient estimate in the specification with individual fixed effects does not change much, this addresses concerns that the individuals in the treatment and control groups are systematically different.

4.2. The Regulatory Environment

Since individuals are willing to pay much more for lottery tickets than the actuarially fair value, lotteries can be thought of as a form of voluntary taxation. As a result, national governments set laws and regulations for the administration of any lotteries within its borders. In the United States, the country with the largest developed equity markets, lotteries are run by individual states and territories and are subject to their laws. Six states prohibit lotteries completely. [6] There are games offered by a single state, a group of states, and since 2009, all individual state lotteries also offer Mega Millions and Powerball.

It is illegal in the United States for private companies to offer promotions that require a purchase by participants. The United States Federal Trade Commission (FTC) only allows companies to offer games of chance if any individual can participate for free by filling out an entry form. Other agencies, including the United States Postal Service (USPS) and the Federal Communications Commission (FCC), are also responsible for enforcing these laws. Individual states have laws requiring firms offering sweepstakes to meet certain obligations. If a sweepstakes involves telephone calls, the FTC's Telemarketing Sales Rule requires specific disclosures, such as the odds of winning a prize, how to participate without buying anything, and that no purchase or payment is required to win. An Illinois government

the gambling risk taking questionnaire is: "Betting a day's income at a high stake poker game."

[6] The states that prohibit lotteries are Alabama, Alaska, Hawaii, Mississippi, Nevada, and Utah.

report on government privatization states that as of 2006, no US state has completely privatized its lottery.[7] According to the report, Illinois and Connecticut both had difficulty finding private firms willing and able to make competitive bids to operate lotteries.

While the lottery laws in the United States vary across 50 states and several territories, other countries with developed markets have a more manageable regulatory structure. Five regional organizations oversee the operation of Canadian lotteries. The Atlantic Lottery Corporation oversees lotteries in New Brunswick, Prince Edward Island, Nova Scotia, and Newfoundland and Labrador. Loto-Quebec, Ontario Lottery and Gaming Corporation, and British Columbia Lottery Corporation are the governing bodies for their provinces while Western Canada Lottery Corporation governs the remaining provinces and territories. The five regional organizations jointly own the Interprovincial Lottery Corporation, which is responsible for administering national lottery games. Australian lotteries are operated by government-owned firms and private firms. Lotterywest is a state-owned and operated firm that manages lotteries in Western Australia, while Tattersall is a government-licensed firm that runs several lotteries throughout the remaining states in Australia. Charitable organizations operate lotteries where the prizes include houses, cars, and furniture. The national government sets some regulations, while the individual states grant licenses to lotteries. The Interactive Gambling Act of 2001 prohibits foreign firms from marketing internet gambling to Australian citizens.

The United Kingdom established The National Lottery Commission in 1994 to regulate the UK National lottery. The National Lottery Commission issued the license to operate the lotteries to the Camelot Group in 1994 and the license has been subsequently reissued. The Camelot Group was formed as a consortium of firms with expertise in various areas of administering lotteries, such as information technology, marketing, and retail sales. The

[7] The website of the report is: http://cgfa.ilga.gov/Upload/2006Gov_Privatization_Rprt.pdf

lottery returns approximately half of the proceeds of ticket purchases as prizes, one quarter to charities, and one quarter in taxes, commissions, and management fees. The Camelot Group goes to great efforts to protect players by identifying and supporting potentially compulsive gamblers and by providing independent private wealth management for jackpot winners.[8] In 2010, The Camelot Group was purchased by The Ontario Teachers Pension Plan, an independent private Canadian defined-benefit pension administration firm.

One avenue for implementing stock loan lotteries is by offering the lottery tickets through one of the regional lottery operators, subject to the national laws. Then the centralized exchange could purchase these tickets with the lending fees and issue them to the individual investors. This system is more realistic in countries with relatively few operators, such as Canada, Australia, and the UK. This infrastructure would require that the regional operator verify the individual investors jurisdiction. A simpler option, explored in the next section, is that lottery tickets are structured as a derivative security.

4.3. Stock Loan Lottery Tickets as Derivative Securities

Stock loan lottery tickets could be structured as a derivative with a payoff that is a random variable linked to the price of a portfolio of securities. For example, a "Pennies Series X" derivative has a payoff of 1 if X is the pennies (hundredths) digit of the closing spot price of an equity index on a particular day. Because major equity indices are portfolios of a large number of individual stocks in specified proportions, it is not feasible to manipulate the pennies digit of the index spot price by buying or selling the component securities. Although Harris (1991) and others identify the propensity for round number clustering in the trading prices of individual stocks, this is not likely to translate to round number clustering in a diversified portfolio of these stocks.

[8] The website of The Camelot Group is: http://www.camelotgroup.co.uk/

Table 5 provides evidence suggesting that there is no clustering in the closing spot prices of two major diversified equity indices. The black bars show the frequencies of the pennies digits in the closing prices of the S&P 500 index between December 30, 1927 and April 21, 2017. The S&P 500 index is a capitalization-weighted index of 500 diversified large US public equities. The gray bars show the frequency of each of the pennies digits in the closing prices of the Nikkei 225 index between January 5, 1970 and April 21, 2017. The Nikkei 225 index is a price-weighted index of 225 diversified large US public equities. There are no abnormal frequencies at zero pennies or any other value. This suggests that a centralized exchange could issue stock loan lottery tickets in the form of financial derivatives instead of holding traditional lottery drawings, potentially bypassing regulations.

