The decline of solvency contagion risk

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Abstract

We study solvency contagion risk in the UK banking system from 2008 to 2015. We develop a model that not only accounts for losses transmitted after banks default, but also for losses due to the fact that creditors revalue their exposures when probabilities of default of their counterparties change. We apply our model to a unique dataset of real UK interbank exposures. We show that risks due to solvency contagion decrease markedly from the peak of the crisis to the present, to the point of becoming negligible. By decomposing the change in losses into two main contributions — the increase in banks' capital and the decrease in interbank exposures — we are able to pinpoint the main driver in each year. In some cases we observe that an increase in aggregate capital is associated with a positive contribution to losses. This suggests that the distribution of capital among banks is also important.

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1 Introduction

Financial institutions form interconnections in order to transfer and transform risk. But during a crisis, such links become channels through which distress can spread, turning an idiosyncratic shock into a systemic one. During the 2008 – 2009 financial crisis, one of the most potent of these mechanisms was solvency contagion. Financial institutions had lent large amounts of money to each other, both directly through interbank markets, and indirectly, through derivative markets. As institutions suffered losses, their creditworthiness deteriorated. Their counterparties reassessed the value of those claims and booked losses as a result. In this way, solvency shocks to one firm led quickly to solvency shocks at other firms.

This type of contagion does not require a firm to default in order to spread losses. Indeed, even at the peak of the crisis, there were very few occurrences of cascades of outright defaults. Instead, the fall in the creditworthiness of banks' counterparties was enough to lead to losses as banks revalued exposures that were marked-to-market. The Basel Commitee on Banking Supervision estimates that "roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses[1] and only about one-third were due to actual defaults" (Basel Committee on Banking Supervision, 2011b).

Our main contribution is to show that solvency contagion risks have declined in the UK banking system over the period from 2008 to 2015. Scarcity of data on confidential interbank exposures means that empirical studies of financial contagion are relatively rare. In this paper, we benefit from access to a unique dataset of interbank exposures. This makes it, to the best of our knowledge, the only post-crisis empirical study of solvency contagion in the UK banking system.

We also assess the impact of two key structural changes in the UK banking system, rising bank capital and falling interbank exposures. During most of the period we study, an increase in aggregate capital or a fall in aggregate exposures is associated with a fall in the risk from solvency contagion. However, an increase in aggregate equity between 2009 and 2010, and broadly unchanged aggregate equity between 2011 and 2012, are both associated with a *rise* in the risk of solvency contagion. This happened because, in that period, capital was rising for some banks and falling for others. The distribution of capital had changed in a way which — on its own — increased the risk of solvency contagion.

Most of the literature on solvency contagion assumes that contagion propagates from one firm to another only after an outright bank default. This is consistent with assuming that there is no ex-ante uncertainty about the ability of a bank to repay its debt and therefore on whether or not it defaults. If a bank does not default it will not pass any distress to its counterparties because, by definition, it will have paid its obligations in full. Therefore, in the absence of uncertainty, distress propagates only through default events. However, in the presence of uncertainty in the value of a bank's assets, there will also be uncertainty as to whether a bank will default or not. The creditors of a bank will incorporate the corresponding credit risk into the valuation of their claims towards that bank. In turn, the reduced valuation of claims impacts the total value of creditors' assets, and hence their own probability of default. As a consequence, in the presence of uncertainty, distress can propagate even in the absence of defaults.

Performing the valuation of a contingent claim on a firm is a classic problem of

[1] Credit Valuation Adjustment (CVA) losses are precisely due to incorporating the risk that a counterparty can default into the mark-to-market value of assets.

corporate finance that has been solved by Merton in a simplified setting (Merton, 1974) and many generalisations of which have been extensively studied. Nevertheless, there is no simple recipe to perform such a valuation when firms hold cross-holdings of debt that tangle them in a complex network. Here, in order to properly quantify the risk due to the deterioration of the creditworthiness of counterparties, we adapt the framework presented in Barucca et al. (2016), which tackles precisely this problem. By doing so, in contrast to the majority of previous studies, we are able to account also for pre-default losses, which – as pointed out above – were significant during the crisis.

The paper is organised as follows: in section 2 we discuss the relationship between our approach and the relevant literature, in section 3 we introduce the modelling framework based on valuation functions, in section 4 we describe the data used and explain how to calibrate the model, in section 5 we presents our results, while in section 6 we draw some conclusions and outline some possible policy implications of our work.

2 Related literature

There are two main strands of literature related to our work. The first discusses theoretical models of solvency contagion, while the second deals with the empirical results obtained by calibrating the models using real interbank exposures.

Eisenberg and Noe provide the conceptual framework upon which most of the subsequent theoretical work is based. In their seminal paper (Eisenberg and Noe, 2001) they consider a network of banks with bilateral exposures and determine the conditions under which there exist and it is possible to compute the payments that clear the outstanding positions of all banks. Banks default when they are not able to repay their debt in full. In turn, creditors that fail to receive some payments from their counterparties might not be able to pay their own creditors, potentially triggering a cascade of defaults. Creditors do not necessarily lose the full face value of their claims towards defaulted banks. In other words, the realised recovery rate is, in general, larger than zero and is endogenously determined. The model has been extended by Rogers and Veraart (2013) to the case in which there are bankruptcy costs, by Suzuki (2002)) to the case in which banks have cross-holdings of equity, and by Fischer (2014) to the case in which liabilities have an arbitrary structure of seniorities.

The Eisenberg and Noe model is consistent with the absence of any ex-ante uncertainty on the value of interbank claims or on cash flows, which makes the model naturally applicable precisely at the maturity of claims. However, if the maturity has not been reached yet, banks cannot simply clear their positions but instead they need to estimate the value that their claims towards other banks might have when those claims mature. Performing the ex-ante valuation of claims when firms have cross-holdings of debt can be viewed as an extension of the problem tackled by structural credit models in the classic papers of Merton (1974) for firms that can default only when the maturity of their debt is reached and of Black and Cox (1976) for firms that can default at any point before the maturity.

Here we adopt the approach introduced in Barucca et al. (2016), in which solvency contagion is interpreted in terms of a revaluation of assets. The framework allows to recover the Eisenberg and Noe model and other models of financial contagion[2] as

[2]For specific choices of valuation functions Barucca et al. (2016) also allows to recover the models in Rogers and Veraart (2013), Furfine (2003), and Bardoscia et al. (2015), further generalised in Bardoscia et al. (2016).

special cases. The valuation is fully consistent with the Eisenberg and Noe model, in the sense that, in the limit in which the valuation is performed at the maturity, one recovers the payments computed using the Eisenberg and Noe model. The key aspects of the framework, which we use throughout this paper, are discussed in section 3. An alternative approach, outlined in Elsinger et al. (2006a,b), is to perform a Monte Carlo sampling of the possible realisations of the financial system at maturity, to compute the solutions of the Eisenberg and Noe model for each realisation, and finally to average over all the sampled realisations. Here we opt for the simpler and efficient procedure of iterative revaluation of equities introduced in Barucca et al. (2016), which does not require any Monte Carlo sampling.

