

# Financial Stability, Growth and Macroprudential Policy\*

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## Abstract

Emerging market economies have an incentive to manage boom-bust cycles in capital flows since they might lead to financial crises and persistent output losses when capital flows reverse during bad times. However, ex-ante policy intervention can negatively affect economic growth in normal times even if it reduces the frequency and magnitude of crises. To formally investigate the trade-off between financial stability and growth, this paper introduces endogenous growth into a small open economy model with occasionally binding constraint. Calibrated to countries which have experienced sudden stop episodes, we find that macroprudential policy is desirable but faces a trade-off between growth and stability. With an additional instrument, a social planner can ignore this trade-off and generate larger benefits. This occurs because the economy experiences a temporary spurt in growth and permanent increase in consumption.

**Keywords:** Macroprudential Policy, Sudden Stop, Endogenous Growth

**JEL Classification:** E44, F38, F43, G38, O30

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# 1 Introduction

Boom-bust cycles in international capital flows sometimes are accompanied by severe financial crises and persistent output losses (see Reinhart and Reinhart, 2009). The prolonged and halting nature of recovery from the experience of emerging markets during sudden stop episodes in the late 1990s and advanced economies in the 2008-2009 Great Recession has revived a debate over the benefits and drawbacks of financial globalization.<sup>1</sup> This debate is particularly important for a small open economy (SOE) when it makes decision on whether to integrate itself into the global financial market. On the one hand, the traditional view asserts that international capital markets enhance growth and productivity by allowing capital to flow to its most attractive destination (see Fischer, 1997; Summers, 2000). On the other hand, economies that are integrated into global capital markets are exposed to boom and bust cycles in capital flows, which often ends up with systemic financial crises in downturns (see Calvo et al., 2006; Mendoza, 2010). Moreover, the cycle might be excessive and inefficient from a social perspective due to the intrinsic distortions in the financial markets (see Lorenzoni, 2008; Bianchi, 2011; Jeanne and Korinek, 2010b). As a result, there might be a case to regulate capital flows even if the overall effect from global financialization is positive.<sup>2</sup> Indeed, many countries that have liberalized their capital markets have adopted prudential policies in tranquil periods in order to reduce financial instabilities. For example, Brazil imposed taxes on its foreign flow in 2009 and changed the policy actively during the financial crises. Moreover, empirical evidence suggests that drops in output during crisis episodes are significantly lower if capital controls existed before the crisis (Ostry et al., 2010). From this perspective, prudential policy intervention could reduce the financial fragility and output loss in crisis. Furthermore, this policy intervention has gained popularity after the “Great Re-

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<sup>1</sup>See Cerra and Saxena (2008); Rogoff and Reinhart (2009) and Ball (2014), which document the persistent output level effect of financial crisis. Meanwhile, Rancière et al. (2008) and Rancière and Tornell (2016) argue that financial liberalization leads to an increase in both the incidence of financial crisis and average economic growth rate.

<sup>2</sup>Rancière et al. (2006) decompose the effect of financial liberalization on economic growth and the incidence of crises and find that financial liberalization has led to a faster long-run growth even if it is accompanied by financial fragility and occasional financial crises. For an overview of prudential capital controls, see Jeanne (2012), Korinek (2011) and IMF (2012).

cession” and is usually termed as “macroprudential policy”. However, such intervention might potentially reduce economic growth since many productive projects are financed by external borrowing. Therefore, a trade-off between financial stability and economic growth exists for the ex-ante capital flows management, and it is quantitatively unclear how much impact macroprudential policy has on economic growth.

Motivated by these questions, this paper introduces endogenous growth into a SOE model with a financial friction of a type that has been used in the literature to make the case for macroprudential regulation of capital flows. The way of modeling endogenous growth in my framework is generic and encompasses many models in the literature. Specifically, we assume that private agents have access to a technology that increases total factor productivity (TFP) in the future. The input in the technology is termed as “growth-enhancing expenditure” and intends to capture all the expenditures that are conducive to long-term growth, such as physical and human capital expenditures, research and development expenditures, etc.<sup>3</sup> In each period, the economy could finance its consumption and growth-enhancing expenditure through external borrowing by posting collaterals, whose value depends on the price of productive assets owned by the private agent (see Mendoza, 2010; Jeanne and Korinek, 2010b; Bianchi and Mendoza, forthcoming). However, the presence of asset price in the collateral and incomplete market imply that there are “pecuniary externalities” in the economy, and competitive equilibrium is thus suboptimal (see Greenwald and Stiglitz, 1986; Geanakoplos and Polemarchakis, 1985; Korinek, 2011; Dávila and Korinek, 2017). Specifically, private agents reduce all the expenditures when the economy experiences a big negative shock, which is accompanied by a decline in asset prices. Given that the collateral value depends on asset prices, private agents have to cut expenditures further due to a decline in collateral value and external borrowing limit. However, this vicious cycle through debt and asset prices is not internalized by atomistic private agents who take the asset price as given, providing a rationale for policy interventions.

A constrained social planner facing the same collateral constraint, can improve social

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<sup>3</sup>Here the physical capital expenditure refers to the part that enhances TFP rather than the part that is an input in the production function.

welfare by restricting external borrowing ex-ante and thus reducing the frequency and magnitude of financial crises. Here we consider two types of constrained social planners depending on the availability of policy instruments to implement their allocations. We are interested in two sets of policy instruments which are accompanied by lump-sum transfers: tax/subsidy on external bond holdings (“capital controls”) and growth-enhancing expenditure (“growth policies”). We consider these instruments for the following reasons. Firstly, pecuniary externalities affect private agents’ choice of growth-enhancing expenditure and external bond holdings simultaneously. To completely implement the social planners’ allocation, we need two types of instruments. However, macroprudential policy typically refers to the policy intervention on financial market, corresponding to tax/subsidy on the external bond holdings in our model. Therefore, we start from defining a constrained social planner who can only intervene in the bond market but respect private agents’ choice of growth-enhancing expenditure. It is also interesting to define a constrained social planner who can choose both bond holdings and growth-enhancing expenditure on behalf of the private agents. For the sake of comparison, we denote the first social planner by a macroprudential social planner and the second one by a multi-instrument social planner. Both social planners are constrained social planners since we focus on the second-best allocation following the literature and the social planners face the same collateral constraint as private agents. Secondly, these two types of constrained social planners have a natural connection to policymakers who have only ex-ante policy instrument and those who have both ex-ante and ex-post policy tools. For the macroprudential social planner, she chooses different allocations from the competitive equilibrium in normal periods since she cannot change the allocations when the collateral constraint binds, which corresponds to policymakers who can only intervene ex-ante. However, for the multi-instrument social planner, she can choose allocations no matter whether the constraint is slack or not, which corresponds to policymakers who can intervene both ex-ante and ex-post.

For the macroprudential social planner, we find that she chooses more bond holdings (less borrowings) than the private agents in normal periods. As a result, the economy

ends up with a lower probability of crisis: both the magnitude and frequency of crises are reduced with macroprudential policy. However, the intervention comes at a cost of reducing growth in normal periods, and quantitative results suggest that average growth is reduced. Therefore, a trade-off between financial stability and growth exists for the macroprudential policy, which can be also termed as a trade-off between average and volatility of growth since the volatility of growth mainly comes from financial instability. However, this trade-off should not be taken for granted. Macroprudential policy increases growth rate in crises at the cost of reducing growth rate in normal periods: it is unclear ex-ante whether the average growth rate is lower or higher. In a one-shock model, we show that the trade-off vanishes when the benefit of macroprudential policy is large enough. For example, when the probability of crises is sufficiently high, or the size of pecuniary externalities is large, it is very beneficial to intervene ex-ante, and the macroprudential social planner can increase both financial stability and average growth.

For the multi-instrument social planner, she could bypass the policy trade-off due to the availability of an additional instrument. Different from the private agents who have to cut external borrowing when the collateral constraint binds, the multi-instrument social planner optimally chooses to shift resources from the growth-enhancing expenditure to consumption, which raises asset price and thus relaxes the borrowing constraint. The intervention can be interpreted as “ex-post” intervention since it is imposed in the time of crises, which comes at a second-order cost but achieves a first-order gain: the economy is able to borrow more than the original borrowing limit and financial crisis is not that costly as before. As a result, the economy hits the collateral constraint more frequently and average growth is reduced.

We calibrate our model to countries which have experienced sudden stop episodes using data in 55 countries from 1961-2015. Quantitative analysis suggests that macroprudential policy can generate welfare gains at the same magnitude as in the literature, i.e. around 0.1 % permanent increase in consumption. The gains come from a higher financial stability and a higher increase in consumption in the long run. However, as a cost macroprudential policy marginally reduces growth at around 0.01 %. With more instruments,

the social planner can generate a larger welfare gains, around 0.2 % permanent increase in consumption. The larger gains come from a temporary spurt in growth and permanent increase in consumption. Growth reverses into a lower level in steady state since the economy borrows more in the long run and expenditures are used to serve a higher level of debt. In our baseline calibration, the temporary growth spurt lasts for 18 years, and the multi-instrument social planner reduces growth by 0.02 %. To implement the macroprudential social planner's allocation, we need 1.28 % capital control tax in steady state, while only 1.00 % capital control tax and 1.00 % subsidy on growth-enhancing expenditure are needed to implement the multi-instrument social planner's allocation. Therefore, our baseline results suggest that the existence of ex-post intervention reduces the size of ex-ante intervention (see Jeanne and Korinek, 2013).

### **Relation to Literature**

This paper contributes to several strands of literature. First, this paper presents a theoretical model to investigate the trade-off between financial stability and economic growth, which is related to the trade-off between average and volatility of growth. Empirical evidence finds mixed results on financial stability and economic growth. For example, Rancière et al. (2008) find a positive relationship between financial instability and economic growth while papers from Ramey and Ramey (1995) argues for a negative link between growth and volatility (See Levine (2005) for a summary). Indeed, we find that a trade-off between stability (volatility) and growth exists for the macroprudential social planner in our framework, which is consistent with Rancière et al. (2008). However, such trade-off depends on parameters that govern the benefits of policy interventions; there exists situations when the trade-off vanishes, consistent with Ramey and Ramey (1995).

Second, this paper contributes to the literature on short-run fluctuations and growth. Previous literature incorporates business fluctuations and economic growth into a fully-fledged DSGE model and investigates the role of short-run fluctuations on medium-run economic growth. For example, Comin and Gertler (2006), Queraltó (2015) and Guerron-Quintana and Jinnai (2014) have built DSGE models to study the relationship between

financial shocks and endogenous growth through the effect of financial constraint on innovation investment. Their mechanisms for endogenous growth follow the work by Romer (1990). My work belongs to the same class of models in the sense that I have the same mechanisms through which short-run fluctuations have a persistent effect on output. Different from their work, I provide a normative framework to analyze the interaction of financial instability and economic growth. Moreover, my focus is on the characterization of optimal policies in managing boom-bust cycles.

Last, this paper contributes to the literature on optimal macroprudential policy and capital flow management. The theoretical rationale for macroprudential policy includes pecuniary externalities (see Lorenzoni, 2008; Jeanne and Korinek, 2010a; Dávila and Korinek, 2017) and aggregate demand externalities (see Korinek and Simsek, 2016; Farhi and Werning, 2016). The general takeaway from the theory is that ex-ante policy intervention can be welfare-improving since it reduces the financial instability. However, the literature has been silent on the effect of ex-ante intervention on economic growth, which is the main trade-off of this paper. To quantify the effect of policy intervention on long-term economic growth, I provide a DSGE model calibrated to sudden stop episodes, which belongs to the literature of SOE models with occasional binding constraint (see Mendoza, 2010; Bianchi, 2011; Jeanne and Korinek, 2010b). Different from the literature, my model can generate a persistent output level effect accompanied with sudden stop episodes, which is consistent with the empirical evidence. Furthermore, my paper is related to the policy debate on ex-ante versus ex-post intervention (see Benigno et al., 2013; Jeanne and Korinek, 2013). Consistent with the literature, we find that the existence of ex-post intervention reduces the magnitude of ex-ante policy intervention. Different from the literature, we focus on the impact of policy interventions on growth and quantify welfare gains using a calibrated SOE-DSGE framework.

The organization of this paper is as follows: section 2 provides empirical evidence on the output cost of financial crises; section 3 presents a benchmark model; section 4 presents welfare analysis; section 5 presents the policy tradeoff in a one-shock model; section 6 presents quantitative analysis; and section 7 concludes.

## 2 Motivating Evidence

The persistent output level effect of financial crisis has been documented by Cerra and Saxena (2008), Rogoff and Reinhart (2009) and Ball (2014).<sup>4</sup> Figure 1 plots the real GDP per capita of different countries in the 1997-98 Asian Crisis and the 2008-09 Global Financial Crisis. The output dynamics in these events suggests that the damage from financial crises can be long-lasting and accompanied by sustained periods of below-trend output. Moreover, contrary to the predictions of new classical growth theory, the output per capita never returns to its pre-crisis trend. Instead, the economy grows at the same rate as its long-run average rate after crises, which suggests that financial crises have a permanent output level effect.<sup>5</sup>

In this paper, we are interested in sudden stop episodes, which is a special type of financial crises in emerging market. Furthermore, as documented by Aguiar and Gopinath (2007), emerging market economies are more likely to be subject to trend shocks rather than cyclical shocks, which suggests that the output cost in crises is higher in emerging markets than advanced economies. To quantify the magnitude of output cost and investigate whether the findings in Cerra and Saxena (2008) still hold, we conduct the same exercise as they did in their paper. Specifically, we estimate an AR(4) model as follows:

$$g_{it} = \alpha_i + \sum_{j=1}^4 \beta_j g_{i,t-j} + \sum_{s=0}^4 \delta_s D_{i,t-s} + \varepsilon_{it} \quad (1)$$

where  $g_{it}$  is the growth rate of real GDP per capita for country  $i$  at year  $t$  and  $D_{it}$  is a dummy variable indicating a sudden stop episode.<sup>6</sup> For the identification of sudden stop episodes, we use the classification in Calvo et al. (2006) (“Calvo episodes”) and Korinek and Mendoza (2014) (“KM episodes”). In Figure 2, we show the impulse response

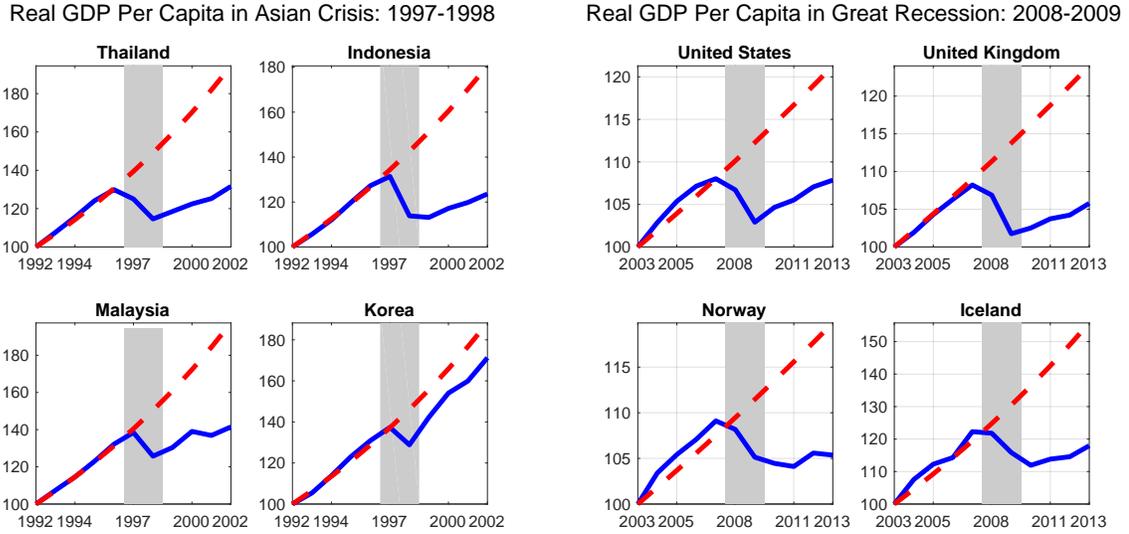
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<sup>4</sup>It is very hard to fully address the causal relationship between financial crisis and output loss in the data. Some work has been done using firm-level data to establish the effect of financial crisis on the output loss through the productivity-enhancing investment (See de Ridder (2016)).