5. Conclusion

The excessive trading of individual investors has large economic consequences. Private firms are motivated to exploit the psychological biases that cause excessive trading. For instance, brokerage firms encourage investors to use mobile apps in order to trade more frequently. Stock loan lotteries have the potential to reduce excessive trading by individual investors. Investors with realization utility need to be compensated to forgo the excessive trading necessary to realize gains. It is cheaper to compensate individual investors with stock loan lottery tickets than with stock loan fees because individual investors with prospect theory preferences overvalue participation in lotteries. Investors participating in stock loan lotteries can experience greater expected utility and earn higher expected returns, while allowing other market participants to earn profits from securities lending and administering the lottery itself. Two promising features of stock loan lotteries are that they provide the greatest utility to the poorest investors and that the benefits increase is a model with market frictions. These results suggest the potential for stock loan lotteries to improve the performance and welfare of individual investors.

The Barberis and Xiong (2009) model estimates expected utility over a one-year horizon. In reality, many investors have financial goals that span a much longer horizon. To the extent that investors care about their children or heirs, a recursive argument suggests investors optimize over an infinite time horizon. Barberis and Xiong (2012) and Henderson (2012) consider the implications of realization utility in an infinite-horizon model framework. Evaluating the implications of introducing stock loan lotteries to one of these infinite-horizon models has the potential to yield more interesting insights about the potential for this financial innovation.

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Table 1: Parameter Values This table lists the important parameters, symbols, and calibrated values for the Barberis and Xiong (2009) two-period model of realization utility. The investor has W_0 in wealth at $t = 0$. He allocates his wealth between the risk-free asset, which has a normalized gross return, $R_f = 1$, and a risky asset, with annualized mean return μ and annualized standard deviation σ. The risky asset return in each period has a binomial distribution with an equal probability of R_u return in the up state and R_d return in the down state, where R_u and R_d are chosen to match μ and σ. The investor chooses x_0, the $t = 0$ risky asset allocation, x_u, the $t = 1$ risky asset allocation following the up state, and x_d, the risky asset allocation following the down state. The investor maximizes $E_0(v)$, the current expected value at $t = 1$ and $t = 2$, and $v()$ is the Tversky and Kahneman (1992) value function applied to realized gains and losses.

Parameter	Symbol	Value(s)
Initial Wealth	W_0	40
Risky Asset: Initial Price	P_0	40
Risky Asset: Annualized Mean Return	μ	1.05-1.15
Risky Asset: Annualized SD Return	σ	0.3
Value Function: Concavity	α	0.88
Value Function: Convexity	β	0.88
Value Function: Sensitivity	λ	2.25

Table 2: Optimal Allocations in the Baseline Model For different values of μ, the annualized expected gross return of the risky asset, this table lists the investor's optimal allocations and best outcome in the baseline Barberis and Xiong (2009) two-period model of realization utility. In all cases, σ, the annualized standard deviation of risky asset returns, is 0.3. Table 1 lists and describes the other important model parameters. The model assumes that there are two periods in a year, and the gross one-period return of the risky asset is modeled as following a binomial distribution with equal probabilities of realizing a return of R_u in the up state and R_d in the down state. The choice variables are the risky asset allocations at $t = 0$ (x_0) and the risky asset allocations at $t = 1$ following the realization of the up (x_u) or down (x_d) states. $E_0(v)$ is expected $t = 0$ cumulative prospect theory utility. The investor experiences utility at $t = 1$ and $t = 2$ if he realizes gains or losses, and the bursts of future prospect theory utility are not discounted by time. The value function for prospect theory utility uses the functional form and parameter values in Tversky and Kahneman (1992).

μ	R_u	R_d	x_0^*	x_u^*	x_d^*	$E_0^*(v)$
1.05	1.230	0.820	0	0	0	0
1.06	1.234	0.826	0	0	0	0
1.07	1.238	0.831	0	0	0	0
1.08	1.241	0.837	0	0	0	0
1.09	1.245	0.843	3.4	2.6	3.4	0.01
1.10	1.249	0.848	3.6	2.8	3.6	1.09
1.11	1.253	0.854	3.7	3.0	3.7	2.22
1.12	1.257	0.860	3.8	5.4	3.8	3.66
1.13	1.261	0.865	4.0	6.0	4.0	5.29
1.14	1.265	0.871	4.1	6.6	4.1	7.01
1.15	1.269	0.876	4.2	7.2	4.4	8.95

Table 3: Optimal Allocations in the Model with Stock Loan Fees For different values of μ, the annualized expected gross return of the risky asset, this table lists the investor's optimal allocations and best outcome in various two-period models of realization utility. The left panel shows results from the baseline (B) model. The choice variables are the risky asset allocations at $t = 0$ (x_0) and the risky asset allocations at $t = 1$ following the realization of the up (x_u) or down (x_d) states. $E_0(v)$ is the expected $t = 0$ cumulative prospect theory utility the investor experiences at $t = 1$ and $t = 2$ if he realizes gains or losses, and the bursts of prospect theory utility are not discounted by time. The value function for prospect theory utility uses the functional form and parameter values in Tversky and Kahneman (1992). The center and right panels show results from models with stock loan fees (F). The investor is not allowed to execute closing trades at $t = 1$. The investor receives a lending fee of f at $t = 2$ for allowing the exchange to lend shares between $t = 0$ and $t = 2$. The center panel shows the investor's optimal allocation and best outcome when f is five basis points. The right panel shows the investor's optimal allocation and best outcome when f is 50 basis points. The asterisks denote cases in which the investor's maximum expected utility in the model with stock loan fees is greater than the investor's maximum utility in the baseline model.

