Financial volatility, currency diversification and banking stability

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Motivation

From the literature:

Evans and McMillan [2009], Rey [2013], Miranda-Agrippino and Rey [2015], Ivashina et al. [2015], Pedrono and Violon [2017]

▷ European banks: a transatlantic asymmetry in international banking (Baba et al. [2009], McGuire and Von Peter [2012])

▷ EA global banks exposed to the global financial cycle:
  ▷ Co-movements between assets \{C, C^*\}
  ▷ Major influence of US monetary policy on credit conditions worldwide \{L, L^*\}

▷ Regarding exchange rate, assets and liabilities: domestic currency appreciation with positive shock on domestic interest rate.
  Engel [1996], Kearns and Manners [2006], Ehrmann et al. [2011]

▷ \{C, C^*, L, L^*, S\} within EA banks’ balance sheet are linked all together.

Aim of this paper:

▷ Link the bank’s exposure to the global financial cycle to the banking volatility
Illustration

(a) International stock market indices

(b) Shadow short rates (SSR), daily returns

(c) Exchange rate

Figure: Financial markets. Sources: Bloomberg, The Reserve Bank of New Zealand.
This paper

Theoretical model:
- Stochastic processes to define assets, liabilities and foreign exchange rate marginal variation
- Equity returns:
  - A residual of total asset and liability marginal variations
- Volatility of equity:
  - Leverage and variance covariance matrix between \{C, C^*, L, L^*, S\}

Data and empirical application:
- Daily data on:
  - International stock market indices
  - US and EA Shadow Short Rate
  - Foreign exchange rate
- Bi-variate DCC GARCH:
  - Conditional variances and correlations
  - Estimation of efficient currency diversification

Key ingredients:
- Differentiating each source of risk within global bank’s volatility
- Identification of the global financial cycle: conditional correlations
Equity

Total assets:

\[ A = C + SC^* \quad \text{with} \quad \frac{C}{A} = (1 - \psi) ; \quad \frac{SC^*}{A} = \psi \]

Total liabilities:

\[ D = L + SL^* \quad \text{with} \quad \frac{L}{D} = (1 - \lambda) ; \quad \frac{SL^*}{D} = \lambda \]

Bank’s equity is defined through \( E \) such that:

\[ E = A - D \]

Bank’s leverage \( l \):

\[ l = D/E \]

Following the Basel III framework, we assume that leverage is defined by authorities. Using definitions of \( l \) and \( E \), we obtain the bank’s equity SDE:

\[
d\tilde{E} = \frac{dE}{E} = (1 + l) \frac{dA}{A} - l \cdot \frac{dD}{D} \\
= (1 + l) \left( (1 - \psi) d\tilde{C} + \psi (d\tilde{C}^* + d\tilde{S}) \right) - l \left( (1 - \lambda) d\tilde{L} + \lambda (d\tilde{L}^* + d\tilde{S}) \right)
\]
Volatility of equity with currency diversification

Introducing 10 covariances \( \{\sigma_{CC^*}, \sigma_{LL^*}, \sigma_{LC}, \sigma_{L^*C^*}, \sigma_{L^*C}, \sigma_{SC}, \sigma_{SC^*}, \sigma_{SL^*}, \sigma_{SL}\} \)

Volatility of equity return:

\[
\text{Var} \left( \frac{d\tilde{E}}{dt} \right) = \sum \text{Var of each component of the BS} : \sigma^2_C \sigma^2_{C^*} \sigma^2_L \sigma^2_{L^*} \sigma^2_S \\
+ \text{The exposure to the global financial cycle : } \sigma_{CC^*}, \sigma_{LL^*} \\\n- \text{The Asset-Debt hedging strategy : } \sigma_{LC} \sigma_{L^*C^*} \sigma_{L^*C} \sigma_{LC^*} \\\n+/- \text{ The FX channel on converted returns and costs : } \sigma_{SC}, \sigma_{SC^*}, \sigma_{SL^*}, \sigma_{SL}
\]

"Efficient" share of foreign asset \( \hat{\psi} \): min. of banking volatility (similarly for \( \hat{\lambda} \))

\[
\hat{\psi} = + \text{ share of } C \text{ in asset-side risk} \\
+ \text{ risk reduction related to part of liability side being also in foreign currency} \\
+ \text{ share of } C^* \text{ risk that can be hedged with } L^* \\
+ \text{ share of } C^* \text{ risk that can be hedged with } L
\]
An application to the US and EA financial markets