Empirical work on financial contagion has been hampered by the general lack of data about bilateral exposures between financial institutions. A partial solution is to adopt reconstruction techniques to infer exposures when only limited aggregate information about them is available. Such techniques have been pioneered in Upper and Worms (2004) and further developed in Anand et al. (2015); Cimini et al. (2015); Gandy and Veraart (2016); Hałaj and Kok (2013).

Most of the empirical papers using real interbank exposures do not account for the ex-ante uncertainty on the value of interbank claims and are based on a simpler contagion mechanism than the one introduced by Eisenberg and Noe. Distress is propagated only through defaults, and the recovery rate is exogenous and in most cases it is set to zero. This means that creditors lose the full face value of their claims towards defaulted banks. The aforementioned mechanism has been applied to several national banking systems: US (Furfine, 2003), Germany (Upper and Worms, 2004), UK (Wells, 2004), Belgium (Degryse et al., 2007), Brazil (Cont et al., 2010), and Italy (Mistrulli, 2011)[3]. Elsinger et al. (2006a,b) apply the Eisenberg and Noe model respectively to the Austrian and to the UK banking system instead[4]. Both the previous empirical studies of the UK banking system use pre-crisis data. Wells (2004) finds that, while idiosyncratic shocks rarely cause cascades of defaults, they can be responsible for material losses to a significant fraction of banks. Elsinger et al. (2006b) show that idiosyncratic shocks can have a much lower impact than systematic shocks.

We refer the reader interested in a more comprehensive account of the literature to Glasserman and Young (2016), which provides an excellent general review, and to Upper (2011), which focuses on summarising empirical results. Here we only mention that, despite the abundance of works investigating the role that the topology of the underlying network of contracts has on the propagation of distress, there is little consenus, for example, as to whether more interconnected networks are more resilent or not (Allen and Gale, 2000; Cifuentes et al., 2005; Freixas et al., 2000). More recent studies find that interconnectedness might have a non-monotonic impact on resilience (Elliott et al., 2014; Gai and Kapadia, 2010; Nier et al., 2007) or point towards an even more nuanced behaviour (Acemoglu et al., 2015; Amini et al., 2013; Bardoscia et al., 2017). Glasserman and Young (2015) follow a different approach that allows to give bounds on losses due to solvency contagion by using only information about single institutions.

[3] Upper and Worms (2004) fully reconstruct bilateral exposures, while Degryse et al. (2007) and Wells (2004) use reconstruction techniques to supplement partial data about actual bilateral exposures.

[4] In both cases bilateral exposures are partially reconstructed.

3 Model

The general framework we adopt models a financial system of n banks as a network. The nodes are n stylised interconnected balance sheets, and the edges represent bilateral exposures between banks. This approach is common to the vast majority of papers modelling financial contagion (Glasserman and Young, 2016). The asset side of the balance sheet of bank i is divided into external assets A_i^e (e.g. loans to households and corporates) and interbank assets (e.g. claims on other banks, such as interbank loans). Similarly, the liability side is comprised of external liabilities L_i^e (e.g. household and corporate deposits), interbank liabilities (e.g. money borrowed on interbank markets), and equity E_i. Interbank exposures are represented by the matrix A, whose element A_{ij} represents the interbank asset of bank i towards bank j. The total interbank assets of bank i are simply $\sum_{j=1}^{n} A_{ij}$. Interbank liabilities are represented by the matrix L whose element L_{ij} represents the interbank liability of bank i towards bank j. The total interbank liabilities of bank i are $\sum_{j=1}^{n} L_{ij}$. Moreover, given that any interbank asset corresponds to an interbank liability (and vice versa), we have that $A_{ij} = L_{ji}$, for all i and j. The diagonal elements of both matrices are equal to zero, i.e. $A_{ii} = L_{ii} = 0$, for all i, simply meaning that a bank cannot have an exposure towards itself. Hence, we can write the balance sheet identity for bank i as follows:

$$A_i^e + \sum_{j=1}^{n} A_{ij} = L_i^e + \sum_{j=1}^{n} L_{ij} + E_i \,. \tag{1}$$

We assume that external assets follow a stochastic process whose value at time t is $A_i^e(t)$ and that all liabilities expire at maturity T. We take interbank liabilities always at their face value. The rationale behind this choice is the following: the fact that a bank might not meet its obligations in full does not imply that the value of its obligations change. In other words, we do not allow the debt between banks to be renegotiated before its expiration. For symmetry, we will also take external liabilities at face value, but we point out that we do so without any loss of generality[5]. By treating interbank assets like interbank liabilities and by taking them always at their face value, we can re-write (1) in the following way:

$$E_i(t) = A_i^e(t) + \sum_{j=1}^{n} A_{ij} - L_i^e - \sum_{j=1}^{n} L_{ij} \,, \tag{2}$$

where the dependence on time has been made explicit. In (2) we can interpret $A_i^e(t)$ as the value of the external assets of bank i, which have been marked-to-market at time t. As a consequence, $E_i(t)$ can be interpreted as the valuation at time t of the equity of bank i, which incorporates the information about the mark-to-market value of external assets.

In order to account for the valuation of interbank assets we adopt the framework introduced in Barucca et al. (2016), which we briefly describe below. Let us imagine that bank j is impacted by an *exogenous* shock to its external assets, i.e. $A_j^e(t) \to A_j^e(t) + \Delta A_j^e$, with $\Delta A_j^e < 0$. As a consequence, also the equity of bank j is reduced by the same

[5] In fact, one could easily consider the difference between external assets and external liabilities as a single stochastic process and put the external liabilities equal to zero. This is the approach taken e.g. in Eisenberg and Noe (2001), in which the cash flow can be interpreted as the difference between external assets and external liabilities.

amount[6], i.e. $E_j(t) \to E_j(t) + \Delta A_j^e$, and therefore its creditworthiness decreases. If bank i is exposed to bank j, it will incorporate the information about the reduced creditworthiness of bank j into the valuation of its interbank asset A_{ij}. The value of the interbank asset will lie between its face value A_{ij} (the case in which the interbank loan is repaid in full) and zero (the case in which the lender is not able to recover anything). Formally, Barucca et al. (2016) introduce the valuation functions $\mathbb{V}_{ij}$ that take value in the interval $[0, 1]$, such that the valuation of the interbank asset A_{ij} is equal to $A_{ij}\mathbb{V}_{ij}$[7]. Hence, valuation functions can be interpreted as discount factors that are equal to one if the interbank loan is repaid in full and that are equal to zero if the lender is not able to recover anything. Moreover, $\mathbb{V}_{ij}$ must be a non-decreasing function of E_j, consistently with the observation that a smaller equity of bank j implies a lower valuation of the interbank assets corresponding to the interbank liabilities of bank j. Eq. (2) becomes:

$$E_i(t) = A_i^e(t) + \sum_{j=1}^{n} A_{ij}\mathbb{V}_{ij}(E_j(t); \ldots) - L_i^e - \sum_{j=1}^{n} L_{ij} \,, \tag{3}$$

where we have explicitly isolated the dependency of valuation functions on equities from the potential dependency on any additional parameters (represented by the ellipses). By doing so, we can now interpret (3) as a set of fixed point equations (one for each bank) for the vector of equities $\mathbf{E}(t)$.