Model	B				F(5)				F(50)			
μ	x_0^*	x_u^*	x_d^*	$E_0^*(v)$	x_0^*	x_u^*	x_d^*	$E_0^*(v)$	x_0^*	x_u^*	x_d^*	$E_0^*(v)$
1.05	0	0	0	0	0	0	0	0	0	0	0	0
1.06	0	0	0	0	0	0	0	0	0	0	0	0
1.07	0	0	0	0	0	0	0	0	0	0	0	0
1.08	0	0	0	0	0	0	0	0	0	0	0	0
1.09	3.4	2.6	3.4	0.01	0	0	0	0	3.5	4.2	3.5	*0.22
1.10	3.6	2.8	3.6	1.09	3.6	4.6	3.6	0.93	3.6	4.7	3.6	*1.48
1.11	3.7	3.0	3.7	2.22	3.7	5.0	3.7	*2.27	3.8	5.2	3.8	*2.87
1.12	3.8	5.4	3.8	3.66	3.8	5.5	3.8	*3.75	3.9	5.7	3.9	*4.39
1.13	4.0	6.0	4.0	5.29	4.0	6.0	4.0	*5.37	4.1	6.2	4.1	*6.07
1.14	4.1	6.6	4.1	7.01	4.1	6.6	4.1	*7.16	4.2	6.9	4.2	*7.92
1.15	4.2	7.2	4.4	8.95	4.3	7.4	4.3	*9.16	4.4	7.6	4.4	*9.99

Table 4: Best Outcomes in the Model with Stock Loan Lotteries For different values of μ, the annualized expected gross return of the risky asset, this table lists the investor's best outcome in various two-period models of realization utility with stock loan lotteries. The choice variables are the risky asset allocations at $t = 0$ (x_0) and the risky asset allocations at $t = 1$ following the realization of the up (x_u) or down (x_d) states. The investor is not allowed to sell any risky-asset holdings at $t = 1$. The values in the table are $E_0^*(v)$, the expected $t = 0$ cumulative prospect theory utility the investor experiences at $t = 1$ and $t = 2$ if he realizes gains or losses, and the bursts of prospect theory utility are not discounted by time. The value function for prospect theory utility uses the functional form and parameter values in Tversky and Kahneman (1992). The investor receives a fee of f for allowing the exchange to lend shares between $t = 0$ and $t = 2$. The fee is pooled into a lottery, and the investor has probability p of winning the lottery. In the left panel, there is a single lottery at $t = 2$ where the investor has probability p of winning $f * P_0 * x_0/p$. In the right panel, there are lotteries at $t = 1$ and $t = 2$ and in each lottery, the investor has probability p of winning $0.5 * f * P_0 * x_0/p$. The asterisks denote cases in which the investor's maximum expected utility in the model with stock loan lotteries is greater than the investor's maximum utility in the baseline model.

p	0.01	0.1	0.01	0.1	0.01	0.1	0.01	0.1
f (BPs)	5	5	50	50	5	5	50	50
Model	L(2)	L(2)	L(2)	L(2)	L(1,2)	L(1,2)	L(1,2)	L(1,2)
$\mu = 1.05$	0	0	0	0	0	0	0	0
$\mu = 1.06$	0	0	0	0	0	0	0	0
$\mu = 1.07$	0	0	0	0	0	0	0	0
$\mu = 1.08$	0	0	0	0	0	0	0	0
$\mu = 1.09$	0	0	*0.13	*0.20	0	0	*0.14	*0.22
$\mu = 1.10$	0.93	0.93	*1.36	*1.44	0.93	0.94	*1.38	*1.46
$\mu = 1.11$	*2.27	*2.27	*2.71	*2.80	*2.27	*2.28	*2.74	*2.82
$\mu = 1.12$	*3.74	*3.74	*4.19	*4.28	*3.74	*3.75	*4.22	*4.31
$\mu = 1.13$	*5.36	*5.36	*5.82	*5.92	*5.36	*5.37	*5.86	*5.95
$\mu = 1.14$	*7.15	*7.15	*7.63	*7.73	*7.15	*7.16	*7.67	*7.76
$\mu = 1.15$	*9.13	*9.14	*9.63	*9.74	*9.13	*9.15	*9.67	*9.77