<sup>5</sup>In Figure 1, it might underestimate the cost of crises by claiming that output returns to its pre-crisis trend since most countries do not. However, we argue that it is true on average by looking at the growth rate in Figure 3.

<sup>6</sup>We use data in 55 countries from 1961 to 2015 to estimate the econometric model and the data source is presented in Appendix A.

Figure 1: Output losses in financial crises

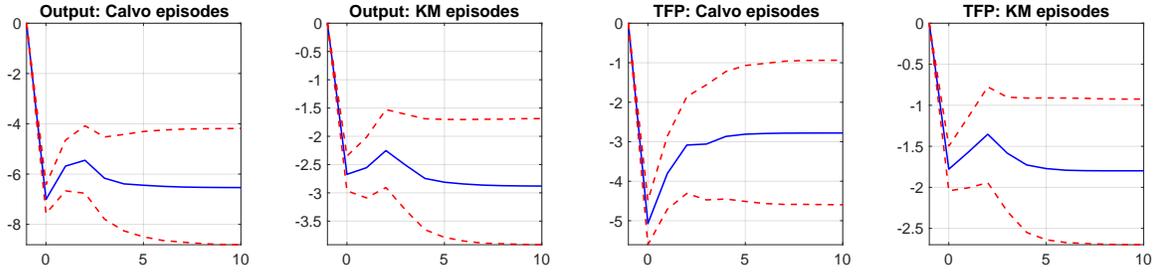


*Note:* The blue line is real GDP per capita normalized to 100 at 5 years before the crises. The red dashed line is a linear extrapolation using data before crisis. The grey area indicates the time of sudden stop episodes.

functions to sudden stop episodes with a one-standard-error band drawn from a thousand Monte Carlo simulation. Consistent with the findings in Cerra and Saxena (2008), we also find a negative and highly persistent impact of sudden stop episodes on output. Depending on the classification of sudden stop episodes, the magnitude differs: 3 percent in “KM” episodes and 6 percent in “Calvo” episodes. This difference is not surprising since Calvo et al. (2006) aim at identifying more severe episodes of crises, which they call “systemic sudden stop episodes”. The magnitude in Cerra and Saxena (2008) is 4 percent, which is similar to our findings. Furthermore, we also conduct the same exercise for total factor productivity (TFP) using Pen World Table data and find that the impulse responses functions for TFP are very similar to that for output, which suggests that TFP and output are highly correlated after crises.

To further quantify the negative effect of crises, we construct 11-year event windows for growth and TFP in Figure 3. Consistent with the findings in the impulse response

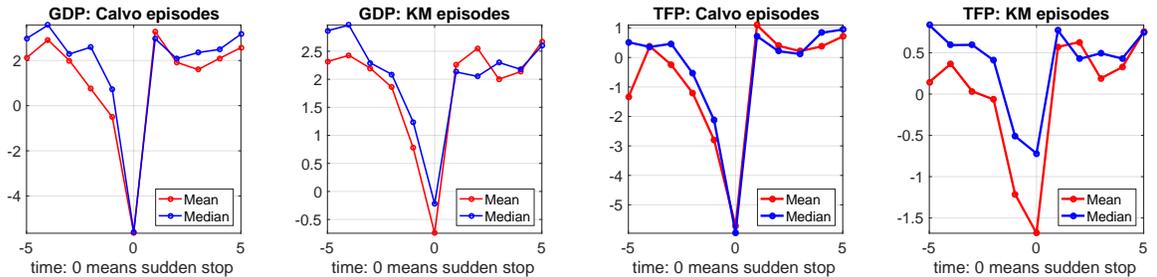
Figure 2: Impulse response functions for output and TFP: sudden stop episodes



*Note:* blue lines are percentage deviation from steady state and red dashed lines are 1 standard deviation error band.

functions, the growth rate of real GDP per capita and TFP experience a large drop in the time of sudden stop episodes. Unfortunately, they only bounce back to their long run average after crises, which is consistent with the phenomenon in Figure 1. One might find that the output cost in Figure 1 is larger than that in Figure 3 since Figure 1 captures the most severe episodes. The event analysis further suggests that the magnitude and the dynamics of output and TFP are very similar, consistent with the implication of endogenous growth literature.

Figure 3: Growth rate in sudden stop episodes (%)



*Note:* The series are constructed using a 11-year window centering on the sudden stop episodes identified by Calvo et al. (2006) and Korinek and Mendoza (2014).

### 3 Model Economy

We consider a SOE model with endogenous growth and collateral constraint to capture the interactions between financial instability and economic growth. The economy is populated by a continuum of identical households, who have access to international capital market and a technology that increases TFP. Due to frictions in the financial market, the maximum amount of external borrowing cannot exceed the value of collaterals in the economy. In normal periods when the collateral constraint is slack, households are able to finance their desired level of expenditures through external borrowing and the economy grows at a normal rate. Financial crises (sudden stop episodes) are modeled as periods when the collateral constraint binds and current account reverses (See Mendoza (2010)). In those periods, households cannot finance enough expenditures for consumption and TFP, which leads to a decline in growth. As a result, sudden stop episodes are accompanied by a decline in TFP and growth.

**Preference** The household is assumed to have the following Constant Relative Risk Aversion (CRRA) preference with Stone-Geary functional form (see Stone, 1954; Geary, 1950):

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t - h_t) \equiv E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t - h_t)^{1-\gamma}}{1-\gamma} \quad (2)$$

where  $\beta \in (0, 1)$  is the discount factor,  $\gamma$  is the coefficient of risk aversion,  $c_t$  is the consumption and  $h_t$  is the subsistence level of consumption. Given that the economy is growing, we assume that  $h_t$  only depends on the level of endogenous TFP  $z_t$ , which is predetermined at time  $t$ . Specifically, it takes the following functional form:

$$h_t = h z_t \quad (3)$$

where  $h > 0$  is a parameter.

**Role of  $h_t$**  There are two roles for  $h_t$  in our framework. First, it increases local risk-aversion in the utility function (2). The presence of  $h_t$  makes the economic agent more risk-averse to falls in consumption  $c_t$  than that in a CRRA utility without Stone-Geary functional form. Second, it reduces the incentive of increasing endogenous TFP in the future. An increase in  $z_{t+1}$  makes agents more risk-averse in period  $t + 1$  since it increases the subsistence level of consumption,  $h_{t+1}$ . When private agents choose  $z_{t+1}$ , they internalize its impact on  $h_{t+1}$ . As a result, the presence of  $h_t$  helps to calibrate the technology that increases TFP.

The second role of  $h_t$  turns out to be important to capture a large fall of growth in crises. We start from a standard CRRA utility and find that the endogenous TFP is not responsive to shocks. This is understandable since in a model with endogenous growth, consumers are very risk-averse to growth uncertainty and tend to cut consumption expenditures following a shock. Only when it is very costly to cut consumptions, growth starts to fall significantly. In other words, to have a large fall in growth, one need to have a large drop in consumption, which is inconsistent with the empirical evidence. One way to deal with this unpleasant feature is to increase the local risk-aversion of utility functions, such as increasing the coefficient of risk aversion  $\gamma$  or introducing Epstein-Zin preference.<sup>7</sup> However, neither a large  $\gamma$  nor Epstein-Zin preference could deliver a large fall of endogenous TFP as in the data, which suggests that the responsiveness of TFP to shocks mainly depends on the technology of choosing  $z_{t+1}$  rather than the degree of risk-aversion.

**Production Function** Production only requires productive asset  $n_t$  as an input and is assumed to take the following form:

$$y_t = A_t n_t^\alpha \tag{4}$$

where  $A_t$  represents the TFP level in the economy and  $\alpha \in (0, 1)$ . Productive asset  $n_t$

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<sup>7</sup>The other way to increase local risk-aversion is to introduce consumption habit. However, such formulation introduces an additional endogenous state variable and significantly increases the computational burden.

is an endowment to the households and normalized to 1. It corresponds to an asset in fixed supply, such as land. In each period, households trade the productive asset  $n_t$  at a market determined price  $q_t$ .

**Endogenous TFP** We assume that the level of TFP  $A_t$  takes the following form:

$$A_t = \theta_t z_t \tag{5}$$

where  $\theta_t$  is an exogenous technology shock, and  $z_t$  is an endogenous technology component chosen by the private agents. The dynamics of the exogenous technology shock  $\theta_t$  is assumed to be as follows:

$$\log \theta_t = \rho \log \theta_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t \sim N(0, \sigma^2) \tag{6}$$

where  $\rho$  and  $\sigma$  are persistence and volatility of the exogenous technology shock.

**Discussion of  $A_t$**  There are two pieces of TFP: exogenous component  $\theta_t$  and endogenous component  $z_t$ . Fluctuations in  $\theta_t$  lead to fluctuations in the economy, including  $z_{t+1}$  and growth rate. One point worth mentioning is that the fall of growth in time of sudden stop episodes represents mostly a fall in  $\theta_t$  since the economy grows at a normal rate before the sudden stop episodes. Without  $\theta_t$ , one cannot explain the negative growth rate in crises.<sup>8</sup> Furthermore, without the endogenous component of  $z_{t+1}$ , one cannot explain the growth rate one period after sudden stop episodes. Otherwise, growth rate after crises should be much higher than its long run average level to offset a decline of  $\theta_t$  in crises. However, this is not true on average, which implies that a decline of endogenous TFP in the economy is at play.

**Source of growth** Households have access to a technology, which costs  $\Psi(z_{t+1}, z_t)$  units

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<sup>8</sup>Admittedly, other shocks such as financial shocks and interest rate shocks are important to understand the sudden stop episodes. However, we focus on the technology shock given that it is crucial to understand the falls of growth rate in time of crises.

of consumption goods in period  $t$  and elevates endogenous technology level from  $z_t$  to  $z_{t+1}$ . We want to interpret  $\Psi(z_{t+1}, z_t)$  as “growth-enhancing expenditure”, which should include all the expenditures that facilitate long-term economic growth. Here we do not take a stand on a particular form of economic growth. Instead, we include all the expenditures that are conducive to growth into  $\Psi$  function. For example, it includes physical capital in the AK growth framework as in Romer (1986), human capital as in Lucas (1988), R&D expenditure as in Romer (1990) and Aghion and Howitt (1992), etc. The only restriction that we make is that there is no externality with the process of choosing  $z_{t+1}$ . In other words, when private agents choose  $z_{t+1}$ , not only they internalize its impact on the subsistence level of consumption  $h_{t+1}$  but also the cost function in the future,  $\Psi(z_{t+2}, z_{t+1})$ . We make this assumption to shut down externalities in endogenous growth but focus on externalities in financial market. This is different from the literature which incorporate endogenous growth into DSGE models, where growth in their framework is suboptimal (See Comin and Gertler (2006) and Kung and Schmid (2015)). Otherwise, externalities in endogenous growth dominate externalities in financial market.<sup>9</sup>

**Assumption 1.** *Cost function  $\Psi(z_{t+1}, z_t)$  is convex and takes the following form:*

$$\Psi(z_{t+1}, z_t) = \left[ \left( \frac{z_{t+1}}{z_t} - \psi \right) + \kappa \left( \frac{z_{t+1}}{z_t} - \psi \right)^\eta \right] z_t,$$

where  $\eta > 1$ ,  $\psi > 0$  and  $\frac{z_{t+1}}{z_t} > \psi$ .

**Financial Friction** In each period, households can purchase  $b_{t+1}$  units of one-period bond from international market, which promises a gross interest rate  $1 + r$  in next period. We assume that domestic economy is atomistic in the international world and takes the interest rate  $1 + r$  as given. Furthermore, bonds are supplied in an infinite elasticity. However, we assume that there is a moral hazard problem between domestic households and international investors: households have the option to invest in a scam that helps to

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<sup>9</sup>One rationale for macroprudential policy comes from externalities in financial market. Furthermore, both externalities in financial market and externalities in growth typically call for policy interventions to increase national savings. It is hard to disentangle which factor dominates if both externalities are present.

remove future productive asset from the reach of international investors. International investors, however, cannot coordinate to punish the households by excluding them from the market. What they can do is to take households to court before the scam is completed. By doing so, they can seize a fraction  $\phi \in (0, 1)$  of productive assets and sell them to other households at the prevailing market price  $q_t$ . As a result, rational international investors demand collaterals from domestic households and the only acceptable collateral is the productive asset. Furthermore, they will restrict the amount of external borrowing up to  $\phi q_t$ . The collateral constraint can be written as:

$$-b_{t+1} \leq \phi q_t \quad (7)$$

**Budget Constraint** In each period, households make expenditure plans for consumption  $c_t$ , growth-enhancing expenditure  $\Psi(z_{t+1}, z_t)$ , purchase of productive asset  $q_t n_{t+1}$  and bond holdings  $b_{t+1}$ . Their income comes from the output  $y_t$ , sale of productive asset  $q_t n_t$  and bond holding  $(1+r)b_t$ . As a result, the budget constraint can be written as follows:

$$c_t + \Psi(z_{t+1}, z_t) + q_t n_{t+1} + b_{t+1} = y_t + q_t n_t + (1+r)b_t, \quad (8)$$

**Market Clear** There are two markets in the economy: final good market and productive asset market. Given that productive asset is in fixed supply and owned by the households, equilibrium condition implies that

$$n_{t+1} = n_t = 1 \quad (9)$$

The final good market can be pinned down by aggregating the budget constraint for each household and applying the equilibrium condition (9) for asset market.

$$c_t + \Psi(z_{t+1}, z_t) + b_{t+1} = y_t + (1+r)b_t, \quad (10)$$

### 3.1 Competitive Equilibrium (CE)

**Competitive Equilibrium** in this economy consists of a stochastic process  $\{c_t, z_{t+1}, n_{t+1}, b_{t+1}\}_{t=0}^{\infty}$  chosen by the households and asset price  $\{q_t\}_{t=0}^{\infty}$  given the initial values  $\{b_0, z_0\}$  such that utility (2) is maximized, constraints (7) and (8) are satisfied, and productive asset market and good market clear, i.e. conditions (9) and (10) are satisfied.