In Barucca et al. (2016) it is shown that the set of equations (3) admits a solution that is simultaneously optimal for all banks (the *greatest* solution), in the sense that all banks attain the equity whose value is maximal across all the solutions. Eq. (3) can be solved iteratively by choosing a starting point $\mathbf{E}^{(0)}(t)$ for the vector of equities and by repeatedly applying (3) to compute the subsequent iterations $\mathbf{E}^{(1)}(t), \mathbf{E}^{(2)}(t), \ldots$, until convergence is reached. In particular, the iterative solution converges to the greatest solution if the starting point $\mathbf{E}^{(0)}(t)$ is the vector of equities computed by taking all interbank assets at their face value, i.e. the equities computed from (2).

In section 5 we will apply the model by carrying out simplified stress test exercises. The basic idea is to assume again that an exogenous shock hits external assets:

$$\mathbf{A}^e(t) \to \mathbf{A}^e(t) + \Delta\mathbf{A}^e \,, \tag{4}$$

where $\Delta A_i^e < 0$, for all i. As discussed above, shocks in external assets will be absorbed by equity in the first instance generating the equity loss $\Delta\mathbf{E}^{\text{shock}} = \Delta\mathbf{A}^e$. In order to compute the additional losses due to the revaluation of interbank assets one simply has to compute the fixed point $\mathbf{E}^*(t)$ of (3) using the post-shock vector of equities as starting point. The total loss in equity will therefore include two contributions, the exogenous shock $\Delta\mathbf{E}^{\text{shock}}$ and the losses due to the contagion mechanism, i.e. $\Delta\mathbf{E}^{\text{cont}} = \mathbf{E}^{(0)}(t) - \mathbf{E}^*(t)$.

Interestingly, the losses due to contagion can be further decomposed by means of the following observation. $\mathbf{E}^{(1)}(t)$, i.e. the vector of equities computed after iterating (3) once, incorporates the valuations performed by each bank of its own direct counterparties. $\mathbf{E}^{(2)}(t)$, the vector of equities computed after iterating (3) twice, captures

[6]Even though in principle the shock in external assets could be larger than the equity, we always have $|\Delta A_j^e| < E_j(t)$. In fact, since our main purpose is to study contagion mechanisms, we are not interested in the case in which the exogenous shock alone is sufficient to make banks default.

[7]Barucca et al. (2016) also introduce valuation functions for external assets, which can be used e.g. to model the impact of bankruptcy costs on external assets, similarly to Rogers and Veraart (2013).

subsequent revaluations performed by each bank's counterparties of their own counterparties. As a consequence, by measuring $\mathbf{E}^{(1)}(t)$ one can isolate the effects of direct losses, i.e. the losses caused by direct counterparties, from the losses genuinely caused by additional rounds of contagion propagation, i.e. by the amplification due to the network topology.

In summary, by denoting the vector of equities before the exogenous shock hits with $\mathbf{E}^{\text{pre-shock}}(t)$, there are three different contributions to the total equity loss:

$$\Delta\mathbf{E}^{\text{shock}} = \mathbf{E}^{\text{pre-shock}}(t) - \mathbf{E}^{(0)}(t) \tag{5a}$$

$$\Delta\mathbf{E}^{\text{direct}} = \mathbf{E}^{(0)}(t) - \mathbf{E}^{(1)}(t) \tag{5b}$$

$$\Delta\mathbf{E}^{\text{amp}} = \mathbf{E}^{(1)}(t) - \mathbf{E}^{*}(t)\,, \tag{5c}$$

while the losses due to contagion are simply $\Delta\mathbf{E}^{\text{cont}} = \Delta\mathbf{E}^{\text{direct}} + \Delta\mathbf{E}^{\text{amp}}$.

3.1 Valuation functions

The key part of the model is the choice and calibration of valuation functions. Moreover, the valuation function will depend also on the kind of interbank exposures. For example, if bank i holds part of the equity of bank j, then a $x\%$ fall in the value of bank j equity leads to the same $x\%$ fall in the value of the corresponding interbank asset held by bank i. However, equity holdings in other banks are deducted from measures of regulatory capital resources, which play a key role in determining whether a bank defaults or not. Hence, whilst falls in the value of equity claims would affect the market value of banks' equity, they will not bring a bank closer to default and so would not transmit through to the value of debt claims.

Other interbank exposures, such as unsecured loans or holdings of tradable debt securities, are not deducted from a bank's capital resources and, as a consequence, could potentially act to transmit contagion. In addition, such exposures have different levels of seniority in default, and therefore their valuation will be dramatically different from the valuation of pure equity claims.

While the ex-ante valuation of the debt of a firm is a classic problem tackled by structural credit models, its extension to the case in which firms have cross-holdings of debt (as in the case of interbank exposures) is not straightforward. The starting point is to observe that *at* the maturity T (i.e. in absence of ex-ante uncertainty) each bank has either defaulted or not defaulted. If bank j has not defaulted, its equity will be positive and it will be able to repay its obligations in full. Therefore, the valuation functions of all the interbank assets corresponding to its interbank liabilities will be equal to one. Conversely, if bank j has defaulted, its equity will be equal to (or smaller than) zero and it will be able to repay only a fraction r_j of its obligations, which is the recovery rate realised by bank j's creditors. Therefore, assuming that all its interbank creditors have the same seniority, the valuation functions of all the interbank assets corresponding to its interbank liabilities will be equal to r_j. However, the precise value of r_j will depend on how much greater bank j's liabilities are than bank j's assets, i.e. on bank j's (negative) equity. As a consequence, we can write the valuation functions at the maturity in the following way:

$$\mathbb{V}_{ij}(E_j(T);\ldots) = \begin{cases} 1 & \text{for } E_j(T) > 0 \\ r_j(E_j(T);\ldots) & \text{for } E_j(T) \le 0 \end{cases}\,, \tag{6}$$

where the dependence of the recovery rate on the equity of creditors has been made explicit. More specifically, if now bank j holds interbank assets corresponding to the interbank liabilities of bank k, the fraction r_j of obligations that it will be able to repay to its creditors will also depend on the payment it gets from bank k, which in turn will depend on whether bank k is in default or not, i.e. on bank k equity, and so on. Hence, recovery rates are endogenously determined in the sense that, once the fixed point $\mathbf{E}^*(T)$ of (3) has been computed, the realised recovery rates are computed as $r_j(E_j^*(T);\dots)$, for all j.