Data

- $C$: log returns of the Eurostoxx50 index
- $C^*$: log returns of the S&P500 index
- $L$: EA SSR changes for EA monetary tightening
- $L^*$: US SSR changes for US monetary tightening
- $S$: the USD/EUR FX

Identifying variances of $\{C, C^*, L, L^*, S\}$ and correlations between the different components:

- 10 bivariate DCC GARCH(1,1) using daily data from 2000 to 2015

Compared to cointegration analysis:

- Capture the potential change in financial integration as mentioned by Evans and McMillan [2009].
Main results from DCC GARCH

- Identification of financial distress:
  - the subprime crisis (2008-2009)
  - global volatility surge in 2008

- Assets are more volatile
  - $\{\sigma_C, \sigma_C^*\} > \{\sigma_S\} > \{\sigma_L, \sigma_L^*\}$

- US Vs EA volatility
  - $\sigma_C > \sigma_C^*$ except for 2008
  - $\sigma_L > \sigma_L^*$ for 2000, 2003, 2009 and since 2011

- Confirm the global financial cycle:
  - $\{\rho_{CC^*}, \rho_{LL^*}\}$, all positive with some dynamics
  - $\{\rho_{LC}, \rho_{L^*C^*}, \rho_{L^*C}, \rho_{LC^*}\}$, all positive with dynamics

- Correlations regarding FX:
  - $\{\rho_{SC}, \rho_{SC^*}, \rho_{SL^*}, \rho_{SL}\}$: positive to negative dynamic depending on sub-period
Efficient diversification

2008: peak in vol. but large $\rho_{LC}$ and $\rho_{L*C*}$, and FX compensation with $\rho_{SC} = -\rho_{SC*}$ and $\rho_{SL} = -\rho_{SL*}$:
- Currency diver. still stabilizing

2009-2012: large compensation effect with $\rho_{SC*} < 0$, plus $\rho_{LC*} > \rho_{L*C*}$:
- Currency mismatch is optimal

After 2012, $\rho_{LC} < \rho_{L*C}$ and $\rho_{SC*}$ increases and becomes positive:
- Currency mismatch is absorbed

Figure: Efficient currency diversification of bank’s balance sheet (2000-2015): $\psi$ and $\lambda$ are defined as to minimize the volatility of bank’s equity.
⇒ Link the bank’s exposure to the global financial cycle to the banking stability.

▷ An application to the US and EA financial markets
  ▷ Identification of the global financial cycle
  ▷ Diversification reduces equity volatility even during large financial distresses such as 2008.

⇒ The currency dimension of banks’ balance sheet then offers an interesting potential regulatory tool to improve the resilience of banks:
  ▷ Possible to hedge FX risk completely.
  ▷ Possible to understand the consequences of banks’ external positions: currency mismatch may improve banking stability.
  ▷ Possible to improve stress test exercises by including FX adjustments.

⇒ Extension:
  ▷ Compared efficient diversification and observed diversification
  ▷ Explain differences in currency diversification
  ▷ Explain conditional correlations.
Equity return volatility

\[
\text{Var}\left(\frac{d\tilde{E}}{dt}\right) = \Sigma_{\text{ortho}}
\]

\[+
2(1 + l)^2 \psi(1 - \psi)\sigma_{CC^*} + \lambda(1 - \lambda)\sigma_{LL^*}
\]

\[\text{global financial cycle risk} + \text{global financial cycle risk}
\]

\[-2(1 + l)l \left[(1 - \psi)\left((1 - \lambda)\sigma_{LC} + \lambda\sigma_{L^*C}\right) + \psi\left(\lambda\sigma_{L^*C} + (1 - \lambda)\sigma_{LC^*}\right)\right]
\]

\[\text{A–D hedging strategies}
\]

\[+2(\psi + l(\psi - \lambda))(1 + l)\left[(1 - \psi)\sigma_{SC} + \psi\sigma_{SC^*}\right]
\]

\[\text{FX channel, asset}
\]

\[-2(\psi + l(\psi - \lambda))l \left[(1 - \lambda)\sigma_{SL} + \lambda\sigma_{SL^*}\right]
\]