**Recursive Formulation** It is easier to define net consumption by  $c_t^h = c_t - h_t$  and write the problem in a recursive formulation. State variables at time  $t$  include endogenous variables  $\{z_t, n_t, b_t\}$  and exogenous variable  $\theta_t$ . We can write the optimization problem as follows:

$$\begin{aligned} V_t^{CE}(z_t, n_t, b_t, \theta_t) &= \max_{c_t^h, z_{t+1}, n_{t+1}, b_{t+1}} u(c_t^h) + \beta E[V_{t+1}^{CE}(z_{t+1}, n_{t+1}, b_{t+1}, \theta_{t+1})] \\ \text{s.t.} \quad c_t^h + h z_t + \Psi(z_{t+1}, z_t) + q_t n_{t+1} + b_{t+1} &= \theta_t z_t n_t^\alpha + q_t n_t + (1+r)b_t, \quad (\lambda_t^{CE}) \\ -b_{t+1} &\leq \phi q_t. \quad (\mu_t^{CE}) \end{aligned}$$

where  $\lambda_t^{CE}$  and  $\mu_t^{CE}$  are the Lagrangian multipliers associated with budget constraint and collateral constraint respectively.

The maximization problem yields the following optimality conditions for each period.

$$\lambda_t^{CE} = u'(c_t^h) \tag{11}$$

$$\lambda_t^{CE} \Psi_{1,t} = \beta E_t [\lambda_{t+1}^{CE} (\theta_{t+1} - h - \Psi_{2,t+1})] \tag{12}$$

$$\lambda_t^{CE} q_t = \beta E_t [\lambda_{t+1}^{CE} (\alpha \theta_{t+1} z_{t+1} + q_{t+1})] \tag{13}$$

$$\lambda_t^{CE} = \mu_t^{CE} + \beta(1+r)E_t [\lambda_{t+1}^{CE}] \tag{14}$$

Condition (11) is the marginal valuation of wealth for households. Conditional (12) is the key equation for growth in this model, where private agents equate marginal cost of choosing  $z_{t+1}$  with marginal benefit. The cost is reflected in the technology cost function  $\Psi_{1,t}$  evaluated at marginal valuation of wealth in period  $t$  while the benefit includes future output  $\theta_{t+1}$  excluding a higher future subsistence level of consumption  $h$ , and

growth-enhancing expenditure  $\Psi_{2,t+1}$ . The third condition (13) is a standard asset pricing function, where holding productive asset  $n_{t+1}$  yields a dividend income  $\alpha\theta_{t+1}z_{t+1}$  and capital gains  $q_{t+1}$ . The last condition (14) is the Euler equation for holding bonds. The additional term  $\mu_t^{CE}$  here captures the effect of collateral constraint on external borrowing. When the collateral constraint (7) binds, the marginal benefit of borrowing to increase consumption exceeds the expected marginal cost by an amount equal to the shadow price of relaxing the collateral constraint  $\mu_t^{CE}$ .

**Deflated Economy** The economy grows at the same rate as the endogenous variable  $z_t$ . To solve for a stationary equilibrium, we deflate all the endogenous variables by  $z_t$  and denote them by hat. Specifically, we denote  $\hat{x}_t = \frac{x_t}{z_t}$ , where  $x_t = \{c_t^h, b_t, q_t, V_t^{CE}, \dots\}$  and  $g_{t+1} = \frac{z_{t+1}}{z_t}$ . The deflated equilibrium conditions are given in Appendix B.

## 4 Welfare Analysis

There is a case for public intervention since the collateral constraint depends on the asset price  $q_t$ , which is an endogenous variable and private agents take as given. To see how it works intuitively, consider a situation when the collateral constraint marginally binds. We can substitute collateral constraint (7) into the budget constraint (8) as follows:

$$c_t + \Psi(z_{t+1}, z_t) = y_t + (1+r)b_t + \phi \mathcal{Q}(c_t, z_{t+1}, z_t, \theta_t, b_t)$$

where  $\mathcal{Q}(c_t, z_{t+1}, z_t, \theta_t, b_t)$  is the asset price given by the equation (13).

Suppose there is a reduction in net wealth  $y_t + (1+r)b_t$ , which could be due to a drop in exogenous technology shock  $\theta_t$ , or a drop in bond holding  $b_t$ , or both. Given that the collateral constraint has already been marginally binding, private agents could not smooth through external borrowing. Instead, they need to cut all the expenditures including consumption  $c_t$ , growth-enhancing expenditure  $\Psi(z_{t+1}, z_t)$  and bond purchase  $b_{t+1}$ . However, there is a second round reduction in expenditures simply because asset price  $q_t$  falls with the expenditures and collateral values are further driven down. Ex-ante,

private agent does not internalize the second round effect and prepares less precautionary buffer to hedge against the bad events. Differently, a social planner, who internalizes the general equilibrium effect, finds it optimal to choose *more* buffers ex-ante and thus reduces the probability of binding constraints.

The difference between private agents and social planner is due to the presence of price dependent collateral constraint and incomplete market, which leads to “pecuniary externalities” in the economy.<sup>10</sup> To formally investigate the magnitude of inefficiency, we proceed by defining a social planner’s problem. We formulate the social planner’s problem similar to the primal approach in optimal policy analysis. Namely, the planner can choose allocations subject to resource, implementability and collateral constraints. This formulation allows us to see the “wedge” between social planner and private agents in choosing allocations and understand the inefficiencies in the economy. To implement the allocation, we consider taxes/subsidies with lump-sum transfers that are needed to close the wedge between two allocations.

Specifically, we consider a social planner who chooses allocations on behalf of the representative household subject to the same constraints as private agents, but lacks the ability to commit to future policies (see Bianchi and Mendoza, forthcoming). Importantly, we assume that asset price  $q_t$  remains market-determined and the Euler equation of asset price (13) enters into the social planner’s problem as an implementability constraint. The implicit rationale is that the social planner cannot directly intervene the asset price but internalize how the allocations affect asset price and thus collateral constraint.<sup>11</sup>

Other than consumption, there are two important allocations: growth-enhancing expenditure  $\Psi(z_{t+1}, z_t)$  and external bond holding  $b_{t+1}$ . As a result, we need to decide which allocations can be chosen by the social planner. It is interesting and useful to define two types of social planners: a social planner who only chooses external bond holding versus

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<sup>10</sup>Pecuniary externalities refer to externalities associated with prices. In an economy with incomplete market, allocations with pecuniary externalities are generically sub-optimal. For a detailed proof, see early contributions by Greenwald and Stiglitz (1986) and Geanakoplos and Polemarchakis (1985).

<sup>11</sup>We do not allow the social planner does not trade asset on behalf of private agents. One rationale is that private agents are better than the planner at observing the fundamental payoffs of financial assets (see Jeanne and Korinek, 2010b).

a social planner who chooses both external bond holding and growth-enhancing expenditure. There are several reasons to do so. Firstly, macroprudential policy refers to the policy intervention in the financial market and it is important to understand the impact of macroprudential policy alone on growth.<sup>12</sup> Secondly, policy interventions for the two social planners' allocations have natural interpretations in the on-going debate on ex-ante versus ex-post intervention (see Benigno et al., 2013; Jeanne and Korinek, 2013). For the social planner who only chooses external bond holding, she can do nothing when the constraint binds (modeled as crisis); instead, she chooses a different allocation when the constraint does not bind (modeled as normal periods). This intervention is considered as ex-ante or macroprudential policy since it is imposed in normal periods. When the social planner can choose both external bond holding and growth-enhancing expenditure, she chooses different allocations even when the constraint binds. Therefore, policy interventions are imposed both in tranquil and crises periods, corresponding to ex-ante and ex-post policies respectively. In other words, to implement the social planner's allocation, we implicitly empower her with both ex-ante and ex-post policy interventions.

For simplicity, we refer to the social planner who can only choose bond holding as *macroprudential* social planner and use superscript "MP" to identify her allocations. The social planner who can choose both bond holding and growth-enhancing expenditure is named *multi-instrument* social planner and her allocations are augmented by superscript "SP". One thing worth mentioning is that both social planners are constrained social planners since they face the same collateral constraint as the private agents and cannot intervene in the asset market.

**Further Discussion on Policy Implementation** In our framework, we abstract the difficulty of implementation issues with our optimal policies and evaluate their implications in a perfect world. Furthermore, we focus more on the role of macroprudential policy rather than growth policy. One reason is that macroprudential policy might be more ef-

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<sup>12</sup>It might be more realistic and relevant for emerging market economies to consider only macroprudential policy for two reasons. First, it is easier to implement capital control taxes/subsidies. Second, policies on growth-enhancing expenditure are hard to implement. In our framework, we apply a reduced-form function for growth. In reality, it is hard to identify technologies that are conducive to long run growth.

fective and thus easier to implement. For example, empirical results on the effectiveness of macroprudential policy are mostly supportive. Lim et al. (2011) and Bruno et al. (2015) have estimated the effectiveness of macroprudential tools using a comprehensive data and argue that such tools are effective in reducing pro-cyclicality of shocks.<sup>13</sup> However, empirical results on the effectiveness of growth policy are less satisfactory. Little evidence has been found on the positive effects of policies on productivity even if there are evidence on the link between policies and innovative activities (see Westmore, 2013).

## 4.1 Macroprudential Social Planner (MP)

The *macroprudential* social planner chooses  $b_{t+1}$  on behalf of private agent subject to the resource constraint (10), collateral constraint (7) and two implementation constraints. Specifically, her recursive problem is defined as follows

$$\begin{aligned}
V_t^{MP}(z_t, b_t, \theta_t) &= \max_{c_t^h, z_{t+1}, b_{t+1}, q_t} u(c_t^h) + \beta E[V_{t+1}^{MP}(z_{t+1}, b_{t+1}, \theta_{t+1})] \\
\text{s.t.} \quad & c_t^h + h z_t + \Psi(z_{t+1}, z_t) + b_{t+1} = \theta_t z_t + (1+r)b_t, (\lambda_t^{MP}) \\
& -b_{t+1} \leq \phi q_t, (\mu_t^{MP}) \\
& u'(c_t^h) q_t = \underbrace{\beta E_t [u'(c_{t+1}^h) (\alpha \theta_{t+1} z_{t+1} + q_{t+1})]}_{G(z_{t+1}, b_{t+1})}, (\xi_t^{MP}) \quad (15) \\
& u'(c_t^h) \Psi_{1,t} = \underbrace{\beta E_t [u'(c_{t+1}^h) (\theta_{t+1} - h - \Psi_{2,t+1})]}_{I(z_{t+1}, b_{t+1})}, (\nu_t^{MP}) \quad (16)
\end{aligned}$$

where equation (15) and (16) are two implementation constraints for the Euler equations of choosing productive asset and TFP.  $\lambda_t^{MP}$ ,  $\mu_t^{MP}$ ,  $\xi_t^{MP}$  and  $\nu_t^{MP}$  are the Lagrangian multipliers associated with budget constraint, collateral constraint and two implementation constraints respectively. We write the implementation constraints as functions of future endogenous state variables  $z_{t+1}$  and  $b_{t+1}$  since we want to solve for time-consistent policy

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<sup>13</sup>There are exceptions. For example, Fernández et al. (2013) cast some doubts on the effectiveness of macroprudential policies since they find the instruments are acyclical, which counters the theoretical predictions for prudential tools. Arguably, the effectiveness of policies depends crucially on their characterization. There are issues which might affect the effectiveness. For example, Bengui and Bianchi (2014) investigate the issue of leakage, and Dogra (2014) investigates the issue of private information.

functions for the social planner as in Bianchi and Mendoza (forthcoming).

The maximization problem implies the following optimality conditions for each period.

$$\lambda_t^{MP} = u'(c_t^h) - \xi_t^{MP} u''(c_t^h) q_t - \nu_t^{MP} u''(c_t^h) \Psi_{1,t} \quad (17)$$

$$\begin{aligned} & \lambda_t^{MP} \Psi_{1,t} - \xi_t^{MP} G_{1,t} - \nu_t^{MP} [I_{1,t} - u'(c_t^h) \Psi_{11,t}] \\ & = \beta E_t [\lambda_{t+1}^{MP} (\theta_{t+1} - h - \Psi_{2,t+1}) - \nu_{t+1}^{MP} u'(c_{t+1}^h) \Psi_{12,t+1}] \end{aligned} \quad (18)$$

$$\phi \mu_t^{MP} = \xi_t^{MP} u'(c_t^h) \quad (19)$$

$$\lambda_t^{MP} = \mu_t^{MP} + \xi_t^{MP} G_{2,t} + \nu_t^{MP} I_{2,t} + \beta(1+r) E_t [\lambda_{t+1}^{MP}] \quad (20)$$

**Economic interpretation** Condition (17) is the marginal valuation of wealth for the social planner. Compared with condition (11) in competitive equilibrium, there are two extra terms due to the presence of implementation constraints: the first term is  $-\xi_t^{MP} u''(c_t^h) q_t$ , which is positive due to the condition (19), and the second term is  $-\gamma c_t^{-\gamma-1} \Psi_{1,t} \nu_t^{MP}$ , which captures the inability of changing  $z_{t+1}$ . The wedge  $-\xi_t^{MP} u''(c_t^h) q_t - \nu_t^{MP} u''(c_t^h) \Psi_{1,t}$  is positive quantitatively, reflecting a higher valuation of wealth for the social planner. Due to this wedge, the social planner chooses different allocations from private agents. Condition (18) and (20) are the Euler equations of TFP and bond holding for the social planner. Given that the social planner has to respect private agents' optimality condition of choosing TFP, condition (18) defines the Lagrangian multiplier  $\xi_t^{MP}$ . For condition (20), the key difference between social planner and private agents is reflected in the difference of the marginal valuation of wealth. In the period when the collateral constraint is slack, i.e.  $\mu_t^{MP} = 0$ , the social planner chooses a higher level of bond than the private agents due to a higher valuation of wealth  $E_t [\lambda_{t+1}^{MP}]$ .