In (7) whether bank j has defaulted or not depends only the value of its equity at maturity. However, banks' equity levels are constantly monitored by regulators, and if they fall below a certain buffer, the bank can be put into resolution (Bank of England, 2014b). Moreover, since banks are reliant on their ability to roll-over their liabilities, banks with little equity would run into problems in funding markets before they were truly insolvent. This was most clearly demonstrated during the crisis by the failure of Northern Rock, and explained in Shin (2009). As a consequence, defaults occur whenever equities become equal to zero, which can happen at any point in time, also before the maturity. Therefore, a more realistic definition of default of bank j should depend not only on the value of its equity at maturity, but on whether its equity has become negative at any time before T[8]. If bank j's equity has been positive *until* maturity, then bank j has not defaulted and $\mathbb{V}_{ij}$ will be equal to one. Conversely, if at any point in time $\bar{s}$ bank j's equity has reached zero, then bank j has defaulted and the valuation function will be equal to the recovery rate at that point in time, i.e. $r_j(E_j(\bar{s});\dots) = r_j(0;\dots)$. Eq. (7) becomes:

$$\mathbb{V}_{ij}(E_j(T);\dots) = \begin{cases} 1 & \text{for } E_j(s) > 0,\ \forall s < T \\ r_j(0;\dots) & \text{otherwise} \end{cases} . \tag{7}$$

In Barucca et al. (2016) the ex-ante valuation functions, i.e. the valuation functions for $t < T$, are derived in the context of Arbitrage Pricing Theory. From this point of view, equities at the maturity are random variables whose realisation depends on the realisation of the stochastic process followed by external assets. Under the assumptions that no arbitrage opportunities exist and that the market is complete, a risk-neutral investor would perform the valuation of equities at $t < T$ by computing their expectation with respect to the (unique) Equivalent Martingale Measure $\mathbb{Q}$, conditional on the information available at time t, i.e. conditional on the realisation at time t of the stochastic process followed by external assets[9]: $\mathbf{E}(t) = \mathbb{E}^{\mathbb{Q}}\left[\mathbf{E}(T)|\mathbf{A}^e(t)\right]$, or by using (3):

$$E_i(t) = A_i^e(t) + \sum_{j=1}^{n} A_{ij}\mathbb{E}^{\mathbb{Q}}\left[\mathbb{V}_{ij}(E_j(T);\dots)|\mathbf{A}^e(t)\right] - L_i^e - \sum_{j=1}^{n} L_{ij}\,. \tag{8}$$

In order to compute the expectation of the valuation function we re-write (7) by using indicator functions:

$$\mathbb{V}_{ij}(E_j(T);\dots) = \mathbb{1}_{\substack{E_j(s)>0 \\ \forall s\in[t,T]}} + r_j(0;\dots)\left(1 - \mathbb{1}_{\substack{E_j(s)>0 \\ \forall s\in[t,T]}}\right), \tag{9}$$

[8] This approach is in line with the Black and Cox model, which extends the Merton model precisely to the case in which defaults can happen also before maturity.

[9] In order to keep the notation light, we do not explicitly distinguish between random variables and their values. For example, $\mathbb{E}^{\mathbb{Q}}\left[A_j^e(T)|A_j^e(t)\right]$ is the expectation of the *random variable* $A_j^e(T)$ conditional on the *realisation* of the random variable $A_j^e(t)$.

where the first term is equal to one if bank j has defaulted, and it is equal to zero otherwise, while the second term is equal to zero if bank j has defaulted and it is equal to one otherwise. The previous equation implies:

$$\begin{aligned}\mathbb{E}^{\mathbb{Q}}\left[\mathbb{V}_{ij}(E_j(T);\ldots)|\mathbf{A}^e(t)\right] = \mathbb{E}^{\mathbb{Q}}&\left[\mathbb{1}_{\substack{E_j(s)>0\\ \forall s\in[t,T]}}|\mathbf{A}^e(t)\right] \\ &+\mathbb{E}^{\mathbb{Q}}\left[r_j(0;\ldots)\left(1-\mathbb{1}_{\substack{E_j(s)>0\\ \forall s\in[t,T]}}\right)|\mathbf{A}^e(t)\right].\end{aligned} \tag{10}$$

The first term of (10) is, by definition, the probability of survival, i.e. the probability that bank j's equity is positive from time t until maturity, conditional on the value of external assets at time t. In order to compute the second term, we need to assume a specific functional form for the recovery rate r_j. We make the following choice:

$$r_j(E_j(T)|\ldots) = \rho\left(1+\frac{E_j(T)}{\sum_k L_{jk}}\right)^+ , \tag{11}$$

with $\rho \in [0,1]$. This choice is motivated by the fact that, as shown in Barucca et al. (2016), for $\rho = 1$ and in the limit in which the valuation is performed at the maturity, the fixed points of (3) coincide with the equities implied by the Eisenberg and Noe model, meaning that the valuation reduces to a clearing procedure. When $\rho < 1$, ρ can be interpreted as a parameter encoding the presence of bankruptcy costs that diminish the realised recovery rate, similarly to Rogers and Veraart (2013). From (11) we can see that $r_j(0|\ldots) = \rho$. In the Appendix we provide some additional details about the computation of conditional probabilities of default under the assumption that external assets follow independent geometric Brownian motion. In particular, we show that the conditional probability of default of bank j is a function of bank j's equity at time t. Hence, by introducing the short-hand $p_j^D(E_j(t))$ for the conditional probability of default of bank j, we can re-write (10) as:

$$\mathbb{E}^{\mathbb{Q}}\left[\mathbb{V}_{ij}(E_j(T);\ldots)|\mathbf{A}^e(t)\right] = 1 - p_j^D(E_j(t)) + \rho\, p_j^D(E_j(t)) . \tag{12}$$

The last term of (12) is simply the loss given default of the interbank asset A_{ij}, i.e. the recovery rate times the probability of default of bank j. As a consequence, (12) simply means that the valuation of the interbank asset A_{ij} is the sum of two contributions, the face value of the asset weighted by the survival probability of bank j and the recovery value weighted by the probability of default. It is worth noting that, if $\rho_j = 1$, the valuation function $\mathbb{V}_{ij}$ is equal to one, meaning that bank j's counterparties will not re-evaluate their claims towards bank j because they expect to recover the full face value of their claims, even after bank j defaults. This corresponds to the situation in which all the proceeds from the liquidation of bank j's assets are enough to pay back all its liabilities, meaning that the liquidation process does not entail any additional bankruptcy costs. Therefore, ρ can be interpreted as an exogenous recovery rate that allows us to account precisely for the bankruptcy costs involved in the liquidation process.

4 Data and model calibration

Here we consider a subset of the UK banking system composed of the banks that have been consistently part of the concurrent stress test carried out by the Bank of

England[10]. Those banks provide approximately 80% of the regulated lending to the UK real economy (Bank of England, 2017). From (3) we can see that, in order to compute the fixed point for the vector of equity, one needs external assets and liabilities of all banks, the matrix of interbank assets, and possibly the additional parameters on which valuation functions depend.

We use two different sources for interbank assets. The first is regulatory data on large exposures that banks have to their counterparties. Banks were required to calculate and report the maximum loss that they would suffer if a counterparty (or group of connected counterparties) failed to meet their obligations. A "large exposure" was defined as any exposure that exceeded 10% of the bank's capital resources (total Tier 1 and Tier 2 capital after deductions). Banks were prohibited from having exposures greater than 25% of capital resources (Basel Committee on Banking Supervision, 2014), although exceptions were made for certain counterparties. Due to the aforementioned threshold, these data will tend to *underestimate* interconnectedness. Nevertheless, they represent the only data about mutual exposures between UK banks that go back until before the peak of the crisis. We can see that, while this entails that the exposures reported are likely to be the output of some valuation performed by the banks, their definition matches (3), given that the maximum loss is realised precisely when the valuation function is equal to zero. Large exposures were collected every quarter from 2008 Q2 to 2013 Q4; for consistency with data taken from balance sheets (see below) here we use the exposures of the last quarter of each year. The second data source is covers the period from 2014 to 2015. It includes those seven banks' holdings of each other's senior unsecured and subordinated debt securities. In contrast to the "large exposures" data there is no reporting threshold so we have a more complete picture of the network. On the other hand, it does not include other sources of exposure, such as counterparty credit risk on derivatives contracts.