Table 5: Best Outcomes: Stock Loan Lotteries with Decision Weights For different values of μ, the annualized expected gross return of the risky asset, this table lists the investor's best outcome in various two-period models of realization utility. Specification 1 is the baseline model in Barberis and Xiong (2009). The choice variables are the risky asset allocations at $t = 0$ (x_0) and the risky asset allocations at $t = 1$ following an up (x_u) or down (x_d) return in the first period. Specifications 2 and 3 are models where the investor chooses a single risky asset allocation at $t = 0$ (x_0) and is not allowed to trade shares at $t = 1$. The investor receives a fee of f for allowing the exchange to lend shares between $t = 0$ and $t = 2$. Specifications 4 through 11 are models where the stock loan fee is pooled into a lottery, and the investor has probability p of winning the lottery. In specifications 4 through 7, there is a single lottery (L) at $t = 2$ where the investor has probability p of winning $f * P_0 * x_0/p$. In specifications 8 through 11, there are lotteries at $t = 1$ and $t = 2$ and in each lottery, the investor has probability p of winning $0.5 * f * P_0 * x_0/p$. The values in the table are $E_0(v)$, the expected $t = 0$ cumulative prospect theory utility the investor experiences at $t = 1$ and $t = 2$ if he realizes gains or losses, and the bursts of prospect theory utility are not discounted by time. The value function for prospect theory utility uses the functional form and parameter values in Tversky and Kahneman (1992). In the top panel, the investor maximizes expected prospect theory utility using objective probabilities. In the bottom panel, the investor maximizes prospect theory utility using decision weights with the functional form and parameter values in Tversky and Kahneman (1992).

	1	2	3	4	5	6	7	8	9	10	11
p				0.01	0.1	0.01	0.1	0.01	0.1	0.01	0.1
f (BPs)		5	50	5	5	50	50	5	5	50	50
Model	B	F(5)	F(50)	L(2)	L(2)	L(2)	L(2)	L(1,2)	L(1,2)	L(1,2)	L(1,2)
$\mu = 1.05$	0	0	0	0	0	0	0	0	0	0	0
$\mu = 1.06$	0	0	0	0	0	0	0	0	0	0	0
$\mu = 1.07$	0	0	0	0	0	0	0	0	0	0	0
$\mu = 1.08$	0	0	0	0	0	0	0	0	0	0	0
$\mu = 1.09$	0.01	0	0.22	0	0	0.13	0.20	0	0	0.14	0.22
$\mu = 1.10$	1.09	0.93	1.48	0.93	0.93	1.36	1.44	0.93	0.94	1.38	1.46
$\mu = 1.11$	2.22	2.27	2.87	2.27	2.27	2.71	2.80	2.27	2.28	2.74	2.82
$\mu = 1.12$	3.66	3.75	4.39	3.74	3.74	4.19	4.28	3.74	3.75	4.22	4.31
$\mu = 1.13$	5.29	5.37	6.07	5.36	5.36	5.82	5.92	5.36	5.37	5.86	5.95
$\mu = 1.14$	7.01	7.16	7.92	7.15	7.15	7.63	7.73	7.15	7.16	7.67	7.76
$\mu = 1.15$	8.95	9.16	9.99	9.13	9.14	9.63	9.74	9.13	9.15	9.67	9.77
$\mu = 1.05$	0	0	0	0	0	0	0	0	0	0	0
$\mu = 1.06$	0	0	0	0	0	0	0	0	0	0.40	0
$\mu = 1.07$	0	0	0	0	0	1.11	0	0	0	1.84	0.55
$\mu = 1.08$	0	0	0	0	0	2.50	0.77	0	0.49	3.36	2.51
$\mu = 1.09$	0	0	0.09	0.41	0.82	4.01	2.48	1.04	2.57	5.01	4.65
$\mu = 1.10$	1.11	0.92	1.56	1.96	2.63	5.65	4.34	2.71	4.83	6.81	6.97
$\mu = 1.11$	2.43	2.48	3.17	3.66	4.61	7.44	6.36	4.54	7.30	8.77	9.51
$\mu = 1.12$	4.08	4.20	4.94	5.52	6.77	9.40	8.59	6.55	10.02	10.93	12.32
$\mu = 1.13$	5.92	6.08	6.90	7.57	9.16	11.56	11.03	8.76	13.00	13.30	15.41
$\mu = 1.14$	8.01	8.17	9.06	9.84	11.82	13.89	13.74	11.21	16.30	15.89	18.67
$\mu = 1.15$	10.34	10.50	11.50	12.34	14.71	16.56	16.74	13.92	19.97	18.76	22.45

Figure 1: Potential Utility Gains from Lottery Payoffs This figure presents the potential utility gains from providing prospect theory investors with lottery payoffs. Tversky and Kahneman (1992) estimate the weighting function for nonnegative gambles as a two-part power function: $w^+(p) = \frac{p^\gamma}{(p^\gamma+(1-p)^\gamma)^{1/\gamma}}, \gamma = 0.61$. If an investor with cumulative prospect theory preferences receives a certain gain of one dollar, his utility is $v(1) = 1^{0.88} = 1$, where $v()$ indicates the Tversky and Kahneman (1992) value function. On the other hand, suppose the investor receives a free lottery ticket with an expected payoff of one dollar. This lottery ticket can be written as a Kahneman and Tversky (1979) gamble $(1/p, p; 0, 1-p)$ with expected utility $E_0(v|p) = w^+(p) * v(1/p)$. The dotted line shows the expected utility as a function of p. For values of p such that $E_0(v|p) > 1$, the investor prefers the lottery payout to a sure gain.