\[\text{FX channel, liability}
\]

where:

\[
\Sigma_{\text{ortho}} = ((1 + l)(1 - \psi))^2 \sigma_C^2 + ((1 + l)\psi)^2 \sigma_{C^*}^2 + (\psi + l(\psi - \lambda))^2 \sigma_S^2 + (l(1 - \lambda))^2 \sigma_L^2 + (l\dot{\lambda})^2 \sigma_{L^*}^2
\]
Efficient asset diversification

\[ \frac{\partial \Sigma_{global}^2}{\partial \psi} = 0 \mid \lambda \text{ constant} \]

\[ \hat{\psi}_{global} = \frac{\sigma_C^2 - \sigma_{CC^*} - \sigma_{SC}}{\sigma_C^2 + \sigma_{C^*}^2 + \sigma_S^2 - 2 (\sigma_{CC^*} + \sigma_{SC} - \sigma_{SC^*})} \]

\[ + \lambda \left( \frac{l}{1 + l} \right) \frac{\sigma_S^2 + \sigma_{SC^*} - \sigma_{SC}}{\sigma_C^2 + \sigma_{C^*}^2 + \sigma_S^2 - 2 (\sigma_{CC^*} + \sigma_{SC} - \sigma_{SC^*})} \]

\[ + \lambda \left( \frac{l}{1 + l} \right) \frac{\sigma_{SL^*} + \sigma_{L^*C^*} - \sigma_{L^*C}}{\sigma_C^2 + \sigma_{C^*}^2 + \sigma_S^2 - 2 (\sigma_{CC^*} + \sigma_{SC} - \sigma_{SC^*})} \]

\[ \text{share of } C^* \text{ risk that can be hedged with } L^* \]

\[ + (1 - \lambda) \left( \frac{l}{1 + l} \right) \frac{\sigma_{SL} + \sigma_{LC^*} - \sigma_{LC}}{\sigma_C^2 + \sigma_{C^*}^2 + \sigma_S^2 - 2 (\sigma_{CC^*} + \sigma_{SC} - \sigma_{SC^*})} \]

\[ \text{share of } C^* \text{ risk that can be hedged with } L \]

(2)
Efficient liability diversification

Similarly for the "Efficient" share of foreign liability $\lambda^*$:

$$
\frac{\partial \Sigma^2_{global}}{\partial \lambda} = 0 \mid \psi \text{ constant}
$$

$$
\hat{\lambda}_{global} = \frac{\sigma_L^2 - \sigma_{LL^*} - \sigma_{SL}}{\sigma_L^2 + \sigma_{L^*}^2 + \sigma_S^2 - 2(\sigma_{LL^*} + \sigma_{SL} - \sigma_{SL^*})}
$$

$$
+ \psi \left( \frac{1 + l}{l} \right) \frac{\sigma_S^2 + \sigma_{SL^*} - \sigma_{SL}}{\sigma_L^2 + \sigma_{L^*}^2 + \sigma_S^2 - 2(\sigma_{LL^*} + \sigma_{SL} - \sigma_{SL^*})}
$$

$$
+ \psi \left( \frac{1 + l}{l} \right) \frac{\sigma_{L^*C^*} + \sigma_{SC^*} - \sigma_{LC^*}}{\sigma_L^2 + \sigma_{L^*}^2 + \sigma_S^2 - 2(\sigma_{LL^*} + \sigma_{SL} - \sigma_{SL^*})}
$$

share of $L^*$ risk that can be hedged with $L^*$

$$
+ (1 - \psi) \left( \frac{1 + l}{l} \right) \frac{\sigma_{SC} + \sigma_{L^*C} - \sigma_{LC}}{\sigma_L^2 + \sigma_{L^*}^2 + \sigma_S^2 - 2(\sigma_{LL^*} + \sigma_{SL} - \sigma_{SL^*})}
$$

share of $L^*$ risk that can be hedged with $C$

(3)
The FX channel

On the asset side

- Following empirical literature (Ehrmann et al. [2011]) : \( \sigma_{SC^*} > 0 \) and \( \sigma_{SC} < 0 \)

Assuming that \( \sigma_{SC} = -\sigma_{SC^*} \) and :