As stated previously, the matrix of interbank liabilities is obtained by transposing the matrix of interbank assets. External liabilities are simply computed as the difference between total liabilities and interbank liabilities. Total liabilities are obtained from banks' published accounts in their annual report.

Analogously, external assets can be computed as the difference between total assets and interbank assets, also obtained from the annual reports. The starting level of equity computed through the accounting identity (2) would then correspond to the book value of shareholders equity, i.e. total assets minus total liabilities. Another option would be to interpret the equity of the model discussed in section 3 as the buffer of regulatory capital that banks hold over and above their capital requirements. From 2014, regulators introduced Common Equity Tier 1 (CET1) as the primary measure of regulatory capital, and banks may be put into resolution if their level of CET1 falls below the minimal amount[11] that regulators require them to hold (Bank of England, 2014b), i.e. if their *usable capital* becomes equal to zero. Interpreting the equity of our model as the usable capital would therefore be consistent with the fact that in our model banks default as soon their equity becomes equal to zero. In this scenario (2) would not hold any more if external assets were still computed as the difference between total assets and interbank assets. Hence, external assets must be adjusted and computed by inverting (2), after

[10] The banks are: Barclays, HSBC, Lloyds Banking Group, Nationwide Building Society, The Royal Bank of Scotland Group, Santander UK, and Standard Chartered.

[11] We use the international minimum standard capital requirement as set out in Basel 3, which corresponds to 4.5% of risk weighted assets Basel Committee on Banking Supervision (2011a).

the initial usable capital has been specified.

Here we switch between the two possible interpretations, depending on the specific analysis. During the period ranging from 2008 to 2015, detailed definitions of capital were changing and the level of requirements was not made public, so we have chosen to use shareholder equity, which gives us a simple and consistent measure over the whole time period. Starting from 2013 we produce an additional set of analysis based on the results of the Bank of England's concurrent stress tests that were performed in 2014, 2015, and 2016 (Bank of England, 2014a, 2015, 2016). These exercises were performed using the balance sheets of the final quarter of the previous year as their starting point. Each year the stress test consists in assuming a given macroeconomic stress scenario and in projecting the amount of usable capital for each bank under the assumed stress scenario. The starting point, year by year and bank by bank, is the actual value of usable capital, which we interpret as the pre-shock value of equity $\mathbf{E}^{\text{pre-shock}}(t)$. In the stress scenario, usable capital falls both because asset values fall (e.g. banks make credit losses) and this affects capital directly, and because risk-weighted asset rise, which increases a banks' capital requirements. In order to estimate the additional potential losses due to solvency contagion, we use the projected amount of remaining usable capital for each bank envisaged by the results of stress tests as the post-shock equity $\mathbf{E}^{(0)}(t)$ of our model[12]. More specifically, within the time horizon of the projection, we take the post-shock value of usable capital from the point when each bank is most vulnerable, i.e. when its ratio between CET1 and risk weighted assets is the smallest[13].

The additional parameters needed to compute valuation functions, as we can see from (12) and from (A.2), are the exogenous recovery rate, the maturity of claims, and the volatility of external assets. For the recovery rate, we use either $\rho = 0$, which leads to an upper bound on the losses, or $\rho = 0.6$, roughly the value implied by table 3a of Acharya et al. (2003) for senior unsecured exposures. We approximate the maturity of liabilities as one year (Webber and Willison, 2011). In order to estimate the volatility of external assets, we follow the standard approach used in the Merton model and its generalisations, which consists in assuming that equities follow the same stochastic process as the external assets (albeit with a different volatility). We refer readers interested in the details to the Appendix. The volatility of equities are estimated from the time series of log returns of banks' stocks; the time series span the last 30 days of the year under analysis[14].

In figure 1 we plot the calibrated valuation functions for all banks, for the years 2008 and 2015, and for $\rho = 0$. The right endpoint of all curves correspond to the starting values of equities, as computed from (2). In 2015 the initial value of the valuation functions is close to one for all banks. In this regime, the fixed point of (3) will be close to the initial equities computed from (2), meaning that the losses implied by marking-to-market the interbank assets will be very small. Most banks would need to suffer large exogenous shocks to their equity to switch into the regime in which valuation functions for interbank assets are materially smaller than one. For example, HSBC would need

[12] This method of using the stress test results published by the Bank of England will not be possible in the future, as the stress test results will include losses due to solvency contagion as well.

[13] The values of CET1 and risk weighted assets can be found in the Annex 1 of Bank of England (2014a, 2015, 2016) under the columns "Actual" (pre-shock) and "Minimum stressed ratio (after the impact of 'strategic' management actions)" (post-shock).

[14] The only exception is Nationwide, which is not publicly traded. Given the similar asset portfolio of the two banks, we use the Lloyds volatility rescaled by the ratio of the external leverages (i.e. the ratio between external assets and equities) of the two banks.

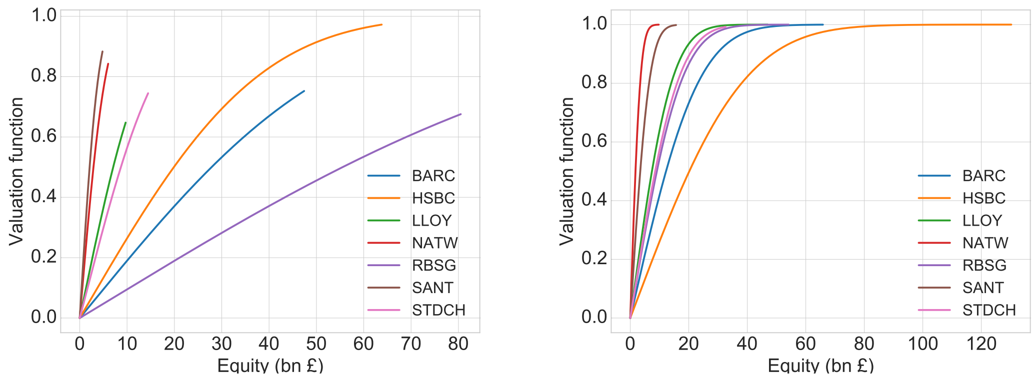

Figure 1: Valuation functions for interbank assets as function of equities in 2008 (left panel) and in 2015 (right panel), $\rho = 0$. The parameters of valuation functions have been calibrated by using data from banks' annual reports and by estimating volatilities of banks' equities from market prices of their stock.

to lose at least 40% of its initial equity. Even if HSBC's equity reduced to £70bn, the slope of its valuation function would be still small, meaning that to any additional loss of £1bn it would correspond a much smaller relative variation in the valuation its interbank assets. The picture painted by 2008 plot is drastically different. The starting value of the valuation functions of most banks is significantly smaller than one. For example, RBS initial equity already implies a devaluation of its interbank assets of 32% and, given that its valuation function is approximately linear, any additional equity loss of £1bn (about 1.2% of its initial equity) translates into a relative devaluation of its interbank assets of about 84 basis points. Lloyds is in a similar situation and any additional equity loss of £1bn (about 10% of its initial equity) corresponds to a relative devaluation of its interbank assets of about 670 basis points.