**Implementation** We assume that the social planner has access to tax/subsidy  $\tau_t^{MP,b}$  on bond holding and a lump-sum transfer  $T_t^{MP}$ . The budget constraint for the private agents becomes

$$c_t^h + h_t + \Psi(z_{t+1}, z_t) + q_t n_{t+1} + \left(1 + \tau_t^{MP,b}\right) b_{t+1} = y_t + q_t n_t + (1+r) b_t + T_t^{MP}$$

where  $T_t^{MP} = \tau_t^{MP,b} b_{t+1}$ . To implement the macroprudential social planner's allocation, we compare the deflated optimality conditions of private agents and macroprudential social planner (See Appendix B), and find that

$$\begin{aligned} \tau_t^{MP,b} &= \frac{\gamma \phi \hat{\mu}_t^{MP} \hat{q}_t (\hat{c}_t^h)^{-1} + \gamma \hat{\nu}_t^{MP} (\hat{c}_t^h)^{-\gamma-1} \Psi_{1,t} - \phi \hat{\mu}_t^{MP} g_{t+1}^{-\gamma} \hat{G}_{2,t} (\hat{c}_t^h)^\gamma - \hat{\nu}_t^{MP} g_{t+1}^{-1-\gamma} \hat{I}_{2,t}}{(\hat{c}_t^h)^{-\gamma}} \\ &- \frac{\beta g_{t+1}^{-\gamma} (1+r) E_t \left[ \gamma \phi \hat{\mu}_{t+1}^{MP} \hat{q}_{t+1} (\hat{c}_{t+1}^h)^{-1} + \gamma \hat{\nu}_{t+1}^{MP} (\hat{c}_{t+1}^h)^{-\gamma-1} \Psi_{1,t+1} \right]}{(\hat{c}_t^h)^{-\gamma}} \end{aligned}$$

## 4.2 Multi-instrument Social Planner (SP)

The *multi-instrument* social planner chooses  $b_{t+1}$  and  $z_{t+1}$  on behalf of private agent subject to the resource constraint (10), collateral constraint (7) and one implementation constraint. Specifically, her recursive problem is defined as follows:

$$\begin{aligned} V_t^{SP}(z_t, b_t, \theta_t) &= \max_{c_t^h, z_{t+1}, b_{t+1}, q_t} u(c_t^h) + \beta E[V_{t+1}^{SP}(z_{t+1}, b_{t+1}, \theta_{t+1})] \\ \text{s.t.} \quad &c_t^h + h z_t + \Psi(z_{t+1}, z_t) + b_{t+1} = \theta_t z_t + (1+r)b_t, \quad (\lambda_t^{SP}) \\ &-b_{t+1} \leq \phi q_t, \quad (\mu_t^{SP}) \\ &u'(c_t^h) q_t = \underbrace{\beta E_t[u'(c_{t+1}^h) (\alpha \theta_{t+1} z_{t+1} + q_{t+1})]}_{G(z_{t+1}, b_{t+1})}, \quad (\xi_t^{SP}) \end{aligned}$$

where the last constraint is the Euler equation of choosing productive asset.  $\lambda_t^{SP}$ ,  $\mu_t^{SP}$  and  $\xi_t^{SP}$  are the Lagrangian multipliers associated with budget constraint, collateral constraint and implementation constraint respectively.

The maximization problem implies the following optimality conditions for each period.

$$\begin{aligned} \lambda_t^{SP} &= u'(c_t^h) - \xi_t^{SP} u''(c_t^h) q_t \\ \lambda_t^{SP} \Psi_{1,t} &= \xi_t^{SP} G_{1,t} + \beta E_t [\lambda_{t+1}^{SP} (\theta_{t+1} - h - \Psi_{2,t+1})] \end{aligned} \quad (21)$$

$$\begin{aligned} \phi \mu_t^{SP} &= \xi_t^{SP} u'(c_t^h) \\ \lambda_t^{SP} &= \mu_t^{SP} + \xi_t^{SP} G_{2,t} + \beta(1+r) E_t [\lambda_{t+1}^{SP}] \end{aligned} \quad (22)$$

**Economic interpretation** Same as the *macroprudential* social planner, the *multi-instrument* social planner values wealth more than the private agents, captured by the term  $-\xi_t^{SP} u''(c_t^h) q_t$ . Conditions (21) and (22) are the Euler equations of TFP and bond holding. Here, the social planner is able to choose TFP, unlike the macroprudential social planner. In the period when the collateral constraint is slack, she borrows *less* from international credit market and allocates *more* to growth-enhancing expenditure compared with the private agents. This precautionary motive is the same as the macroprudential social planner. However, she behaves differently when the constraint binds. Unlike the macroprudential social planner who chooses the same allocation as the private agents, the multi-instrument social planner tends to allocate more resources from growth-enhancing expenditure to consumption. By doing so, she could increase the asset price and thus relaxes the collateral constraint. As a result, the crisis is not that costly for the multi-instrument social planner and she actually ends up to borrow more.

**Implementation** We assume that there is a combination of taxes/subsidies  $\{\tau_t^{SP,z}, \tau_t^{SP,b}\}$  on growth-enhancing expenditure and bond, and lump-sum transfers  $T_t^{SP}$  to implement the allocation of multi-instrument social planner. The budget constraint of private agents changes into

$$c_t^h + h_t + \left(1 + \tau_t^{SP,z}\right) \Psi(z_{t+1}, z_t) + q_t n_{t+1} + \left(1 + \tau_t^{SP,b}\right) b_{t+1} = y_t + q_t n_t + (1 + r) b_t + T_t^{SP}$$

where  $T_t^{SP} = \tau_t^{SP,z} \Psi(z_{t+1}, z_t) + \tau_t^{SP,b} b_{t+1}$ .

Compare the deflated optimality conditions between private agents and social planner

(See Appendix B),  $\{\tau_t^{SP,z}, \tau_t^{SP,b}\}$  have to satisfy

$$\begin{aligned}\tau_t^{SP,z} &= \frac{\gamma\phi\hat{q}_t(\hat{c}_t^h)^{-1}\hat{\mu}_t^{SP}\Psi_{1,t} - \phi\hat{\mu}_t^{SP}(\hat{c}_t^h)^\gamma g_{t+1}^{-\gamma}\hat{G}_{1,t}}{\Psi_{1,t}(\hat{c}_t^h)^{-\gamma}} \\ &- \frac{\beta g_{t+1}^{-\gamma} E_t \left[ \hat{c}_{t+1}^{-\gamma} \tau_{t+1}^{SP,z} \Psi_{2,t+1} + \gamma\phi\hat{\mu}_{t+1}^{SP}\hat{q}_{t+1}(\hat{c}_{t+1}^h)^{-1}(\theta_{t+1} - h - \Psi_{2,t+1}) \right]}{\Psi_{1,t}(\hat{c}_t^h)^{-\gamma}}, \\ \tau_t^{SP,b} &= \frac{\gamma\phi\hat{q}_t(\hat{c}_t^h)^{-1}\hat{\mu}_t^{SP} - \phi\hat{\mu}_t^{SP}(\hat{c}_t^h)^\gamma g_{t+1}^{-\gamma}\hat{G}_{2,t} - \beta g_{t+1}^{-\gamma}(1+r) E_t \left[ \gamma\phi\hat{q}_{t+1}(\hat{c}_{t+1}^h)^{-1}\hat{\mu}_{t+1}^{SP} \right]}{(\hat{c}_t^h)^{-\gamma}}\end{aligned}$$

## 5 A One-Shock Model

To better understand the policy trade-off between financial stability and growth, we simplify our model to the case where there is only one binary shock in the second period. Specifically, we assume that there is only one shock in the intermediate period,  $\theta_2 = 0.9$  with probability  $p \in [0, 1]$ . From period 3 onwards, there is no shock and  $\theta_t = 1$  for  $t > 2$ . We can solve our model almost analytically to get intuitions. Depending on the parameters, starting from period 3 onwards, the economy converges to a steady state whether it is constrained or unconstrained. It is important to differentiate these two cases and we provide calibrations such that in the long run the economy is either unconstrained (Unconstrained Steady State) or constrained (Constrained Steady State).

### 5.1 Unconstrained Steady State

We first consider a set of parameters such that the economy is in unconstrained steady state from period 3 onwards. In particular, we choose parameters such that  $\beta(1+r)g_{ss}^{-\gamma} = 1$ , where  $g_{ss}$  is the growth rate in the steady state and endogenously determined by the Euler equation (12). The benchmark parameter values are given in Table 1. For the specific calibration methods, we explain in detail in section 6.

Table 1: Parameter values in one shock model

$r$	$\gamma$	$\eta$	$\psi$	$\beta$	$\phi$	$\alpha$	$p$
6 %	2	2	0.9	0.9873	0.0543	0.2	5.5 %

Starting from period 3 onwards, the economy in competitive equilibrium is in steady state since there is no shock anymore. In period 2, depending on the initial wealth  $\theta_2 + (1 + r)\hat{b}_2$ , the economy might be constrained. When the constraint is slack, the economy borrows to smooth the shock and growth rate  $g_3$  is unaffected. The growth rate is reduced, however, when the constraint starts to bind. In period 1, we assume that there is no constraint and we solve allocations depending on the initial wealth at period 1, denoted by  $w_1$ .

It is interesting to compare the private agents' allocations with the macroprudential social planner and multi-instrument social planner. For the macroprudential social planner, she chooses the same allocation as the competitive equilibrium in period 2 since she needs to respect private agents' choice of growth rate. However, she values wealth differently from the private agents due to the presence of pecuniary externalities, and thus chooses a different allocation in period 1. The multi-instrument social planner, however, is able to choose a different allocation from the private agents in period 2 when the constraint binds. Specifically, she chooses a higher consumption  $\hat{c}_2$  and lower growth  $g_3$  so as to increase the asset price  $\hat{q}_2$ , collateral value and external borrowing  $\hat{b}_3$ . As a result, the multi-instrument social planner borrows more in period 1 due to less precautionary motives.

Policy interventions have implications for growth. To see the difference between private agents and social planners, we report a chart showing the average and volatility of growth on different parameters.<sup>14</sup> In the first two graphs of Figure 4, we use the benchmark parameters in Table 1 and plot the average and volatility of growth against the

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<sup>14</sup> To construct this statistics, we run two 100-period simulations: one is that  $\theta_2 = 0.9$  (state "L") and the other is  $\theta_2 = 1$  (state "H"). Growth rate for each simulation is calculated as follows:

$$G^i = (\Pi_{t=1}^{100} g_{t+1})^{\frac{1}{100}}, \text{ where } i \in \{H, L\}$$

Therefore, the statistics are calculated as follows

$$\begin{aligned} \text{Average}_G &= p * G^L + (1 - p) * G^H \\ \text{Vol}_G &= \sqrt{p * (G^L - \text{Average}_G)^2 + (1 - p) * (G^H - \text{Average}_G)^2} \end{aligned}$$

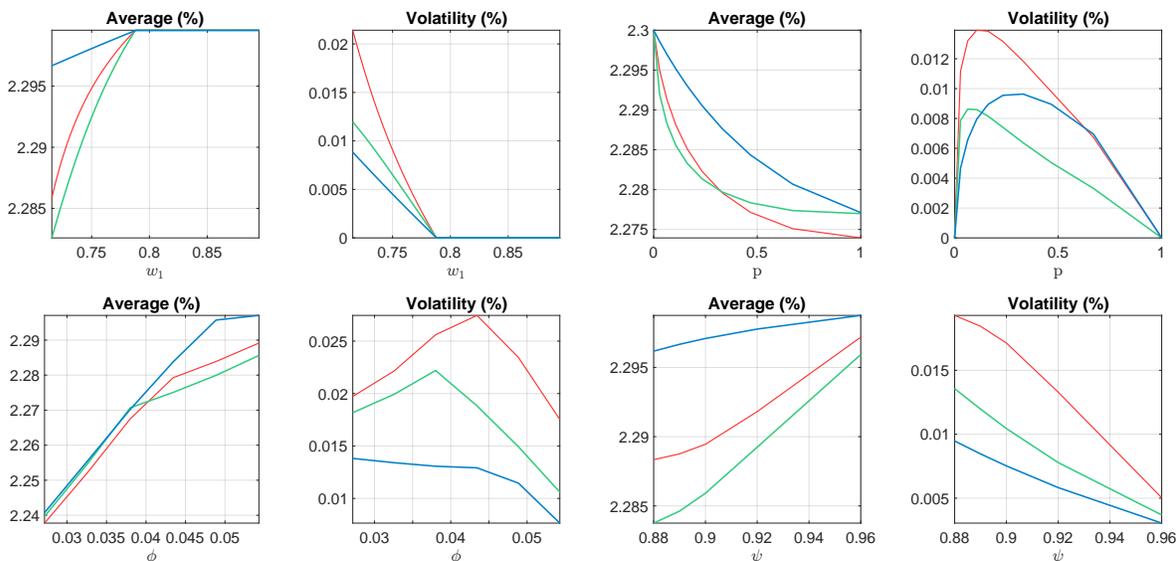
initial wealth  $w_1$  in period 1. One can see that there is a trade-off between average and volatility of growth for the macroprudential social planner: she reduces the volatility of growth at the cost of reducing average growth. However, this trade-off should not be taken for granted and it depends on the parameter values.<sup>15</sup> In particular, it depends on parameters that affect the pecuniary externalities in the model, such as probability of negative shock,  $p$ , and parameter in the collateral constraint,  $\phi$ . Intuitively, the macroprudential social planner wants to reduce the volatility of growth since agents are risk averse to growth volatility. The impact on the growth rate is ambiguous since macroprudential social planner increases the growth rate  $g_3$  in the low state ( $\theta_2 = 0.9$  with probability  $p$ ) but reduces the growth rate  $g_2$  in period 1. When the pecuniary externalities are large (a higher probability  $p$ , or a tighter collateral constraint  $\phi$ ), it is beneficial for the macroprudential social planner to intervene, which is reflected in a higher average growth than that in competitive equilibrium (See Figure 4). Furthermore, this policy trade-off does not seem to depend on other parameters, such as  $\psi$ .

Interestingly, one can see that the multi-instrument social planner seems to ignore this policy trade-off: she could choose a lower volatility of growth and higher average growth. Furthermore, it is robust to the parameter values. The reason is that the social planner has less precautionary motive in period 1 and thus has a higher growth in period 1,  $g_2$ . Furthermore, she can mitigate the cost of bad shock in period 2 by raising consumption and thus the growth rate in period 2,  $g_3$ . As a result, she is not binded by this policy trade-off.

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<sup>15</sup>To see how other parameter values affect the trade-off, we change the values of relevant parameters, and plot the growth statistics in Figure 4. The initial wealth is fixed at  $w_1 = 0.6891$ .

Figure 4: Policy trade-off: unconstrained steady state



Note: red, green and blue lines represent statistics for CE, MP and SP.