- \( \psi = 0.5 \) : FX channel=0

- \( \psi > 0.5 \) : a positive shock on \( \{r, r^*\} \) goes with a foreign currency appreciation
  - Converted asset returns increase, thus :
    - A relatively low \( \lambda \) increases \( \Sigma^2 \) (i.e when \( \frac{\psi}{\lambda} > \frac{l}{1+l} \)) : no compensation
    - A relatively large \( \lambda \) decreases \( \Sigma^2 \) (i.e when \( \frac{\psi}{\lambda} < \frac{l}{1+l} \)) : compensation

- \( \psi < 0.5 \) : a positive shock on \( \{r, r^*\} \) goes with a foreign currency depreciation
  - Converted asset returns decrease, thus :
    - A relatively low \( \lambda \) decreases \( \Sigma^2 \) (i.e when \( \frac{\psi}{\lambda} > \frac{l}{1+l} \)) : compensation
    - A relatively large \( \lambda \) increases \( \Sigma^2 \) (i.e when \( \frac{\psi}{\lambda} < \frac{l}{1+l} \)) : no compensation

- When \( \psi = \lambda = 0 \), or when \( \frac{\psi}{\lambda} = \frac{l}{1+l} \), FX channel=0
DCC GARCH

Two steps: 1) estimate the conditional volatility of each one of the two series \(\{i, j\}\) from univariate GARCH(1,1); 2) capture from the first step the dynamic correlation between the two series.

Suppose \(r_t\) a 2x1 vector of returns of 2 assets at time \(t\), \(H_t\) a 2x2 matrix of conditional variances of \(r_t\) at time \(t\) and \(z_t\) a 2x1 vector of iid errors such that \(E[z_t] = 0\) and \(E[z_t z_t^T] = I\). Then, univariate GARCH is such that:

\[
    r_t = H_t^{1/2} z_t
\]

Decomposing the covariance matrix \(H_t\) into conditional standard deviation \(D_t\) from univariate GARCH, and a correlation matrix \(R_t\) capturing the dynamic correlation \(\{i, j\}\), the DCC GARCH introduces the following extension:

\[
    H_t = D_t R_t D_t
\]

Where the varying conditional correlation matrix \(R_t\) is defined as:

\[
    R_t = (I \odot Q_t)^{-1/2} Q_t (I \odot Q_t)^{-1/2}
\]

\[
    Q_t = (1 - a - b) \bar{Q} + a \epsilon_t - 1 \epsilon_t - 1^T + b Q_{t-1}
\]

Therefore, the dynamic matrix process \(Q_t\) is a function of \(\bar{Q}\), the unconditional correlation matrix of the standardized errors \(\epsilon_t\). Our results suggest that all correlations are mean-reverting process where \((a + b) < 1\). Additionally, all Wald tests reject the null hypothesis where \(a = b = 0\): conditional correlations are dynamic.
Conditional variance

Identification of financial distress:
- 2001-2002: the bursting of the dotcom bubble
- 2008-2009: the subprime crisis
- 2008: peak in volatility

Assets are more volatile:
- $\{\sigma_C, \sigma_C^*\} > \{\sigma_S\} > \{\sigma_L, \sigma_L^*\}$

US Vs EA volatility:
- $\sigma_C > \sigma_C^*$ except for 2008
- $\sigma_L > \sigma_L^*$ for 2000, 2003, 2009 and since 2011
Conditional variance

<table>
<thead>
<tr>
<th></th>
<th>$\sigma^2_{C^*}$</th>
<th>$\sigma^2_C$</th>
<th>$\sigma^2_S$</th>
<th>$\sigma^2_{L^*}$</th>
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Table: Conditional variances. $S$, $C$, $C^*$, $L$ and $L^*$ refer to the exchange rate, the eurostoxx 50 index, the S&P500 index, the euro Shadow Short Rate and the US Shadow Short Rate respectively.
Conditional correlations: assets and liabilities
Conditional correlations: foreign exchange rate

![Correlation graphs for US SSR, EA SSR, Eurostoxx50, and SP500 with FX rates from 2000 to 2015.](image-url)
### Conditional correlations:

<table>
<thead>
<tr>
<th>Year</th>
<th>$\rho_{LL}^*$</th>
<th>$\rho_{CC}^*$</th>
<th>$\rho_{LC}$</th>
<th>$\rho_{LC^*}$</th>
<th>$\rho_{L^*C}$</th>
<th>$\rho_{SL}$</th>
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