4.1 Data analysis

In section 5 we decompose how amplification changes over time into two contributions, the effect due changes in exposures and the effect due to changes in the level of equities. We do this by "transplanting" either exposures or equities from a certain year into the balance sheets of a different year, thereby building synthetic balance sheets. However, synthetic balance sheets might require some adjustments to make them consistent. First, let us assume that we are transplanting year $y+1$ exposures into year y balance sheets. In order to leave equities unchanged, we offset the difference in interbank assets and liabilities with a corresponding change in external assets and exposures:

$$\begin{aligned} A_i^e(t) &\to A_i^e(t) - \sum_j A_{ij}^{(y+1)} + \sum_j A_{ij}^{(y)} \\ L_i^e &\to L_i^e - \sum_j L_{ij}^{(y+1)} + \sum_j L_{ij}^{(y)} \,. \end{aligned} \tag{13}$$

Second, let us assume that we are transplanting $y+1$ initial equities into year y balance sheets. In this case we "absorb" the change in equities by offsetting external assets:

$$A_i^e(t) \to A_i^e(t) + E_i^{\text{pre-shock},(y+1)}(t) - E_i^{\text{pre-shock},(y)}(t) \,. \tag{14}$$

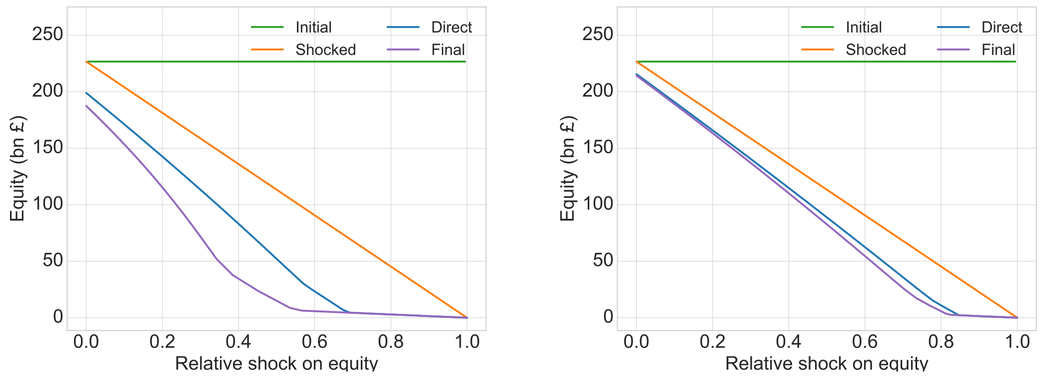

Figure 2: Break down of aggregate losses for a range of homogeneous shocks across banks. 2008 balance sheets and exposures, $\rho = 0$ (left panel) and $\rho = 0.6$ (right panel).

so that exposures are left unchanged.

5 Results

In this section we apply the model presented in section 3 by carrying out simplified stress test exercises. This use of the model can be thought of as a macro-prudential "overlay" on top of a micro-prudential stress test. The initial losses firms suffer should be interpreted as the results of the individual banks' stress test that do not include losses due to contagion. We then use our model to compute the additional losses due to contagion within the system.

We start by exploring the effect of a broad range of shocks. We shock external assets such that the corresponding relative loss of equity $\Delta E_i^{\text{shock}}/E_i^{\text{pre-shock}}(t)$ is the same across all banks. In figure 2 we break down the different contributions in (5) to total losses for 2008, for two different values of the exogenous recovery rate ρ. The green line is simply the aggregate initial equity $\sum_i E_i^{\text{pre-shock}}(t)$. For a given value of the shock, the distance between the green and the orange line corresponds to the aggregate exogenous shock $\sum_i \Delta E_i^{\text{shock}}$, the distance between the orange and the blue line corresponds to the aggregate losses caused by direct exposures $\sum_i \Delta E_i^{\text{direct}}$, while the distance between the blue and the purple line corresponds to the aggregate losses caused by additional rounds of contagion $\sum_i \Delta E_i^{\text{amp}}$. For $\rho = 0$ losses due to contagion can be as large as losses due to the exogenous shock. For $\rho = 0.6$, while smaller, they are still sizeable. From figure 2 we also see that there are losses due to contagion even without any exogenous shock to equities. This happens because, the valuation of interbank assets in the absence of a shock does not necessarily correspond to the face value of interbank assets, i.e. $\mathbb{V}_{ij}(E_j^{\text{pre-shock}}(t))$ might be strictly smaller than one, as for example in the left panel of figure 1.

Figure 3 shows that aggregate losses due to contagion (corresponding to the difference between the orange line and the purple line in figure 2) decreased significantly from 2008 to 2015 (left panel). During this period two key structural changes took place in the UK banking system: aggregate capital increased significantly; and interbank exposures declined sharply (right panel). It is worth reminding the reader that from 2008 to 2013 our dataset contains only exposures that exceed 10% of bank's capital and that, as

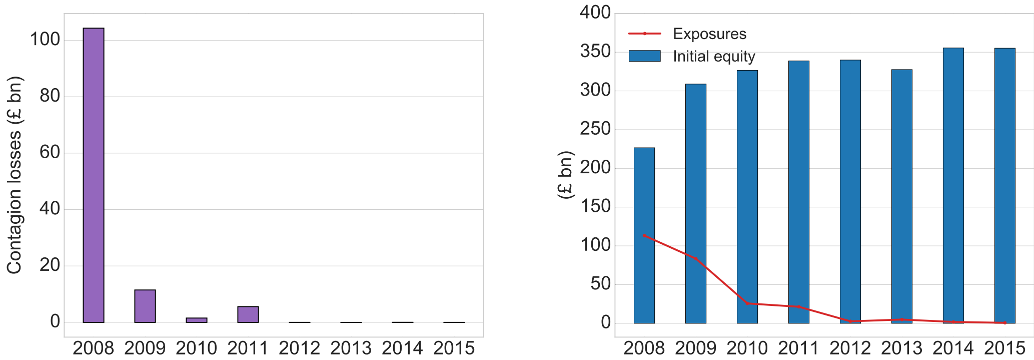

Figure 3: Left panel: Aggregate losses due to contagion from 2008 to 2015, $\rho = 0$, 40% relative shock on equity. Right panel: Capital and interconnectedness from 2008 to 2015.

a consequence, across this period exposures – and hence the effect of contagion – might be underestimated. In particular, the matrix of exposures becomes especially sparse in 2012 and 2013. However, even where we use a more granular dataset, in 2014 and 2015, the aggregate amount of exposures continues to fall, suggesting that the overall trend in contagious losses is not an artefact of the aforementioned threshold.