## 5.2 Constrained Steady State

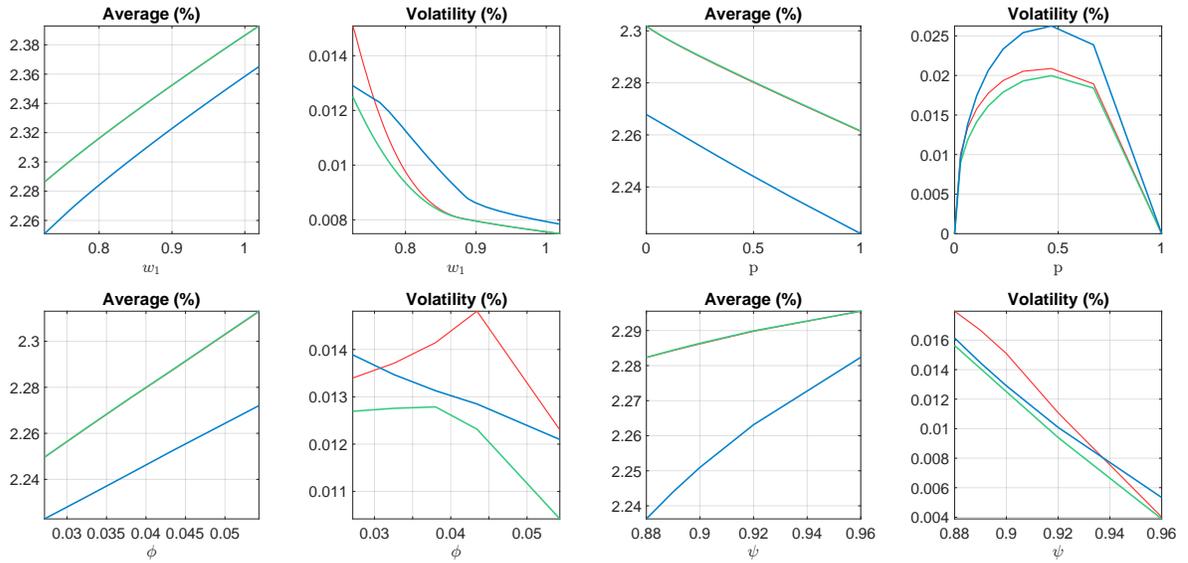
We also consider a set of parameters such that the economy is constrained from period 3 onwards. Specifically, we choose  $\beta = 0.98$  in our benchmark parameter such that  $\beta(1+r)g_{ss}^{-\gamma} < 1$ . Other parameters are the same as that in Table 1. The policy trade-off between average growth and volatility of growth is similar as before. But the impact of introducing macroprudential social planner on average growth is smaller since in the long run the economy is constrained anyway and private agents have a higher precautionary motive than that in the case where the steady state is unconstrained. Figure 5 plots the average and volatility of growth over the relevant variables as that in Figure 4. One can see that macroprudential social planner can marginally increase the average growth and reduce the volatility of growth, which implies that the policy trade-off vanishes.

For the multi-instrument social planner, we find that average growth decreases and volatility of growth increases. To understand the source of welfare gains for multi-instrument social planner to intervene, we plot the impulse response functions for a given initial wealth  $w_1 = 0.6890$ . One can see that multi-instrument social planner can generate

a boom in consumption and growth in the first period. Given that she borrows more to finance consumption and growth, the economy ends up with more debt in the long run, which leads to a reduction in the long run growth. The economy benefits from this intervention since the economy has experienced a temporary burst in growth and permanent increase in consumption (See Figure 6). As a cost, the average growth is reduced and volatility of growth increases.

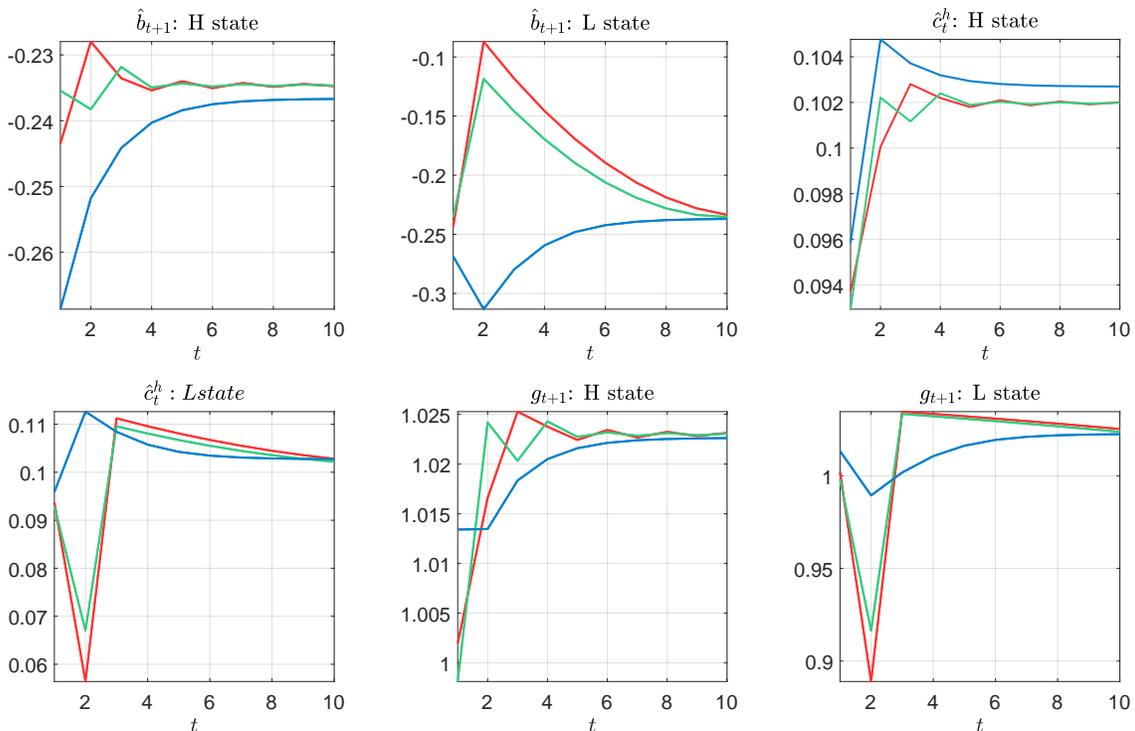
In sum, we find that there is a policy trade-off for the macroprudential social planner, which depends on the size of externalities and whether the economy is constrained or not in the long run. The multi-instrument social planner, however, does not face such trade-off.

Figure 5: Policy trade-off: constrained steady state



*Note:* red, green and blue lines represent statistics for CE, MP and SP.

Figure 6: Impulse response functions: constrained steady state



Note: red, green and blue lines represent statistics for CE, MP and SP. H state refers to the case where  $\theta_2 = 1$  and L state refers to the case where  $\theta_2 = 0.9$ .

## 6 Quantitative Analysis

### 6.1 Calibration

We calibrate our model to annual frequency using 55 countries' data between 1961 and 2015 (See Appendix A for details). The model can be solved using a variant of endogenous gridpoint method as in Carroll (2006) (See Appendix C for details). There is only one shock in the economy, exogenous technology shock  $\theta_t$ , which follows an AR(1) as follows. We discretize the process using Rouwenhorst method as in Kopecky and Suen (2010).

$$\log \theta_t = \rho \log \theta_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t \sim N(0, \sigma^2)$$

We need to assign values to 11 parameters in the model:  $\{\beta, r, \gamma, h, \psi, \eta, \kappa, \alpha, \rho, \sigma, \phi\}$ . The calibration proceeds in two steps. First, some parameter values are taken from the literature. For example, we choose the interest rate  $r$  to be 6 % and the coefficient of risk aversion parameter  $\gamma$  to be 2. For the parameter  $\alpha$ , it equals the share of productive asset income in total income and we choose the value to be 0.2 following Jeanne and Korinek (2010b). For parameter  $\eta$ , we choose  $\eta = 2$  so that cost function is quadratic. Second, given these parameter values, we jointly choose the remaining parameters to match relevant moments as in the data. In particular, we want to match the growth process in the sudden stop episodes as in Figure 3.

The remaining parameters are  $\{\beta, h, \psi, \kappa, \rho, \sigma, \phi\}$ . In the model, the parameter  $\beta$  decides the incentive to borrow and is chosen to match the long run Net Foreign Asset (NFA) to GDP ratio (-30%). The parameter  $\phi$  decides the maximum value of borrowing in the economy and thus the probability of binding constraint.<sup>16</sup> In the model, we define crises episodes as the periods when constraint binds and magnitude of current account reversal exceeds 1 standard deviation of its long run average (see Bianchi, 2011). The parameter  $\phi$  is chosen to match the probability of crisis at 5.5 %, a standard value in the literature (See Bianchi (2011) and Eichengreen et al. (2008)). The parameter  $h$  and  $\kappa$  are jointly chosen to match the average growth rate, 2.3 %, and the share of consumption in GDP, 77.6 %. Specifically,  $h$  and  $\kappa$  have to satisfy the deflated resource constraint, (10) and Euler equation of  $z_{t+1}$ , (12) as follows:

$$\underbrace{\hat{c}_{ss}}_{77.6\%} + \hat{\Psi}(\underbrace{g_{ss}}_{1+2.3\%}) = 1 + \frac{1+r-g_{ss}}{g_{ss}} \underbrace{\hat{b}_{ss}g_{ss}}_{-30\%}$$

$$\Psi_1(g_{ss}) = \beta g_{ss}^{-\gamma} (1 - h - \Psi_2(g_{ss}))$$

---

<sup>16</sup>We calibrate our model such that the collateral constraint marginally binds in the long run, and we have the following relationship in the steady states.

$$\underbrace{-\hat{b}g_{ss}}_{30\%} = \phi \hat{q}$$

$$\hat{q} = \frac{\beta g_{ss}^{1-\gamma}}{1 - \beta g_{ss}^{1-\gamma}} \alpha$$

where the average value of  $\theta$  is normalized at 1, and the value of  $h$  and  $\kappa$  depend on the value of  $\beta$  and  $\psi$ .<sup>17</sup>

We are interested in the growth dynamics in the sudden stop event analysis as in Figure 3.<sup>18</sup> The volatility of exogenous shock,  $\sigma$  governs the minimum level of exogenous shock  $\theta_t$ . Given that  $\theta_t$  is the only shock in our model and it triggers sudden stop episodes, growth in time of sudden stop mainly reflects the drop in  $\theta_t$ . The parameter  $\psi$  decides the minimum level of endogenous growth rate,  $g_{t+1}$ , and thus the fall in growth rate one year after sudden stop. Therefore, it is natural to use  $\sigma$  and  $\psi$  to jointly match the growth rate in time of sudden stop and one year after. Parameter  $\rho$  is chosen to match the correlation of current account and output at -0.25 since we focus on the relationship between capital flows and growth.<sup>19</sup> In sum, given the values of  $\{r, \gamma, \alpha, \eta\}$ , we pick values of  $\{\beta, \psi, \rho, \sigma\}$ , which decides the value of  $\{\phi, \kappa, h\}$ , simulate the model, calculate the moments of simulated data and compare them with the data and event windows.<sup>20</sup> The values of all parameters are reported in Table 2.

Table 2: Calibration

	Value	Source/target
Share of productive asset income	$\alpha=0.2$	Jeanne and Korinek (2010b)
Risk-free interest rate	$r = 6\%$	Standard in the literature
Risk Aversion	$\gamma = 2$	Standard in the literature
Parameters in $\Psi$ functions	$\eta = 2$	Quadratic Form
Volatility of technology shock	$\sigma = 0.04$	Growth rate in the year of sudden stop episodes= -5.65 %
Parameters in $\Psi$ functions	$\psi = 0.95$	Growth rate in the year after sudden stop episodes= 3.28 %
Parameters in $\Psi$ functions	$\kappa = 26.29$	Share of consumption= 77.6 %
Subsistence level parameter	$h = 0.51$	Average growth= 2.3 %
Discount rate	$\beta = 0.968$	Probability of crisis= 5.5 %
Persistence of technology shock	$\rho = 0.83$	Correlation between current account and output= -0.25
Collateral constraint parameter	$\phi = 0.0852$	Average NFA-GDP ratio= -30 %

<sup>17</sup>Here we calibrate our economy so that in the long run the economy is unconstrained and the collateral constraint marginally binds.

<sup>18</sup>We focus on the event window in Calvo's episodes since it displays a larger drop in output than that in "KM" episodes.

<sup>19</sup>In Aguiar and Gopinath (2007), they find that the persistence of shock governs the correlation between current account and output. The correlation is constructed by firstly de-trending the output series with a HP filter and then calculating the correlation between current account to GDP ratio and the cyclical component of output.

<sup>20</sup>In particular, we simulate our model for 11,000 periods and throw away the first 1000 periods. Data moments are calculated based on the remaining 10,000 periods of data.

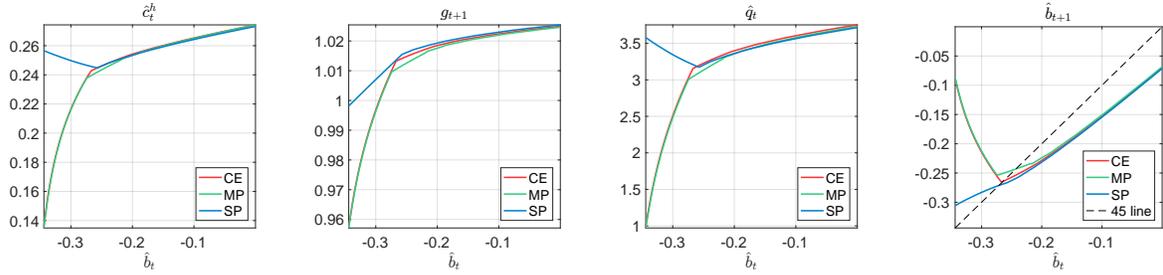
## 6.2 Policy Functions

The differences between the social planners and private agents are captured by the policy functions. In Figure 7, we plot consumption  $\hat{c}_t^h$ , growth  $g_{t+1}$ , asset price  $\hat{q}_t$  and bond holding  $\hat{b}_{t+1}$  for competitive equilibrium (red line), macroprudential social planner (green line) and multi-instrument social planner (blue line) over bond holding  $\hat{b}_t$  when the  $\theta_t$  is 2 standard deviations below its long run average.

There are kinks in all policy functions due to the presence of collateral constraint. We first focus on the difference between competitive equilibrium and the macroprudential social planner. One can see that the macroprudential social planner chooses a higher level of bond than the private agents when constraint is slack. Furthermore, consumption, asset price and growth rate are lower in normal periods. All these behaviors reflect a more precautionary motive and the economy is more resilient to external shocks: the economy becomes binding in a lower level of bond  $\hat{b}_t$  than the competitive equilibrium.