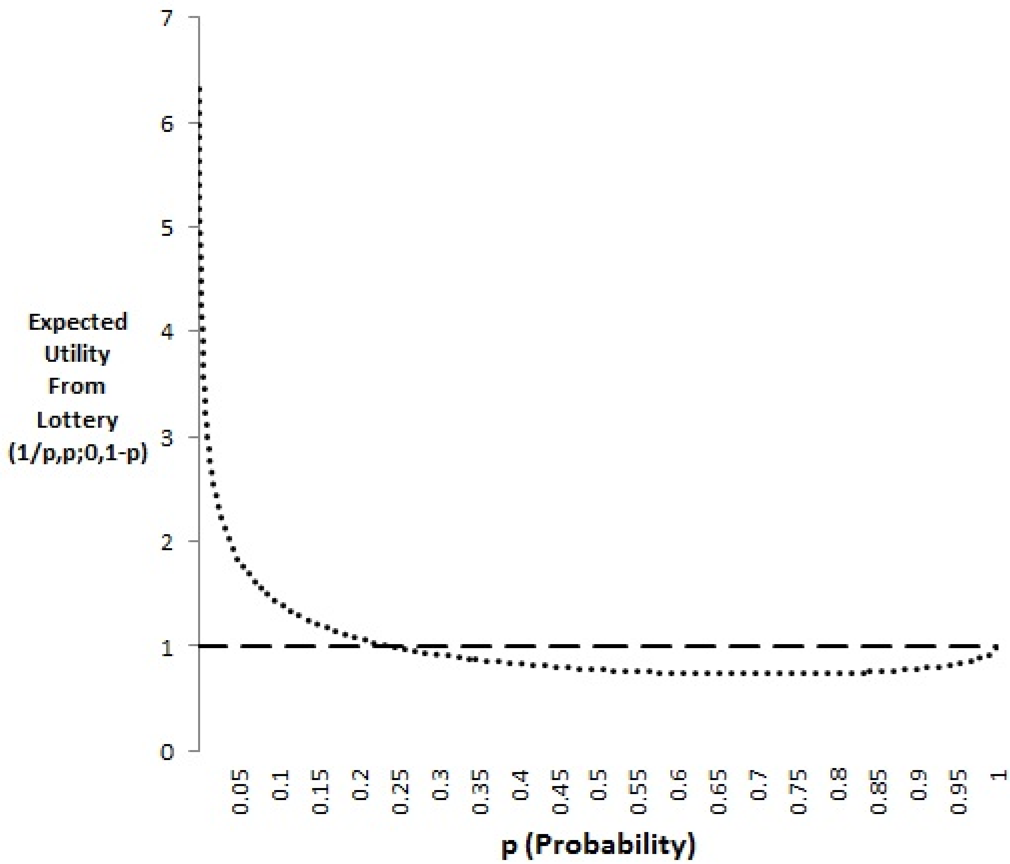


Figure 2: Conditions for Welfare Gains This figure shows conditions in which introducing stock loan fees or stock loan lotteries produces unconditional gains in investor welfare. In any model, an investor chooses allocations to maximize prospect theory utility. An investor has unconditionally greater welfare if his allocations in one model produce higher expected utility and higher expected wealth than his allocations in another model. For all points above the dotted curve, the investor has unconditionally greater welfare in the model with stock loan fees than in the baseline model of realization utility. For all points above the solid line, the investor has unconditionally greater welfare in the model with stock loan lotteries than in the baseline model of realization utility. In the model with stock loan lotteries, there is a single lottery at $t = 2$ with a probability $p = 0.1$ of winning the lottery. The area between the curves are points in (μ, f) space in which introducing stock loan lotteries leads to unconditional improvements in investor welfare and introducing stock loan fees does not.