We proceed to decompose aggregate losses into these key drivers in the following way, focusing initially on the fall between 2008 and 2009. First, we create a first intermediate setting by "transplanting" 2009 interbank exposures into 2008 balance sheets. We then compute the difference between aggregate losses due to contagion in the first intermediate setting and in the "baseline", i.e. the setting in which 2008 exposures and balance sheets are used. This difference captures the effect in the variation of exposures from 2008 to 2009. Second, we build a second intermediate setting on top of the first intermediate setting, by "transplanting" 2009 initial equities. Similarly, we compute the difference between aggregate losses due to contagion in the second intermediate setting and in the first intermediate setting. This difference captures the effect of the variation of initial equities from 2008 to 2009 in addition to the effect of the variation of exposures. In both cases balance sheets are adjusted as explained in section 4 to keep them consistent. We note that there is still a residual difference between the second intermediate setting and using 2009 exposures and balance sheets due to, for example, the difference in external liabilities. One could proceed in the opposite order by building a first intermediate setting in which initial equities are "transplanted" before exposures. If the results obtained in the two cases match, then the effects due to initial equity and exposures can be robustly separated. If not, the interplay between the two effects cannot be determined with confidence.

In figure 4 we decompose this reduction in contagion into these two key drivers. We see that the large decrease in contagion from 2008 to 2009 is driven by both falling exposures and increasing capital — although it is difficult to disentangle the two effects. From 2010 to 2011 the residual contribution outweighs both effects, which suggests that other elements of the system played a more important role.

The decline in contagion from 2008 to 2009 and from 2011 to 2012 is unambiguously due to the fall in interbank exposures. However, if we perform the decomposition by "transplanting" equity first, we see that changes in equity contribute to an *increase* in

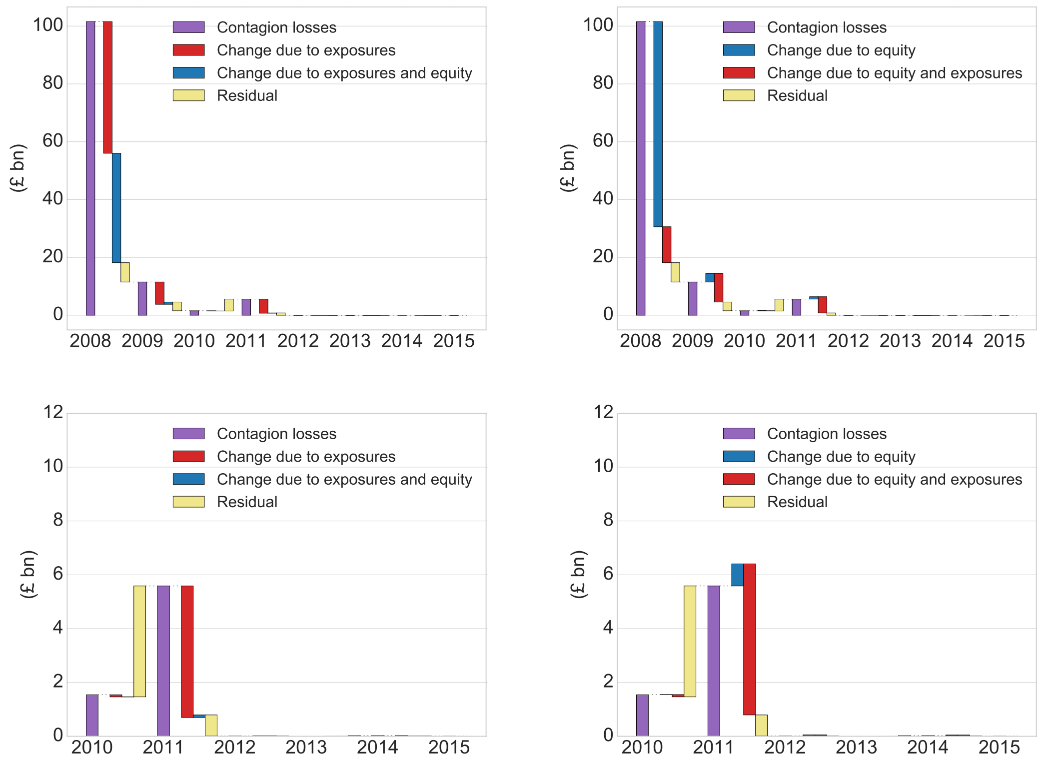

Figure 4: Breakdown of the effects of exposures and initial equity on the aggregate losses due to contagion, $\rho = 0$, 40% relative shock on equity. Left panel: "transplanting" exposures first, then equity. Right panel: "transplanting" equity first, then exposures. Bottom panels zoom into the years 2010 – 2013.

contagion (bottom right panel of figure 4), despite broadly unchanged levels of aggregate equity (figure 3). This happens because, over that period, our measure of equity falls for some banks, and increases for others. Whilst aggregate equity increases between 2009 and 2010 and it is broadly unchanged between 2011 and 2012, the distribution of equity has changed in a way which — on its own — would increase contagion. Finally, aggregate losses due to contagion are negligible from 2012 to 2015. The residual component in figure 4 highlights that there are other dimensions of the system which have important effects on the degree of contagion. In fact, we restrict our analysis to consider just the most important two during the period in question: capital ('bank resilience') and exposures ('interconnectedness').

In figure 5 we study the joint impact of these dimensions on losses due to contagion in periods of high and low contagion (2008 and 2015, respectively). In order to assess the impact of exposures we adjust them in order to produce a set of comparative statics. We reduce all exposures by a factor k (both on the asset side and on the liability side) and we correspondingly increase external assets and liabilities of the same amount, so that the adjustments to exposures does not change the initial equity:

$$\begin{aligned} A_{ij} &\to kA_{ij} \\ A_i^e(t) &\to A_i^e(t) + (1-k)\sum_j A_{ij}\,. \end{aligned} \tag{15}$$

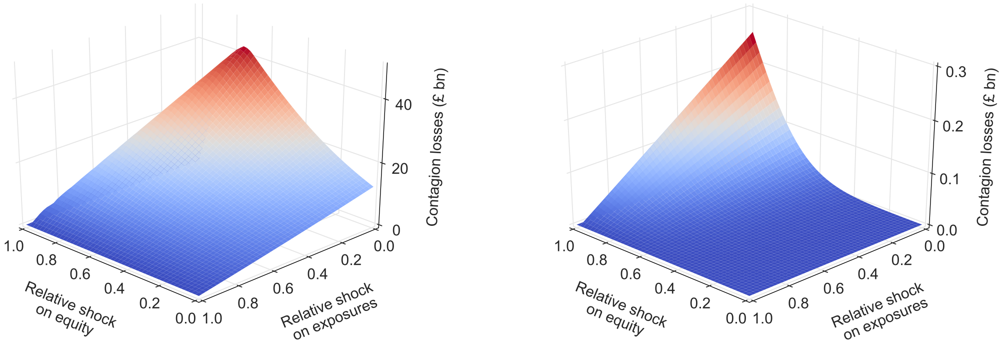

Figure 5: Aggregate losses due to contagion for a broad range of shocks to equity and exposures, homogeneous across banks. Data from 2008 (left panel) and 2015 (right panel) balance sheets and exposures, $\rho = 0.6$.

This analysis highlights the importance of considering the *joint* impact of these two dimensions. We see that the profile of the surface is not symmetrical: meaning that, while both dimensions are important, the marginal impact of changing capital and exposures can be markedly different.