For the multi-instrument social planner, she is able to choose both growth-enhancing expenditure and bond holding. Different from the private agents and macroprudential social planner, she could shift resources from growth-enhancing expenditure to consumption when the collateral constraint binds. This behavior comes at a second-order cost since it distorts the first-order conditions of choosing bond and TFP. However, there is a first-order gain because it increases asset price  $\hat{q}_t$  and thus relaxes the collateral constraint. As a result, the social planner can borrow more even in time of crisis and crisis is not as costly as that in competitive equilibrium: one can see that consumption  $\hat{c}_t^h$ , growth  $g_{t+1}$  and asset price  $\hat{q}_t$  are much higher than that in competitive equilibrium. The multi-instrument social planner's allocation in crisis also has implications for the allocation in normal periods: she actually chooses less bond holdings than the private agent, and the economy becomes binding with a higher level of bond. In other words, the economy ends up with more financial instability when the social planner has two instruments. This is simply because the crisis is not costly and the social planner strikes a new balance between impatience and precautionary motive. Less precautionary motive actually leads to a higher consumption and growth in normal periods.

Figure 7: Policy functions when  $\theta_t$  is 2 standard deviations lower than average



### 6.3 Event Windows

Our quantitative model is able to match growth dynamics in sudden stop episodes. To see this, we plot 11-periods event windows in Figure 8. These event-windows are constructed by simulating our model, identifying sudden stop episodes and calculating the average of variables across different episodes. In the simulated data, sudden stop episodes are identified as periods when the collateral constraint binds and magnitude of current account reversal exceeds 1 standard deviation of its long run average. Not surprisingly, these episodes are periods when the exogenous TFP component  $\theta_t$  declines and the collateral constraint binds ( $\hat{\mu}_t > 0$ ). Current account experiences a large reversal because the economy hits the borrowing limit and has to cut external liability, i.e. an increase in  $\hat{b}_{t+1}$ . Furthermore, these events are accompanied by a decline in economic activities such as consumption  $\hat{c}_t$  and growth-enhancing expenditure (reflected in a decline in growth rate  $g_{t+1}$ ). Asset price  $\hat{q}_t$  drops with consumption and growth rate, which leads to the amplification effect through collateral constraint. As one can see, our model can capture the empirical regularity of sudden stop episodes. In particular, it captures the dynamics of growth rate as in the data: growth rate in time of crises falls with a decline in  $\theta_t$  and it does not bounce back more to compensate the loss of output due to a fall in endogenous growth  $g_{t+1}$  when the large negative shock hits the economy.

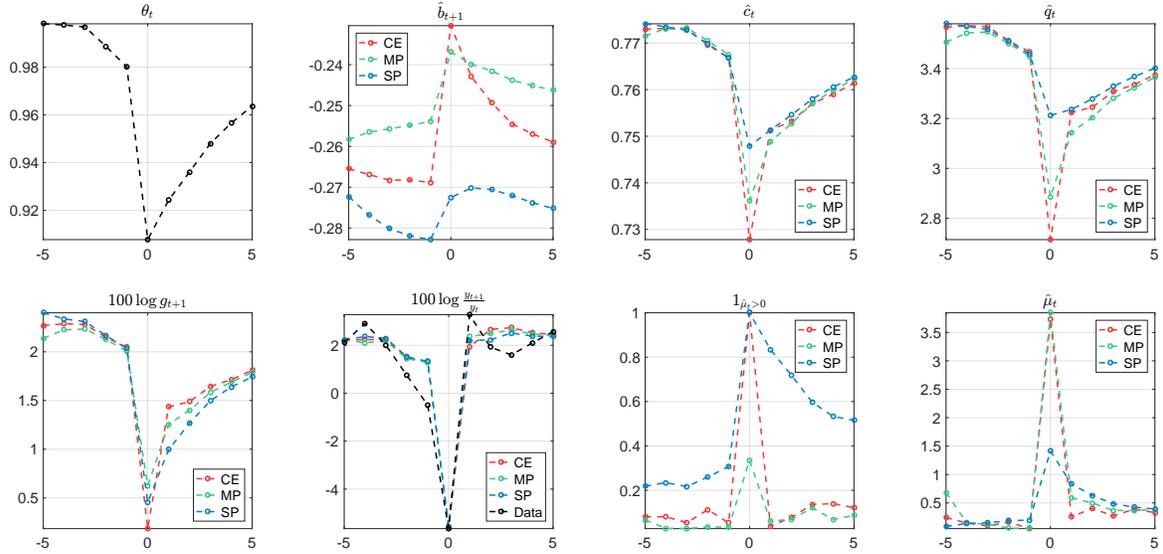
To see the difference between the social planners and competitive equilibrium, we also report their event windows. They are constructed by fixing the same initial bond position

$\hat{b}_t$  and exogenous shock  $\theta_t$  for each episode as in the competitive equilibrium, and deriving the dynamics of relevant variables with the social planners' policy functions. For the macroprudential social planner, one can see her precautionary motive: she chooses more bond  $\hat{b}_{t+1}$  than the private agents 5 periods before the crisis. As a result, when the crisis actually happens, the economy suffers less than the private agents: the declines in consumption  $\hat{c}_t$ , asset price  $\hat{q}_t$  and growth rate  $g_{t+1}$  are smaller. Furthermore, the frequency of crises has been reduced. However, the intervention comes at a cost of constraining economic activities in normal periods: growth and consumption are both reduced.

For the multi-instrument social planner, she behaves differently than the macroprudential social planner and private agents. As we explained before, she is able to choose allocations in time of crises and thus reduces the cost of crises, which is reflected in a much smaller value of Lagrangian multiplier  $\hat{\mu}_t$  for the multi-instrument social planner. Even if she borrows more before crises, she does not have to cut external borrowing and total expenditure as much as private agents when the bad shock is realized. Therefore, the social planner actually ends up borrow more than the private agents and the economy hits the collateral constraint more frequently.

**Growth dynamics** Once introducing the macroprudential social planner, growth rate is reduced due to the precautionary motive, which results in a smaller fall of endogenous growth in time of crisis. For the multi-instrument social planner, growth rate is higher since the social planner has less precautionary motive and can borrow more. When the large bad shock is materialized, she can mitigate the crisis by shifting resources towards consumption. As a result, growth rate does not fall as much as that in competitive equilibrium.

Figure 8: Event windows



## 6.4 Growth and Stability

We report the model moments in Table 3. As one can see, the model is able to match the relevant moments as in the data. In particular, we are able to match the important statistics for growth and probability of crisis. Given that our model is about the role of macroprudential capital controls, we think it is also crucial to match the net foreign asset to GDP ratio and correlation between current account and GDP.

Table 3: Moments

Moments	Data	CE	MP	SP
Average GDP growth (%)	2.30	2.31	2.31	2.29
Prob. of crisis (%)	5.50	6.23	1.89	14.23
NFA-GDP ratio (%)	-30.00	-27.18	-25.78	-28.98
Consumption-GDP ratio (%)	77.6	77.53	77.65	77.58
Correlation between current account and GDP	-0.25	-0.22	-0.37	-0.54

For the macroprudential social planner, one can see that there is a trade-off between growth and stability. Consistent with the event-window analysis, the social planner chooses a higher level of bond for a more precautionary motive and thus reduces the

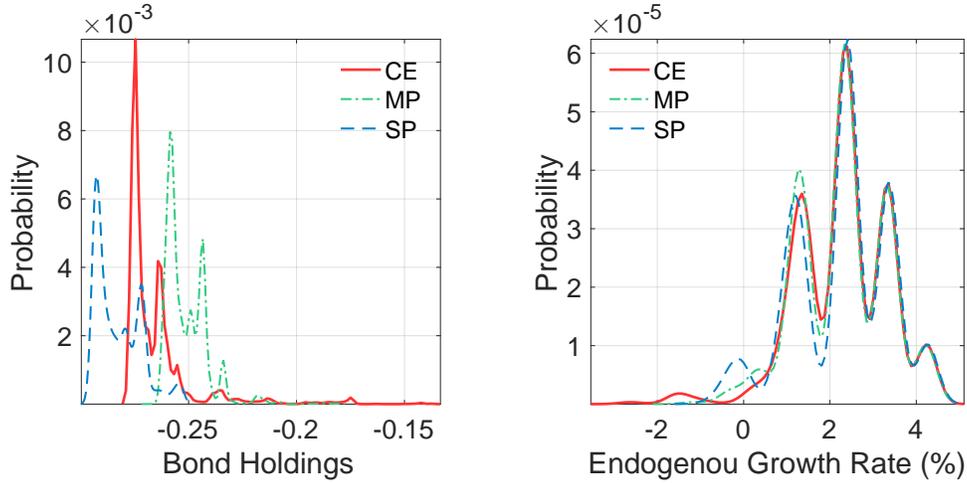
probability of crises in the model. The net foreign asset to GDP ratio increases from -27.18 % to -25.78 % and the probability of crisis decreases from 6.23 % to 1.89 %. In terms of growth, the average growth is marginally reduced: the difference is very small and could not be detected in the two decimal digits. We find the reduction in growth is at the order of 0.01 %.

For the multi-instrument social planner, she could ignore the policy trade-off. Given that the crisis is not costly, she actually borrows more than private agents ex-ante, which leads to a higher probability of crises ex-post. Average growth is reduced since the economy borrows more and resources are used for debt service. Furthermore, the economy hits borrowing constraint more frequently: when it happens, the social planner shifts resources from growth-enhancing expenditure towards consumption to mitigate the crisis. This is optimal behavior since agents are very impatient and the social planner strikes a new balance between impatience and precautionary motive.

The other way to view the difference is to compare ergodic distributions of bond holding and growth rate in Figure 9. Clearly, the macroprudential social planner has more precautionary motive and chooses a higher bond holdings while the multi-instrument social planner is able to borrow more since she can relax the collateral constraint in time of crises. In terms of the ergodic distribution for growth  $g_{t+1}$ , both social planners have less mass in the extremely low growth than the competitive equilibrium. But they achieve this by choosing a lower mass in normal growth. For the multi-instrument social planner, since she chooses a lower level of bond ex-ante and ends up with more constrained states, she has more mass in the lower growth region than the macroprudential social planner. As a result, the growth rate for the multi-instrument social planner ends up even lower than the macroprudential social planner.

**Average and volatility of growth** The other way to look at the trade-off between stability and growth is to see average and volatility of growth since volatility mostly comes from financial instability in our model. To see the impact of introducing two social planners on growth rate, we run 1,000 simulations for 1,100 periods using policy functions

Figure 9: Ergodic Distribution



in competitive equilibrium. The initial 1,000 periods are dropped in each sample and we introduce two social planners starting from period 1,001. The average and volatility of growth<sup>21</sup> can be calculated for competitive equilibrium and two social planners, and the results are reported in Table 4. Consistent with the insight from one-shock model, one can see that there is a trade-off between average and volatility of growth for the macroprudential social planner. Both average and volatility of growth are reduced by 0.01 %. However, the multi-instrument social planner ignores the trade-off: average growth is reduced by 0.02 % but volatility is increased by 0.02 %.

Table 4: Growth statistics (%)

	Average	Volatility
CE	2.33	0.34
MP	2.32	0.33
SP	2.31	0.36

<sup>21</sup>Average and volatility of growth are calculated in a similar way as in footnote 14.

## 6.5 Welfare Impact

Social planners have an incentive to intervene the market since it generates considerable welfare gains. To quantify the magnitude of welfare gains, we define the gains of policy interventions as follows:

$$\Delta^i(\hat{b}_t, \theta_t) = \left( \frac{\hat{V}^i(\hat{b}_t, \theta_t)}{\hat{V}^{CE}(\hat{b}_t, \theta_t)} \right)^{\frac{1}{1-\gamma}-1}, \text{ where } i \in \{MP, SP\} \quad (23)$$

where  $\hat{V}^i(\hat{b}_t, \theta_t)$  are deflated functions for competitive equilibrium ( $i = CE$ ), macroprudential social planner ( $i = MP$ ) and multi-instrument social planner ( $i = SP$ ).

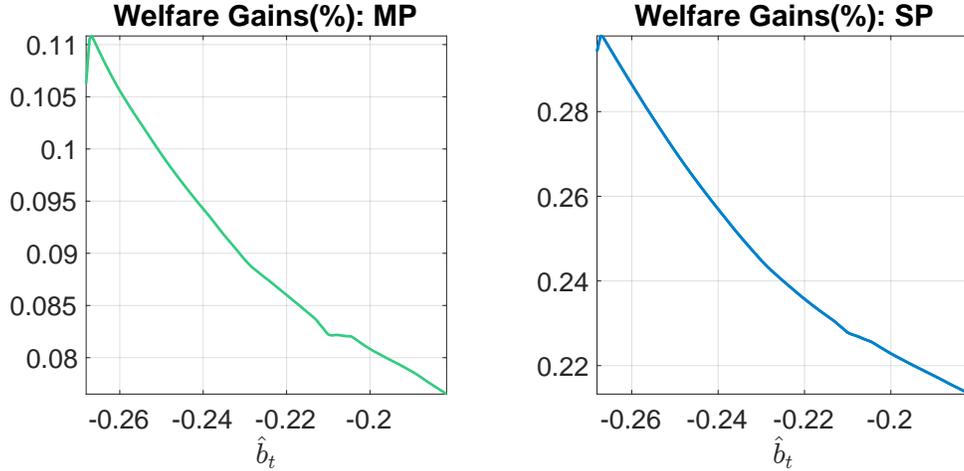
Given the definition in (23),  $\Delta^{MP}(\hat{b}_t, \theta_t)$  and  $\Delta^{SP}(\hat{b}_t, \theta_t)$  are conditional welfare gains and depend on the initial value of bond holding  $\hat{b}_t$  and exogenous shock  $\theta_t$ . We can also define unconditional welfare gains as follows:

$$E[\Delta^i(\hat{b}_t, \theta_t)], \text{ where } i \in \{MP, SP\} \quad (24)$$

where the expectation is taken using the ergodic distribution of competitive equilibrium.