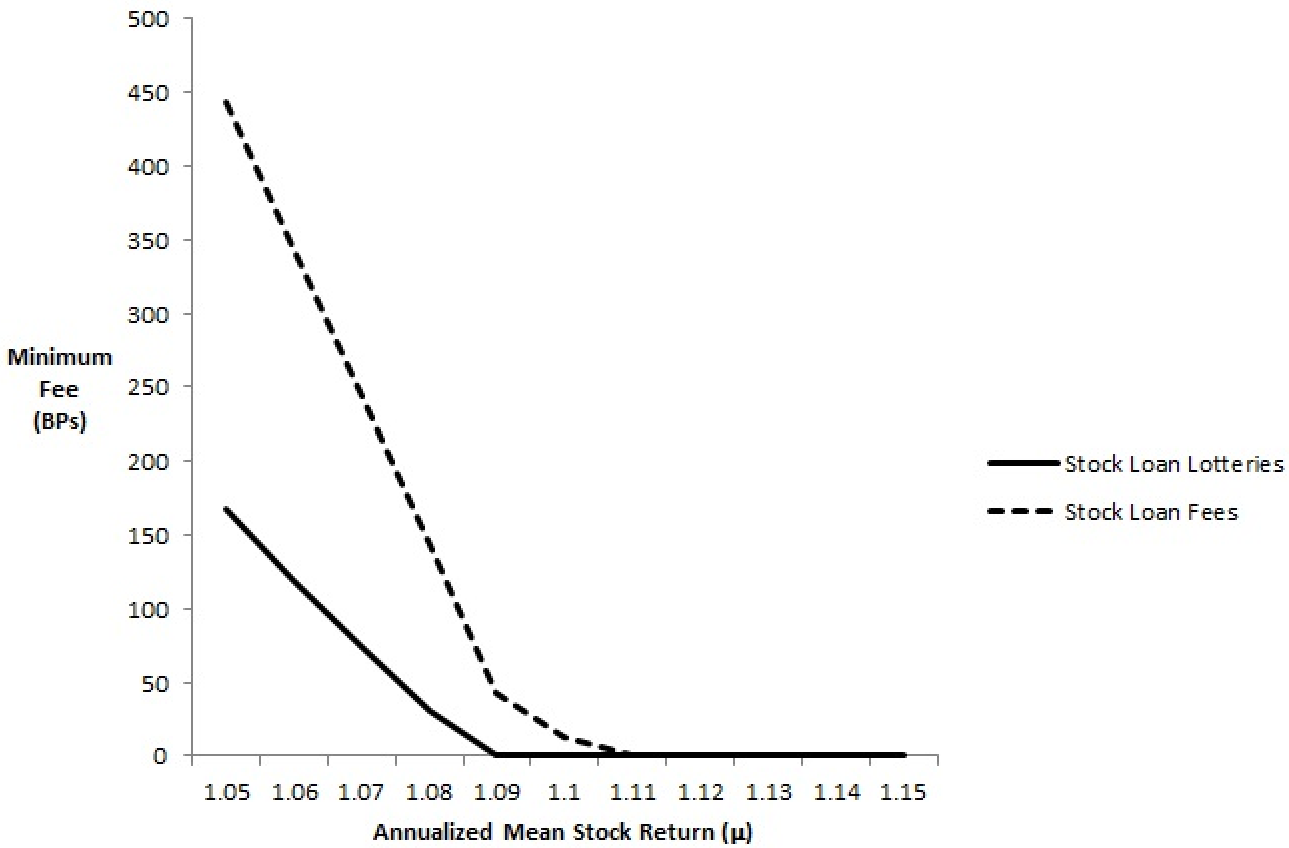


Figure 3: Regressive Features of Stock Loan Lotteries This figure shows how introducing stock loan lotteries is regressive in that it provides disproportionate benefits to poor investors. In a stock loan marketplace where there is a single winner of a single lottery, the probability of winning the lottery is identical to the investor's wealth share. This figure shows how a standardized measure of utility, $E[U(W_i)]/W_i$, varies by the investor's wealth share, in four different two-period models of realization utility. The solid lines show standardized utility for models with a single stock loan lottery at $t = 2$ and stock loan fees of five and 50 basis points. The dotted lines show standardized utility for models with no stock loan lotteries and stock loan fees of five and 50 basis points. For all models, the annualized expected gross return of the risky asset (μ), is 1.12.