In order to gauge the relative importance of the additional losses caused by contagion in a more realistic scenario, we use the shocks implied by the results of the Bank of England's concurrent stress tests for the years 2014, 2015, and 2016[15]. Both the usable capital and the shocks implied by the stress tests for all banks are listed in table 1. We use $\rho = 0.0$, which allows us to give upper bounds for the case in which interbank exposures are unsecured. For all the years the additional losses due to contagion are very small. In 2013, due to the sparsity of the matrix of exposures there are no additional losses due to contagion. In 2014 and in 2015 the aggregate additional losses due to contagion amount to £0.20 billion and to £0.02 billion respectively.

6 Conclusions and policy implications

We show that solvency contagion risk in the UK banking system has declined markedly since the financial crisis. We demonstrate this effect both for homogeneous shocks across all banks and for the shocks implied Bank of England's concurrent stress tests for the years 2014, 2015, and 2016. We account not only for losses due to outright defaults of banks, but also for losses due to the revaluation of interbank exposures performed by creditors when the probability of default of their counterparties change. We also decompose losses due to solvency contagion into two main contributions, the increase in banks' capital and the decrease in interbank exposures. Finally, we show that, rather than the aggregate amount of capital, it is its distribution that plays a crucial role in determining the extent of solvency contagion. That result is a reminder of the importance of distributions — of both capital and exposures — in a complex financial system, because such effects can often by masked when focusing on aggregate

[15] We now interpret the equity of our model as the usable capital, i.e. the capital that exceeds the minimal capital requirements. See section 4 for a detailed explanation of how the starting values of equities are computed.

	2014			2015			2016		
	Usable capital (£m)	Shock (£m)	Shock (%)	Usable capital (£m)	Shock (£m)	Shock (%)	Usable capital (£m)	Shock (£m)	Shock (%)
BARC	20 110	−1 605	7.9	22 910	−10 890	47.5	24 890	−9 890	39.7
HSBC	76 325	−15 900	20.8	78 100	−34 885	44.7	81 365	−21 405	26.3
LLOY	14 760	−11 510	77.9	20 200	−8 090	40.0	18 965	−5 675	29.9
NATW	3 973	−2 210	55.6	5 635	375	6.7	6 425	−355	5.5
RBSG	17 695	−14 650	82.8	23 980	−18 750	78.1	27 065	−21 540	79.6
SANT	5 503	−2 440	44.3	6 110	−1 195	19.6	6 130	−1 225	20.0
STDCH	20 105	−5 510	27.4	20 610	−17 115	83.0	24 365	−14 060	57.7

Table 1: Usable capital and shocks to the usable capital implied by the Bank of England's concurrent stress tests. Both usable capital and shocks have been computed as explained in section 4 using data from the Annex 1 of Bank of England (2014a, 2015, 2016). Years refer to the year in which the stress tests have been carried out. Stress tests are performed by using balance sheets of the final quarter of the previous year.

statistics.

We have also demonstrated how the model can be used as an "overlay" on top of traditional micro-prudential stress tests in order to capture contagion throughout the system. This usage allows macro-prudential regulators to perform stress tests in a way which recognises the system is more than the sum of its parts. Whilst our results show that the risk arising from solvency contagion in the UK does not appear to be substantial at the moment, in reality this risk will interact with other channels of contagion. Such interactions could mutually reinforce effects that would have been small on their own. From a policy perspective, our model should be therefore used within a suite of models that target a wide variety of possible channels of contagion.

Future research could extend the analysis in two key directions. By its nature, financial system interconnectedness is a global phenomenon, and has the potential to transmit risks across borders. However, we are currently able to conduct analysis only on the subset of the UK interbank network for which we have data. Access to global data through data-sharing initiatives with foreign regulators would facilitate more accurate assessment of the extent of solvency contagion risk facing the UK financial system. For the cases in which exposures would not be available, network reconstruction techniques could be employed to fill the gaps in data. Furthermore, we have modelled solvency contagion for only a subset of asset classes through which banks are interconnected. The modelling framework could be extended to assess solvency contagion through a larger set of asset classes, each characterised by a different valuation function and a different recovery rate.

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Source code. The simulations have been performed by using an implementation of the NEVA framework, which we make available here: https://github.com/marcobardoscia/neva.

Appendix

Probability of default

The probability of default conditional on the observation of external assets at time t that bank j defaults *before* the maturity T is:

$$p_j^D(E_j(t)) = \mathbb{E}^{\mathbb{Q}}\left[1 - \mathbb{1}_{\substack{E_j(s)>0 \\ \forall s\in[t,T]}} | \mathbf{A}^e(t)\right] . \tag{A.1}$$

Analogously to Barucca et al. (2016), valuation functions are computed in closed form using the approximation: $E_j(s) \simeq E_j(t) + A_j^e(s) - A_j^e(t)$, for all j and $s \in [t,T]$, implying that the default conditions read: $A_j^e(s) \leq A_j^e(t) - E_j(t)$. By assuming that external assets follow independent geometric Brownian motion, i.e. $dA_j^e(t)/A_j^e(t) = \sigma_j^A dW_j(t)$, we can easily adapt the result in Black and Cox (1976):

$$p_j^D(E_j(t)) = \begin{cases} 1 & \text{for } E_j(t) \leq 0 \\ \Phi\left[\frac{\log\left(1-\frac{E_j(t)}{A_j^e(t)}\right) + \frac{(\sigma_j^A)^2(T-t)}{2}}{\sigma_j^A\sqrt{(T-t)}}\right] & \\ \quad + \frac{A_j^e(t)}{A_j^e(t)+E_j(t)}\Phi\left[\frac{\log\left(1-\frac{E_j(t)}{A_j^e(t)}\right) - \frac{(\sigma_j^A)^2(T-t)}{2}}{\sigma_j^A\sqrt{(T-t)}}\right] & \text{for } 0 < E_j(t) < A_j^e(t) \\ 0 & \text{for } E_j(t) \geq A_j^e(t) \end{cases} , \tag{A.2}$$

where Φ is the cumulative distribution of a Gaussian random variable with mean equal to zero and variance equal to one.

Volatility of external assets

In order to estimate the volatility of external assets we proceed as customary by assuming that also equities follow a geometric Brownian motion, i.e. $dE_j(t)/E_j(t) = \sigma_j^E dW'_j(t)$. Via Ito's lemma one finds:

$$\sigma_j^A A_j^e(t)\frac{\partial E_j(t)}{\partial A_j^e(t)} = \sigma_j^E E_j(t) \qquad \forall j .$$

From (3) we see that $E_j(t)$ depends explicitly on $A_j^e(t)$ only through the first term, which leads to:

$$\sigma_j^A A_j^e(t) = \sigma_j^E E_j(t) \qquad \forall j . \tag{A.3}$$

Normally one would measure the equity (e.g. via the market capitalisation of a firm) and solve (A.3) jointly with (3) in order to find the vectors $\mathbf{A}^e(t)$ and $\boldsymbol{\sigma}^A$. However, we derive (3) from an accounting identity and for the external assets $\mathbf{A}^e(t)$ we use the value implied by the banks' balance sheets. Therefore, (A.3) can be solved independently, leading to $\sigma_j^A = \sigma_j^E E_j(t)/A_j^e(t)$, for all j.

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