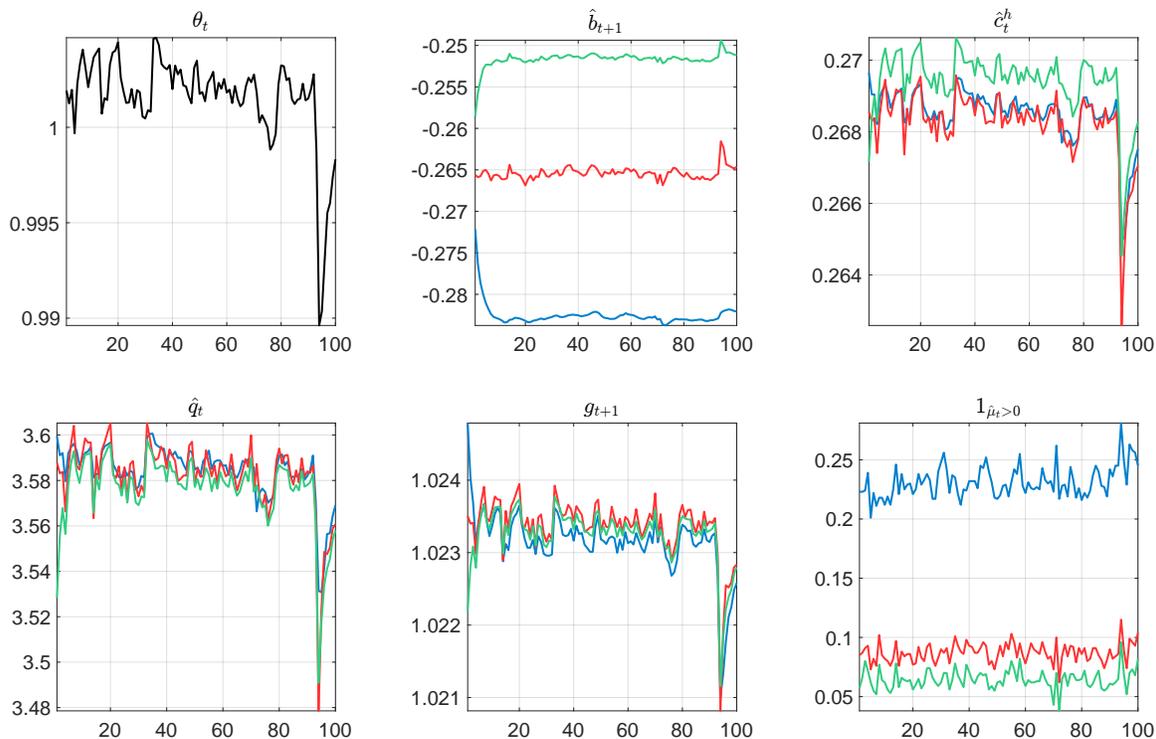
Figure 10 plots the conditional welfare gains of two social planners over the bond space when the shock is 2 standard deviations below its long run average. Consistent with the literature, the gains peak at the region where the size of externalities is maximum. The gains become smaller when the economy has a higher amount of bond holding since the probability of crisis is lower. The gains also become smaller when the economy has a lower amount of bond holding because the macroprudential social planner can do nothing when the constraint binds. In terms of the size of gains, we find that the unconditional welfare gains for the macroprudential social planner and multi-instrument social planner are around 0.07 % and 0.23 % respectively. In the literature such as Bianchi (2011), the gains are at the order of 0.1 %, which is in the same range as our macroprudential social planner. For the multi-instrument social planner, the gains are much larger since the social planner can intervene ex-post and thus reduce the cost of financial crises.

Figure 10: welfare gains (%)



**Source of Welfare Gains** To further understand the source of welfare gains, we run the 1,000 simulations as before and plot the impulse response functions of consumption  $\hat{c}_t^h$ , growth rate  $g_{t+1}$  and other relevant variables in Figure 11. One can see that introducing the macroprudential social planner reduces growth  $g_{t+1}$  and consumption  $\hat{c}_t^h$  for precautionary motive, which is reflected in a larger bond holding  $\hat{b}_{t+1}$  and less probability of crisis in the long run. The gains for the macroprudential social planner come from an increasing financial stability and a larger consumption in the long run. As a cost, the growth is reduced both in the short run and long run. For the multi-instrument social planner, the source of gains come from a different channel. On impact, growth and consumption increases since the planner is able to borrow more for less precautionary motives. Consumption stays at a higher level in the long run while growth converges to a lower level in the long run. On average, it takes around 18 years for the growth spurt periods to end. There are two reasons for the growth rate to converge to a lower level. First, the economy experiences a temporary boom in the short run by borrowing more. In the long run, the economy has to serve a higher level of debt and thus reduces growth in the long run. Second, the economy ends up with more binding constraint periods, and the social planner cuts growth expenditure to smooth the consumption in such episodes.

Figure 11: Simulations



Note: red, green and blue lines represent variables for CE, MP and SP.

In sum, the source of welfare gains come from the consumption channel for both social planners since the economy has a higher and smoother consumption in the long run. One can also see this point from the average consumption expenditures in Table 3. Differently, the multi-instrument social planner can generate a larger welfare gains since she is also able to generate a boom of growth and consumption in the short run. To strengthen this point, we decompose the welfare gains in the following procedure. First, for each simulation, we can construct series  $m_{t+1}^i = m_t^i g_{t+1}^i$  and  $c_t^i = \hat{c}_t^i m_t$ , where  $i \in \{CE, MP, SP\}$ . We can then approximate the welfare by summing up the discounted utility, i.e.  $V^i = \sum_{t=0}^{100} \beta^t \frac{(c_t^i)^{1-\gamma}}{1-\gamma}$ . Second, to control for consumption (growth) effects, we construct a different consumption series using  $\hat{c}_t$  (or  $m_{t+1}$ ) for the competitive equilibrium and  $m_{t+1}$  (or  $\hat{c}_t$ ) for the social planners. Based on this new consumption series, we can calculate welfare gains using equation (23).

The overall welfare gains can be decomposed into two channels: consumption and growth. The results are presented in Table 5. One can see that the source of gains are from the consumption channel: using the deflated consumption series in competitive equilibrium generates a welfare loss since growth has been reduced for both social planners. For the macroprudential social planner, she has a even larger welfare loss since the growth has been lowered in both short run and long run. For the multi-instrument social planner, the loss is lower since she is able to raise growth in the short run that is only reversed in around 18 years.

Table 5: Source of welfare gains (%)

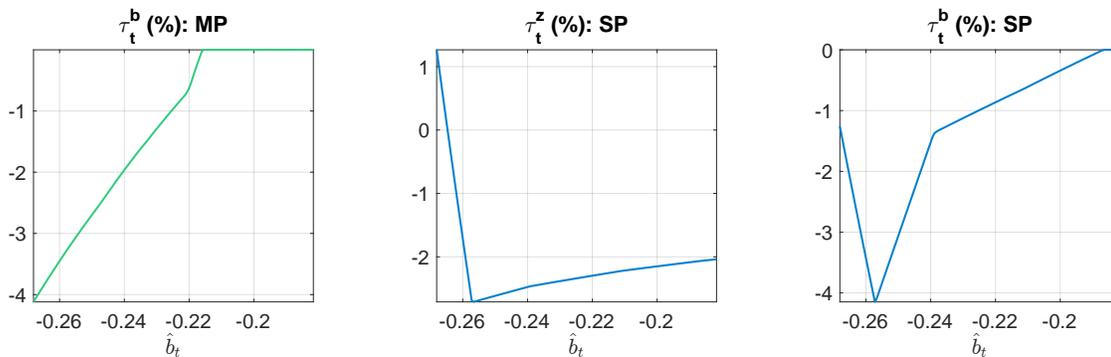
	Overall	Control for consumption	Control for growth
MP	0.06	-0.34	0.40
SP	0.25	-0.13	0.38

## 6.6 Policy Interventions: Ex-ante versus Ex-post

To implement the social planners' allocation, we consider tax/subsidy on growth-enhancing expenditure and bond holdings, and lump-sum transfers. Interestingly, the two social planners correspond to policymakers who can intervene ex-ante only and those who can intervene both ex-ante and ex-post. In Figure 12, we report the tax rate over the bond space when the shock is 2 standard deviations below its long run average.

One can see that for the macroprudential social planner, she cannot intervene ex-post but only subsidize bond holdings ex-ante: tax rate are at the same order of the literature as in Figure 12. On average, it is -1.28 % in the ergodic distribution, meaning the social planner wants to impose a 1.28 % capital control taxes on external borrowings. For the multi-instrument social planner, given that she could intervene ex-post, ex-ante she imposes less restrictions on borrowing. Furthermore, she subsidizes growth in normal periods as show in Figure 12. On average, we find that she imposes 1.00 % capital control taxes and 1.00 % subsidies on growth-enhancing expenditure. In our baseline results, the existence of ex-post intervention reduces the magnitude of ex-ante intervention (see Jeanne and Korinek, 2013).

Figure 12: Taxes



## 6.7 Sensitivity Analysis

We conduct sensitive analysis for different parameters in our model. Similar to our baseline calibration, we firstly give values for 8 parameters, i.e.  $\{\beta, \psi, r, \gamma, \alpha, \eta, \rho, \sigma\}$ : we only change the value of our interested parameter while keep other parameter values the same as in baseline calibration. Given these values, we choose  $\{\kappa, h, \phi\}$  to match average growth, consumption share in GDP ratio and NFA-GDP ratio. If we are interested in the sensitivity of  $\phi$ , we change its value afterwards. We follow this strategy since we want our model to match average growth, which is affected by consumption share in GDP and NFA-GDP ratio. The sensitivity analysis results are presented in Table 6 and we discuss the robustness of our results with respect to parameters. One can see that our results do not change with  $\alpha$  since in our calibration we assume that collateral constraint binds in steady state and  $\phi$  changes with  $\alpha$ .

**Welfare Impact** Our results on welfare gains are robust to various of parameters. In particular, we find that the macroprudential social planner can generate welfare gains equivalent to 0.1 % permanent increase in consumption, while the multi-instrument social planner can generate larger gains, equivalent to 0.2 % permanent increase in consumption. In particular, the size of gains increases with parameters that affect the size of externalities, such as  $\phi$ . The gains also increase with parameters that make growth more sensitive to shocks, such as  $\{\psi, \gamma, \eta\}$ . Given that the social planners smooth the economy, welfare

Table 6: Sensitivity Analysis

	Welfare Gains (%)		Taxes on Debt (%)		Subsidies on Growth (%)	Prob of Crisis (%)			Average Growth			Volatility of Growth			Reverse in Growth
	MP	SP	MP	SP	SP	CE	MP	SP	CE	MP	SP	CE	MP	SP	SP
baseline	0.07	0.23	1.28	1.00	1.00	6.23	1.89	14.23	2.33	2.32	2.31	0.34	0.33	0.36	18
$\beta = 0.93$	0.01	0.08	1.51	0.76	-1.40	13.24	12.39	13.84	2.33	2.33	2.21	0.26	0.26	0.29	2
$\beta = 0.95$	0.03	0.17	1.67	1.17	-0.18	10.83	9.52	13.66	2.33	2.33	2.26	0.30	0.30	0.33	3
$\psi = 0.94$	0.13	0.34	1.65	1.30	1.49	2.86	1.89	15.33	2.34	2.33	2.32	0.41	0.41	0.44	23
$\psi = 0.96$	0.03	0.14	1.04	0.90	0.54	7.32	2.06	12.36	2.32	2.32	2.30	0.27	0.27	0.29	13
$\phi = 0.07$	0.01	0.09	0.81	0.80	0.35	7.27	6.66	11.56	2.32	2.32	2.31	0.33	0.33	0.35	11
$\phi = 0.08$	0.03	0.14	0.94	0.93	0.60	7.43	2.34	12.96	2.33	2.32	2.31	0.33	0.33	0.36	15
$r = 3\%$	0.12	0.44	2.59	2.12	1.26	7.84	6.26	14.29	2.35	2.33	2.26	0.34	0.34	0.37	5
$r = 4\%$	0.11	0.35	1.92	1.23	0.80	7.35	2.49	16.18	2.34	2.32	2.28	0.34	0.34	0.37	7
$\gamma = 3$	0.08	0.33	2.38	2.43	0.64	10.49	7.00	15.33	2.38	2.36	2.27	0.31	0.31	0.34	5
$\gamma = 4$	0.06	0.35	3.03	3.90	-0.65	12.12	10.65	16.24	2.41	2.39	2.17	0.28	0.28	0.32	4
$\alpha = 0.3$	0.07	0.23	1.28	1.12	1.00	6.23	1.89	14.23	2.33	2.32	2.31	0.34	0.34	0.36	17
$\alpha = 0.4$	0.07	0.23	1.28	1.12	1.00	6.23	1.89	14.23	2.33	2.32	2.31	0.34	0.34	0.36	17
$\eta = 3$	0.06	0.15	1.29	0.71	0.36	3.41	1.86	4.52	2.31	2.30	2.29	0.19	0.18	0.20	6
$\eta = 4$	0.02	0.07	1.01	0.46	-0.01	5.14	1.84	1.92	2.30	2.30	2.29	0.12	0.12	0.13	4
$\rho = 0.80$	0.07	0.23	1.37	1.09	0.84	5.93	2.22	15.12	2.33	2.32	2.31	0.29	0.28	0.31	15
$\rho = 0.90$	0.04	0.22	1.35	0.09	0.19	4.72	2.22	12.66	2.35	2.34	2.33	0.51	0.51	0.52	22
$\sigma = 0.02$	0.02	0.09	0.93	0.96	0.40	10.91	8.29	13.95	2.30	2.30	2.27	0.17	0.17	0.18	5
$\sigma = 0.03$	0.04	0.16	1.22	1.18	0.87	7.38	6.75	15.72	2.32	2.31	2.28	0.25	0.25	0.27	9

*Note:* Welfare gains and taxes on debt are calculated by simulating the economy for 10,000 periods. Probability of crisis is defined as periods when the collateral constraint binds and the current account reversal exceeds 1 standard deviation of its long run average. Average growth is calculated using 1,000 simulations of 100 periods.

gains also increase with parameters that govern the risks, such as  $\{\rho, \sigma\}$ .<sup>22</sup> The welfare gains increase with discount rate  $\beta$  and decrease with interest rate  $r$  since they decide private agents' impatience condition given by  $\beta(1+r)g^{-\gamma}$ . Intuitively, when agents are more impatient, i.e. a lower  $\beta$  or  $r$ , the economy borrows more and ends up with more crises. Policy interventions should have more benefits since they mitigate the frequency and severity of crises. Indeed, we find a larger gains with a lower interest rate. However, welfare gains increase with  $\beta$ . This is because  $\beta$  also decides the Euler equation of TFP and the volatility of growth increases with  $\beta$ . Policy interventions can reduce the volatility of growth and thus have more benefits.

**Size of Interventions** In our baseline results, we find that the macroprudential social planner imposes 1.28 % capital control taxes while the multi-instrument social planner imposes 1.00 % capital control taxes and 1.00 % subsidy on the growth-enhancing expenditure. The size of capital control taxes are robust to various parameter values and change with the size of externalities and ergodic distribution of debt. In particular, we

<sup>22</sup>Here, lower  $\rho$  implies a higher risk for the economy since it is more likely to enter into a bad state tomorrow conditional on a good state today.

find that the multi-instrument social planner might also want to tax the growth when she optimally chooses to borrow more and thus has to mitigate the crisis by taxing the growth. Furthermore, the results that the existence of ex-post intervention reduces the size of ex-ante intervention are robust to many parameter other than  $\gamma$ .