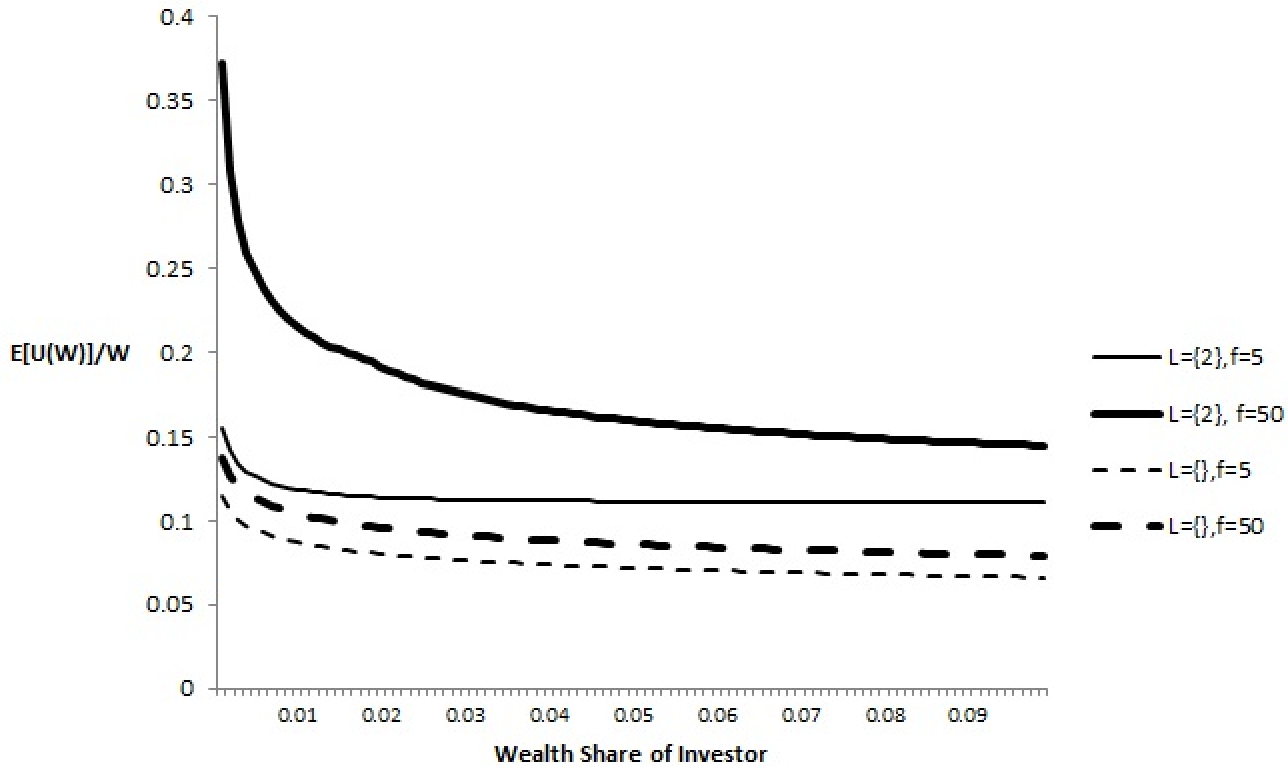


Figure 4: Stock Loan Lotteries with Transactions Costs and Leverage Constraints This figure shows how the conditions for unconditional improvement in welfare change after introducing transactions costs and leverage constraints. An investor has unconditionally greater welfare if the utility-maximizing allocation provides strictly higher expected utility and strictly higher expected wealth. There are three two-period models of realization utility: the baseline Barberis and Xiong (2009) model, a model with fixed stock loan fees, and a model with a single stock loan lottery at $t = 2$ with probability $p = 0.1$ of winning the lottery. For different values of μ, the annualized expected gross return of the risky asset, I solve all three models in a perfect market as well as a market with frictions. The market frictions include 1.3% round-trip transactions costs and a maximum leverage ratio of 2. For each value of μ, in each environment, I calculate f_F and f_L, the minimum fee required to provide investors in the fee and lottery models unconditionally greater welfare than in the baseline model. For each μ, the potential for welfare improvement by introducing stock loan lotteries is $max[f_F(\mu) - f_L(\mu), 0]$. The solid line shows the potential for welfare improvement with market frictions, while the dotted line shows the potential for welfare improvement with perfect markets.

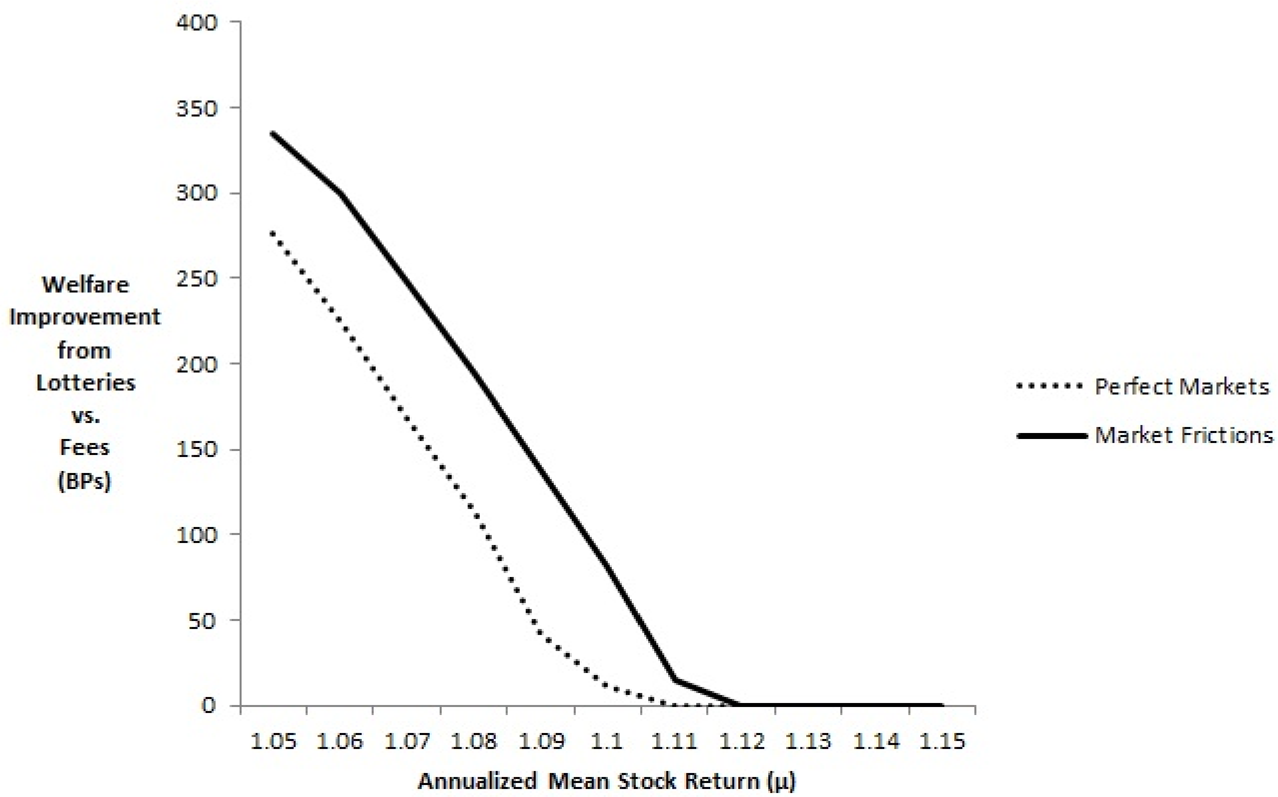
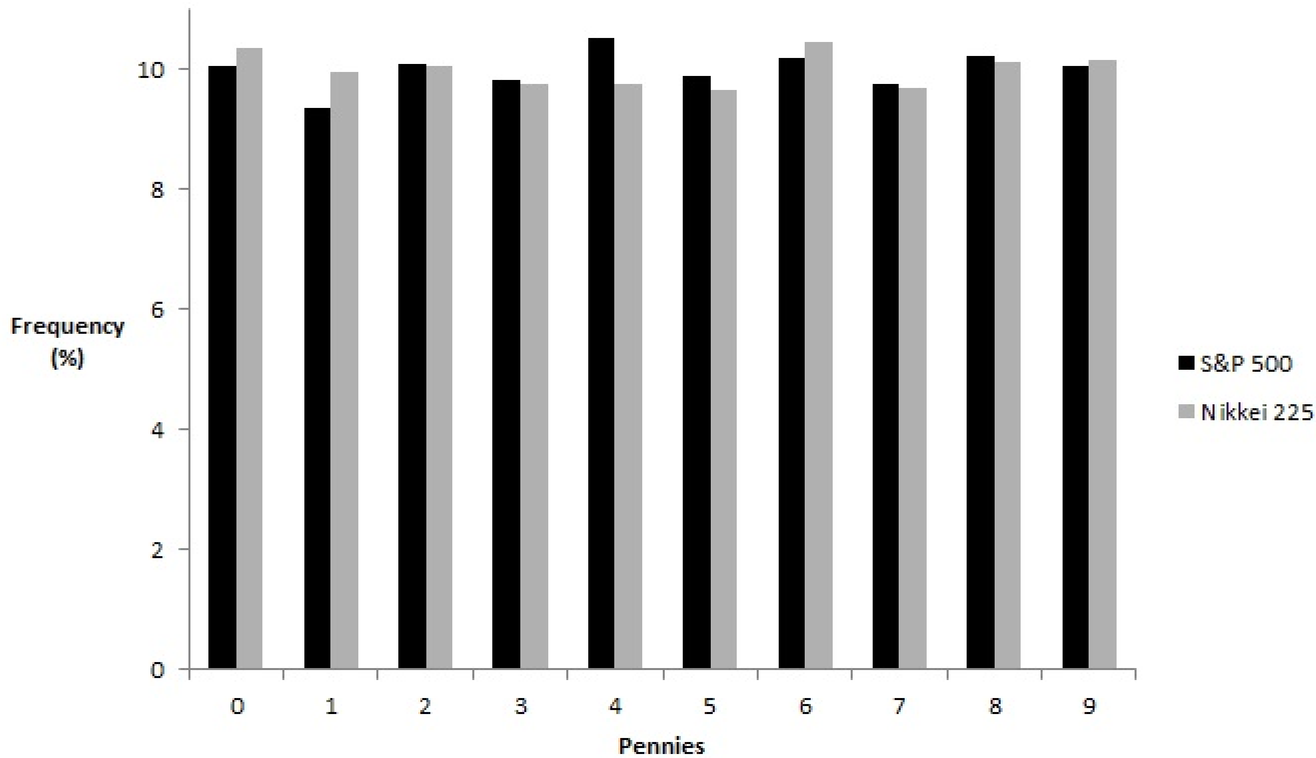