**Impact on growth** The policy trade-off between average and volatility of growth for the macroprudential social planner is very robust to all the parameter values. However, the difference between two allocations is typically small and does not show up in the two decimal digits. For the multi-instrument social planner, she ignores this trade-off and could generate a short run growth spurt. In our baseline results, the duration of growth spurt lasts for 18 years. We find that this number varies with different parameter values. Generally speaking, it increases the size of gains that the multi-instrument social planner can generate. Exceptions include the parameter values  $\gamma$  and  $\rho$  when the number decreases with the gains.

## 7 Conclusion

In this paper, we introduce endogenous growth into a SOE-DSGE framework with occasionally binding constraint to capture the fact that sudden stop episodes are accompanied by a large temporary fall in economic growth. Importantly, growth rate in our model only returns to its long run average level after the sudden stop episodes, consistent with the empirical fact. We derive optimal policy interventions in this framework and find that macroprudential policy faces a trade-off between stability and growth in our baseline calibration. The trade-off is also related to the average and volatility of growth, a long standing policy trade-off identified in the literature. In a one-shock model, we find that this trade-off depends on parameters that affect the size of externalities in the economy and whether the economy is constrained in the long run. Quantitative analysis suggests that a macroprudential social planner faces this trade-off: she reduces the probability of crisis at a cost of marginally reducing average growth. To implement her allocation, we find that policymakers need to impose 1.28 % of capital control tax on average. Such

intervention can generate some benefits since the economy is smoother and consumption increases in the long run. Quantitative analysis suggests that the welfare gains are at the order of 0.1 % permanent increase in consumption.

We also define a social planner who has an additional policy instrument other than the capital control tax. More instruments enable the social planner to bypass the policy trade-off in growth and stability since she could mitigate the cost of crisis by changing consumption and growth even when the collateral constraint binds. On average, we find that policymakers impose 1.00 % capital control tax and 1.00 % subsidy on growth. This result is related to the discussion on ex-ante and ex-post policy intervention (Benigno et al. (2013) and Jeanne and Korinek (2013)). In particular, we also find that the existence of ex-post intervention reduces the size of ex-ante intervention. Furthermore, we find that the benefits of intervention are larger than the macroprudential social planner since the multi-instrument social planner could also generate a short run boom in growth and consumption. Quantitative results suggest that the short run spurt in growth is only reversed in 18 years and consumption boom lasts in the long run. Equivalently, the benefit is around 0.2 % increase in permanent consumption.

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## A Data Source

Our sample includes the following 55 countries:

Algeria	Argentina	Australia	Austria	Belgium
Brazil	Canada	Chile	China	Colombia
Cote d'Ivoire	Croatia	Czech Republic	Denmark	Dominican Republic
Ecuador	Egypt, Arab Rep.	El Salvador	Finland	France
Germany	Greece	Hungary	Iceland	Indonesia
Ireland	Italy	Japan	Korea, Rep.	Lebanon
Malaysia	Mexico	Morocco	Netherlands	New Zealand
Nigeria	Norway	Pakistan	Panama	Peru
Philippines	Poland	Portugal	Russian Federation	South Africa
Spain	Sweden	Thailand	Tunisia	Turkey
Ukraine	United Kingdom	United States	Uruguay	Venezuela, RB

The sources are as follows:

**GDP per capita growth:** the data is from World Development Indicators(WDI) calculated using GDP per capita for constant local currency;

**TFP:** the data is from Pen World Table;

**Consumption share of GDP:** calculated using final consumption expenditure and GDP data in WDI;

**Net Foreign Asset to GDP ratio:** the data is taken from an updated dataset in Lane and Milesi-Ferretti (2007)(See <http://www.philiplane.org/EWN.html>).

## B Deflated Competitive Equilibrium

We deflate our economy by the endogenous variable  $z_t$  and denote the deflated variables by hat. The deflated competitive equilibrium conditions are given by

$$\begin{aligned}
 (\hat{c}_t^h)^{-\gamma} \Psi_{1,t} &= \beta g_{t+1}^{-\gamma} E_t \left[ (\hat{c}_{t+1}^h)^{-\gamma} (\theta_{t+1} - h - \Psi_{2,t+1}) \right] \\
 (\hat{c}_t^h)^{-\gamma} \hat{q}_t &= \beta g_{t+1}^{1-\gamma} E_t \left[ (\hat{c}_{t+1}^h)^{-\gamma} (\alpha \theta_{t+1} + \hat{q}_{t+1}) \right] \\
 (\hat{c}_t^h)^{-\gamma} &= \hat{\mu}_t^{CE} + \beta g_{t+1}^{-\gamma} (1+r) E_t \left[ (\hat{c}_{t+1}^h)^{-\gamma} \right] \\
 \hat{c}_t^h + \hat{\Psi}(g_{t+1}) + \hat{b}_{t+1} g_{t+1} &= \theta_t - h + (1+r) \hat{b}_t \\
 \hat{\mu}_t^{CE} \left( \hat{b}_{t+1} g_{t+1} + \phi \hat{q}_t \right) &= 0, \text{ with } \hat{\mu}_t^{CE} \geq 0.
 \end{aligned}$$

For the macroprudential social planner, the deflated equilibrium conditions are

$$\begin{aligned}
\hat{\lambda}_t^{MP} &= (\hat{c}_t^h)^{-\gamma} + \frac{\gamma \phi \hat{\mu}_t^{MP} \hat{q}_t}{\hat{c}_t^h} + \gamma \hat{\nu}_t^{MP} (\hat{c}_t^h)^{-\gamma-1} \Psi_{1,t} \\
\hat{\lambda}_t^{MP} \Psi_{1,t} - \frac{\phi \hat{\mu}_t^{MP} g_{t+1}^{-\gamma} \hat{G}_{1,t}}{(\hat{c}_t^h)^{-\gamma}} - \hat{\nu}_t^{MP} \left[ g_{t+1}^{-1-\gamma} \hat{I}_{1,t} - (\hat{c}_t^h)^{-\gamma} \hat{\Psi}_{11,t} \right] \\
&= \beta g_{t+1}^{-\gamma} E_t \left[ \hat{\lambda}_{t+1}^{MP} (\theta_{t+1} - h - \Psi_{2,t+1}) - \hat{\nu}_{t+1}^{MP} (\hat{c}_{t+1}^h)^{-\gamma} \hat{\Psi}_{12,t+1} \right] \\
\hat{\lambda}_t^{MP} &= \hat{\mu}_t^{MP} + \frac{\phi \hat{\mu}_t^{MP} g_{t+1}^{-\gamma} \hat{G}_{2,t}}{(\hat{c}_t^h)^{-\gamma}} + \hat{\nu}_t^{MP} g_{t+1}^{-1-\gamma} \hat{I}_{2,t} + \beta(1+r) g_{t+1}^{-\gamma} E_t \left[ \hat{\lambda}_{t+1}^{MP} \right]
\end{aligned}$$

where

$$\begin{aligned}
I(z_{t+1}, b_{t+1}) &= z_{t+1}^{-\gamma} \hat{I}(\hat{b}_{t+1}), \\
I_{1,t} &= (-\gamma) z_{t+1}^{-\gamma-1} \hat{I}(\hat{b}_{t+1}) + z_{t+1}^{-\gamma} \hat{I}'(\hat{b}_{t+1}) \frac{-b_{t+1}}{z_{t+1}^2} = -z_{t+1}^{-\gamma-1} \left[ \gamma \hat{I} + \hat{I}' b_{t+1} \right], \\
I_{2,t} &= z_{t+1}^{-\gamma-1} \hat{I}'.
\end{aligned}$$

and

$$\begin{aligned}
G(z_{t+1}, b_{t+1}) &= z_{t+1}^{1-\gamma} \hat{G}(\hat{b}_{t+1}), \\
G_{1,t} &= (1-\gamma) z_{t+1}^{-\gamma} \hat{G}(\hat{b}_{t+1}) + z_{t+1}^{1-\gamma} \hat{G}'(\hat{b}_{t+1}) \frac{-b_{t+1}}{z_{t+1}^2} = z_{t+1}^{-\gamma} \left[ (1-\gamma) \hat{G} - \hat{G}' b_{t+1} \right], \\
G_{2,t} &= z_{t+1}^{-\gamma} \hat{G}'.
\end{aligned}$$

For the multi-instrument social planner, the deflated equilibrium conditions are

$$\begin{aligned}
\hat{\lambda}_t^{SP} &= (\hat{c}_t^h)^{-\gamma} + \frac{\gamma \phi \hat{\mu}_t^{SP} \hat{q}_t}{\hat{c}_t^h} \\
\hat{\lambda}_t^{SP} \Psi_{1,t} &= \frac{\phi \hat{\mu}_t^{SP} g_{t+1}^{-\gamma} \hat{G}_{1,t}}{(\hat{c}_t^h)^{-\gamma}} + \beta g_{t+1}^{-\gamma} E_t \left[ \hat{\lambda}_{t+1}^{SP} (\theta_{t+1} - h - \Psi_{2,t+1}) \right] \\
\hat{\lambda}_t^{SP} &= \hat{\mu}_t^{SP} + \frac{\phi \hat{\mu}_t^{SP} g_{t+1}^{-\gamma} \hat{G}_{2,t}}{(\hat{c}_t^h)^{-\gamma}} + \beta(1+r) g_{t+1}^{-\gamma} E_t \left[ \hat{\lambda}_{t+1}^{SP} \right]
\end{aligned}$$

## C Numerical Methods

We first create a grid space  $\mathcal{G}_b = \{\hat{b}^0, \hat{b}^1, \dots\}$  for bond holding  $\hat{b}_t$  and a grid space  $\Theta = \{\theta_1, \dots, \theta_5\}$  for exogenous technology shock  $\theta_t$ . The discretization method for the AR (1) process of  $\theta_t$  follows the Rouwenhorst method as in Kopecky and Suen (2010). We apply endogenous gridpoint method as in Carroll (2006) to iterate the first order conditions in CE, SP and MP. We keep iteration until the policy functions converge. The policy functions that we are interested include: consumption  $\mathcal{C}(\hat{b}_t, \theta_t)$ , growth  $\mathcal{G}(\hat{b}_t, \theta_t)$ , asset price  $\mathcal{Q}(\hat{b}_t, \theta_t)$  and bond holding  $\mathcal{B}(\hat{b}_t, \theta_t)$ . Denote the iteration step by  $j$  and start from arbitrary policy functions  $\mathcal{C}^0(\hat{b}_t, \theta_t)$ ,  $\mathcal{G}^0(\hat{b}_t, \theta_t)$ ,  $\mathcal{Q}^0(\hat{b}_t, \theta_t)$  and  $\mathcal{B}^0(\hat{b}_t, \theta_t)$ , where 0 means the iteration step  $j = 0$ . Given policy functions in iteration step  $j$ , we solve the policy functions as follows:

1. For any  $\theta_t \in \Theta$  and  $\hat{b}_{t+1} \in \mathcal{G}_b$ , we can solve  $\{\hat{c}_t^h, g_{t+1}, \hat{q}_t\}$  using the equilibrium conditions. Using the budget constraint, these allocations imply a unique  $\hat{b}_t$ . Then we have a combination of  $\{\hat{b}_t\}$  and corresponding allocations  $\{\hat{c}_t^h, g_{t+1}, \hat{q}_t, \hat{b}_{t+1}\}$ . We can update the policy functions using these combinations. In this process, we need to deal with the collateral constraint. Specifically, we assume that the constraint is slack and then check whether this condition is satisfied.
2. We firstly assume that the constraint is slack and the allocations  $g_{t+1}, \hat{c}_t^h, \hat{q}_t$  can be solved using the following conditions.

$$\Psi_t(g_{t+1}) = \frac{E_t \left[ \left( \mathcal{C}^j(\hat{b}_{t+1}, \theta_{t+1}) \right)^{-\gamma} \left( \theta_{t+1} - h - \Psi_2 \left( \mathcal{G}^j(\hat{b}_{t+1}, \theta_{t+1}) \right) \right) \right]}{(1+r)E_t \left[ \left( \mathcal{C}^j(\hat{b}_{t+1}, \theta_{t+1}) \right)^{-\gamma} \right]}$$

$$\hat{c}_t^h = g_{t+1} \left[ \beta(1+r)E_t \left[ \left( \mathcal{C}^j(\hat{b}_{t+1}, \theta_{t+1}) \right)^{-\gamma} \right] \right]^{-\frac{1}{\gamma}}$$

$$\hat{q}_t = (\hat{c}_t^h)^\gamma \beta g_{t+1}^{1-\gamma} E_t \left[ \left( \mathcal{C}^j(\hat{b}_{t+1}, \theta_{t+1}) \right)^{-\gamma} (\alpha \theta_{t+1} + \mathcal{Q}(\hat{b}_{t+1}, \theta_{t+1})) \right]$$

3. If the collateral constraint  $-\hat{b}_{t+1}g_{t+1} \leq \phi \hat{q}_t$  is satisfied, we proceed to solve  $\hat{b}_t$  using

budget constraint.

$$\hat{b}_t = \frac{\hat{c}_t^h + h + \hat{\Psi}(g_{t+1}) + \hat{b}_{t+1}g_{t+1} - \theta_t}{1+r}$$

4. If the constraint is violated, we can solve the allocations  $\{\hat{q}_t, \hat{c}_t^h, g_{t+1}\}$  using the following equations.

$$\begin{aligned} (\hat{c}_t^h)^{-\gamma} \Psi_t(g_{t+1}) &= \beta g_{t+1}^{-\gamma} E_t \left[ \left( \mathcal{C}^j(\hat{b}_{t+1}, \theta_{t+1}) \right)^{-\gamma} \left( \theta_{t+1} - h - \Psi_2 \left( \mathcal{G}^j(\hat{b}_{t+1}, \theta_{t+1}) \right) \right) \right] \\ -\hat{b}_{t+1}g_{t+1} &= \phi \hat{q}_t \\ \hat{q}_t &= (\hat{c}_t^h)^\gamma \beta g_{t+1}^{1-\gamma} E_t \left[ \left( \mathcal{C}^j(\hat{b}_{t+1}, \theta_{t+1}) \right)^{-\gamma} (\alpha \theta_{t+1} + \mathcal{Q}(\hat{b}_{t+1}, \theta_{t+1})) \right] \end{aligned}$$

5. Policy functions can be updated using the combinations of  $\hat{b}_t$  and  $g_{t+1}, \hat{c}_t^h, \hat{q}_t, \hat{b}_{t+1}$ .
6. We keep iterating until policy functions in two consecutive iterations are close enough.

To solve policy functions for two social planners, we need to solve additional policy functions of Lagrangian multipliers, such as  $\mu(\hat{b}_t, \theta_t)$  and  $\nu(\hat{b}_t, \theta_t)$ .