Figure 5: Stock Loan Lotteries as Derivative Securities This figure shows how lottery tickets could be structured as derivative securities. For a diversified equity index, a "Pennies Series X" derivative has a payoff of 1 if the pennies digit of the closing price of the index on a particular day is X. This histogram shows that there is no evidence of clustering in the pennies digit of major diversified equity indices. The black bars show the frequency of each of the pennies digits in the closing prices of the S&P 500 index between December 30, 1927 and April 21, 2017. The S&P 500 index is a capitalization-weighted index of 500 diversified large US public equities. The gray bars show the frequency of each of the pennies digits in the closing prices of the Nikkei 225 index between January 5, 1970 and April 21, 2017. The Nikkei 225 index is a price-weighted index of 225 diversified large Japanese public equities. Daily closing prices are from Bloomberg.



Appendix A: Mathematical Properties of the Value Function

The investor maximizes $E_0 v(x)$, where $v(x)$ has the functional form:

$$\begin{cases} v(x) = x^{\alpha} & x \geq 0, 0 < \alpha < 1 \\ v(x) = -\lambda(-x)^{\alpha} & x < 0, 0 < \alpha < 1, \lambda > 1 \end{cases}$$

Suppose the investor accepts some gamble (G) with potential gains $(g_1, g_2 \dots g_m)$ and potential losses $(l_1, l_2 \dots l_m)$. This implies:

$E_0 v(x; G) = \sum_{i=1}^{m} p(g_i) v(g_i) + \sum_{j=1}^{n} p(l_j) v(l_j) > 0$

Applying the functional form of the value function:

$E_0 v(x; G) = \sum_{i=1}^{m} p(g_i)(g_i)^{\alpha} - \lambda \sum_{j=1}^{n} p(l_j)(-l_j)^{\alpha} > 0$

Consider a proportionately larger gamble $(kG, k > 1)$. This gamble has expected value:

$E_0 v(x; kG) = \sum_{i=1}^{m} p(g_i) v(kg_i) + \sum_{j=1}^{n} p(l_j) v(kl_j)$

Applying the functional form of the value equation to the larger gamble:

$E_0 v(x; kG) = k^{\alpha} \sum_{i=1}^{m} p(g_i)(g_i)^{\alpha} - \lambda k^{\alpha} \sum_{j=1}^{n} p(l_j)(-l_j)^{\alpha}$

$E_0 v(x; kG) = k^{\alpha} [\sum_{i=1}^{m} p(g_i)(g_i)^{\alpha} - \lambda \sum_{j=1}^{n} p(l_j)(-l_j)^{\alpha}] = k^{\alpha} E_0 v(x; G)$

Since $k > 1$, $0 < \alpha < 1$, and $E_0 v(x; G) > 0$,

$E_0 v(x; Kg) > E_0 v(x; g) > 0$

So given that the investor accepts G, he always prefers kG. Therefore, an investor who chooses a positive risky asset allocation always chooses to exhaust the nonnegative wealth constraint.

QED